Chapter 11

Modulation

RADIO SIGNALS AND COMPLEX NOTATION

Excellent Tutorial source:
www.fourier-series.com
(time and frequency domains)

Simple model of a radio signal

- A transmitted radio signal can be written

$$s(t) = A\cos(2\pi f t + \phi)$$

Amplitude (A) Frequency (f) Phase (ϕ)

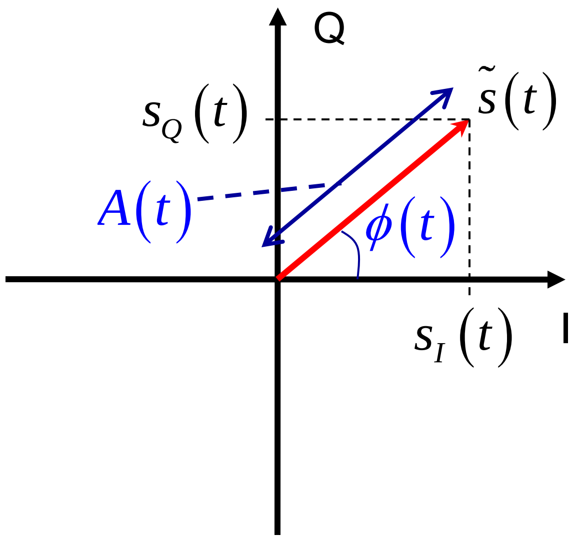
- By letting the transmitted information change the amplitude, the frequency, or the phase, we get the three basic types of digital modulation techniques.
  - **ASK** (Amplitude Shift Keying)
  - **FSK** (Frequency Shift Keying)
  - **PSK** (Phase Shift Keying)

FSK and PSK: Constant amplitude

$M = 2^K$ K bits mapped into one symbol BPS = (K)(symbol rate) for multilevel modulation

Interpreting the complex notation

Complex envelope (phasor)



Polar coordinates:

$$\tilde{s}(t) = s_I(t) + j s_Q(t) = A(t) e^{j\phi(t)}$$

Euler's Formula: $e^{ix} = \cos x + i \sin x$

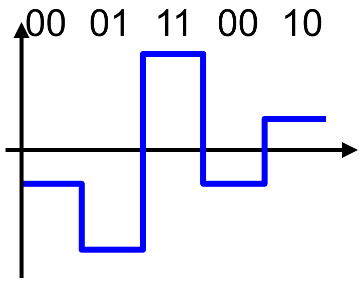
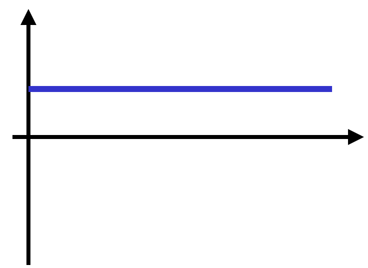
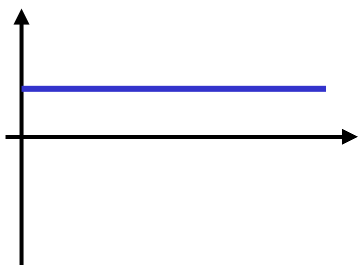
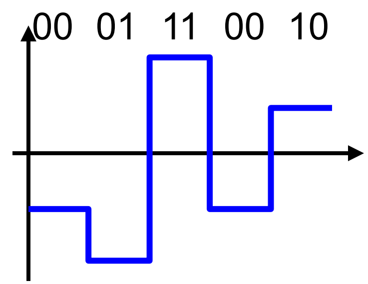
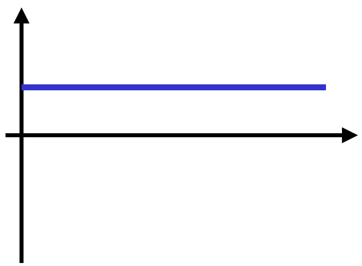
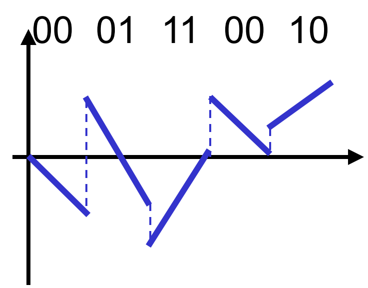
Transmitted radio signal

$$\begin{aligned} s(t) &= \mathrm{Re}\left\{\tilde{s}(t) e^{j2\pi f_c t}\right\} \\ &= \mathrm{Re}\left\{A(t) e^{j\phi(t)} e^{j2\pi f_c t}\right\} \\ &= \mathrm{Re}\left\{A(t) e^{j(2\pi f_c t + \phi(t))}\right\} \\ &= A(t)\cos\left(2\pi f_c t + \phi(t)\right) \end{aligned}$$

By manipulating the amplitude $A(t)$ and the phase $\phi(t)$ of the complex envelope (phasor), we can create any type of modulation/radio signal.

Example: Amplitude, phase and frequency modulation

$$s(t) = A(t)\cos\left(2\pi f_c t + \phi(t)\right)$$

	$A(t)$	$\phi(t)$	Comment:
4ASK			- Amplitude carries information - Phase constant (arbitrary)
4PSK			- Amplitude constant (arbitrary) - Phase carries information
4FSK			- Amplitude constant (arbitrary) - Phase slope (frequency) carries information

MODULATION BASICS

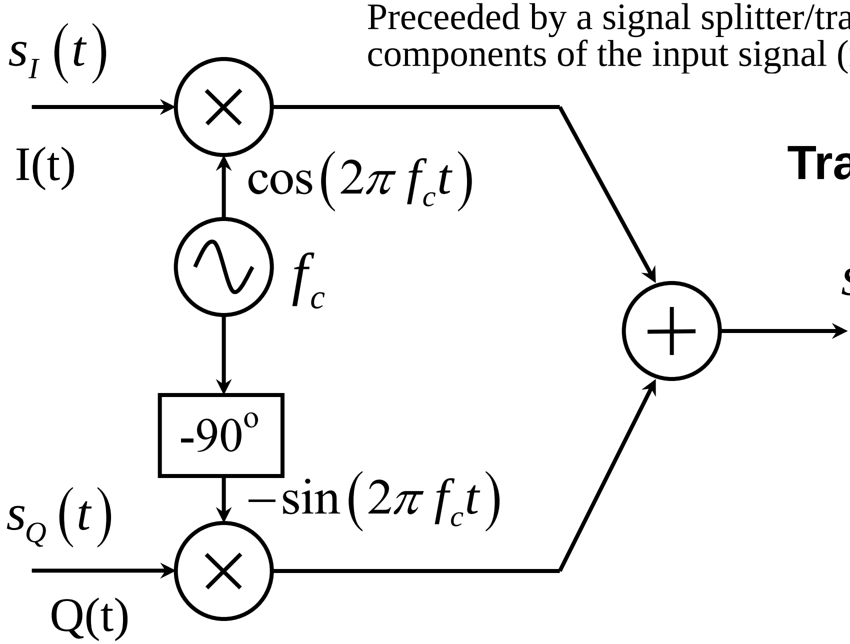
The IQ modulator

One modulation technique that lends itself to digital processes is called "IQ Modulation". In its various forms, IQ modulation is an efficient way to transfer information and it works well with digital formats. An IQ modulator can create AM, FM and PM.

Preceeded by a signal splitter/transform that produces the sin & cosine components of the input signal (not shown)

I-channel
(in-phase)

Q-channel
(quadrature)



Transmited radio signal

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

Take a step into the complex domain:

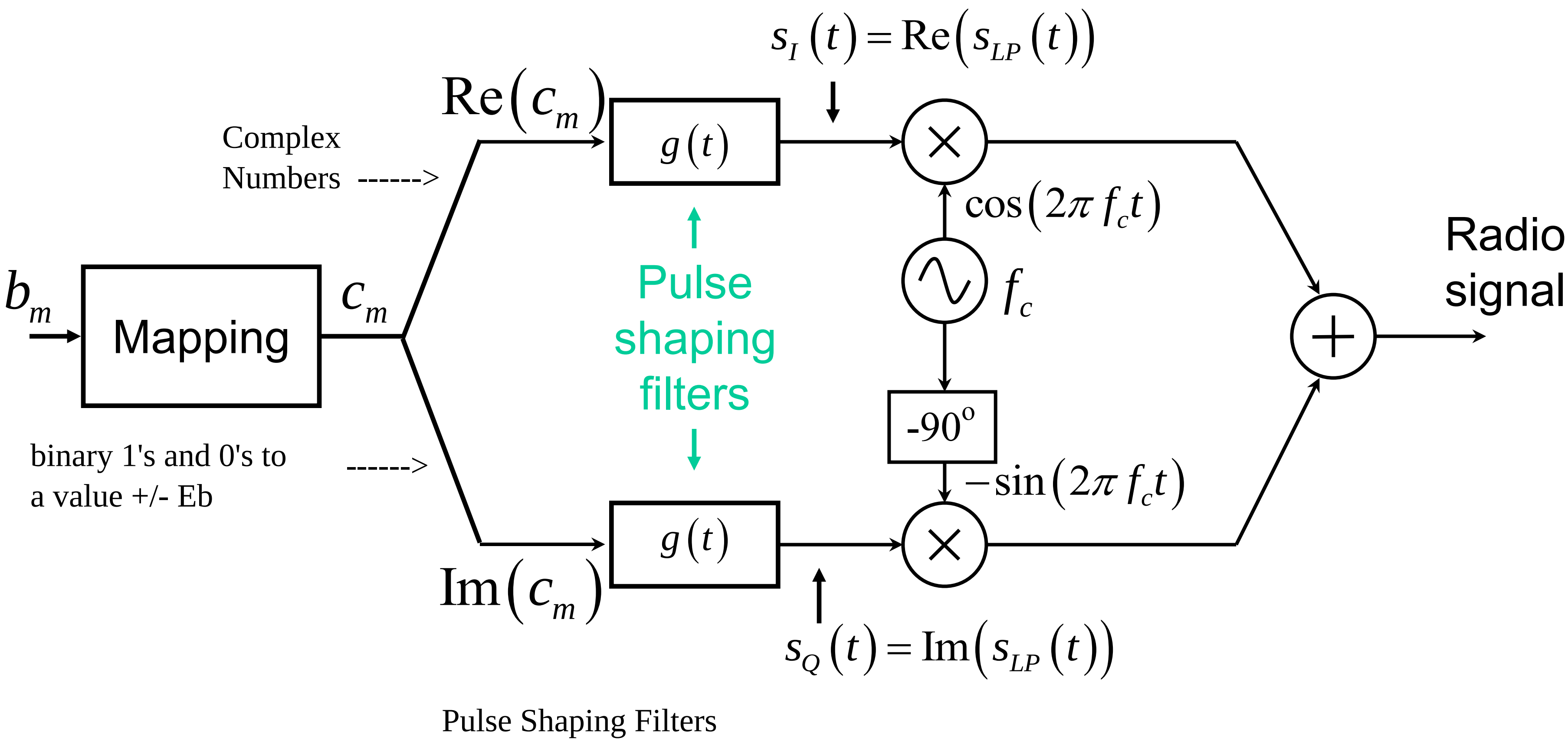
Complex envelope $\tilde{s}(t) = s_I(t) + j s_Q(t)$

Carrier fc $e^{j2\pi f_c t}$

$$\Rightarrow \quad s(t) = \mathrm{Re}\left\{\tilde{s}(t)e^{j2\pi f_c t}\right\}$$

Pulse amplitude modulation (PAM) Interpretation as IQ-modulator

For real valued basis functions $g(t)$ we can view PAM as:



(Both the rectangular and the root- / raised-cosine pulses are real valued.)

IMPORTANT MODULATION FORMATS

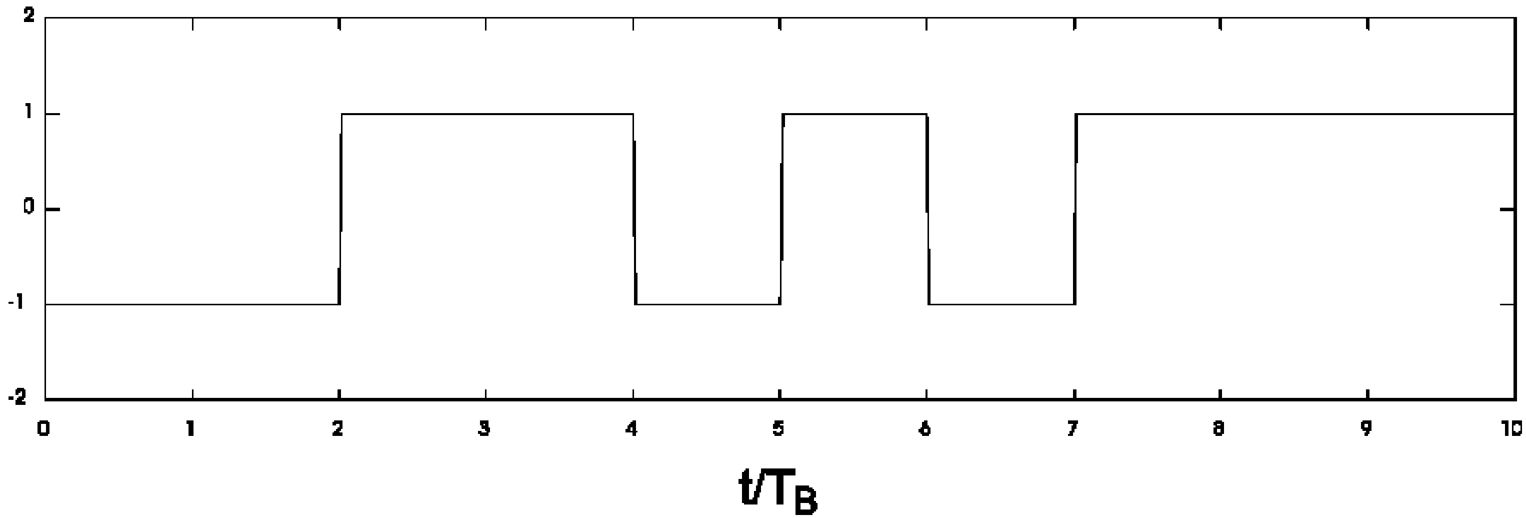
Spectral Efficiency - bits per symbol (high order modulation format)
Adjacent Channel Interference
Sensitivity wrt Noise (somewhat counter to spectral efficiency-low order modulation format)
Robustness wrt Delay and Doppler Dispersion (filtering adds delay)
Efficient Generation (Class C/E/F vs Class A/B)

a lot of RF transmission protocols will switch modulation formats depending on channel performance to optimize data rates based on dynamic channel conditions and the receiver's performance

Binary phase-shift keying (BPSK) Rectangular pulses

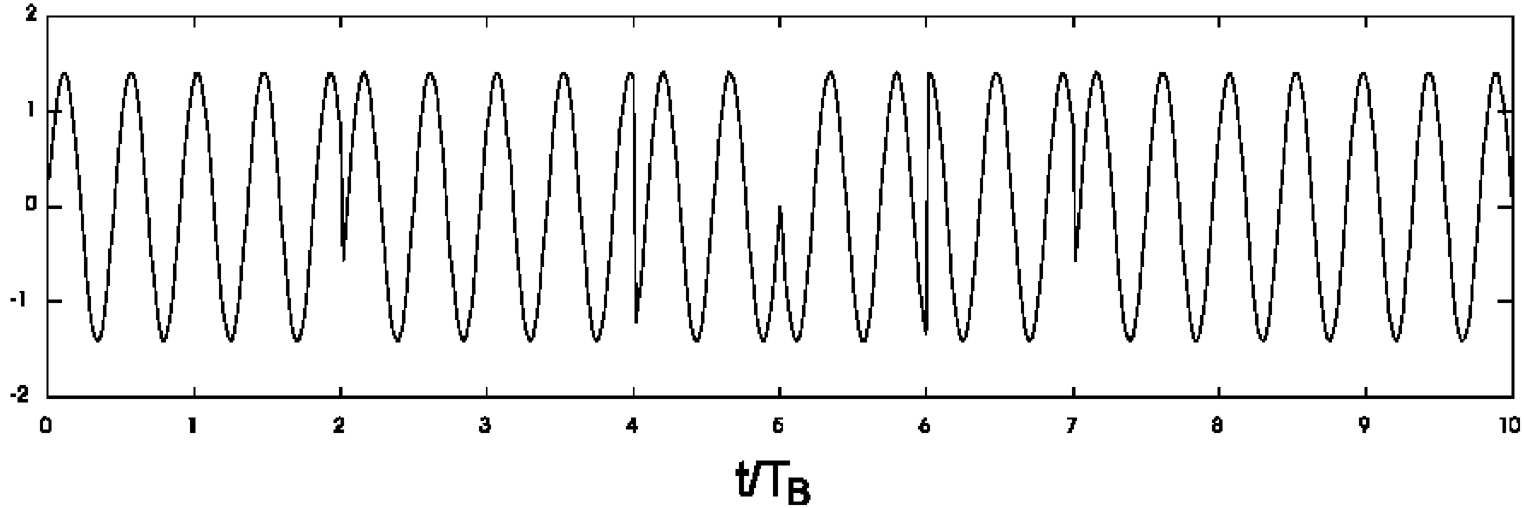
Base-band

(signal)



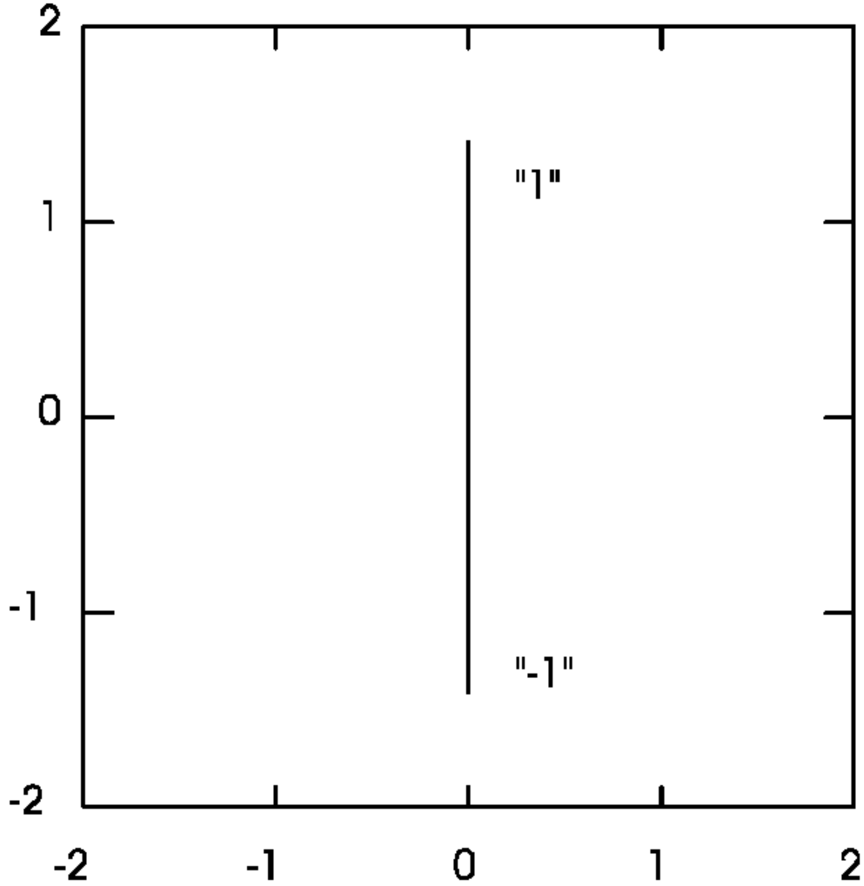
Radio signal

(showing carrier fc) phase modulation

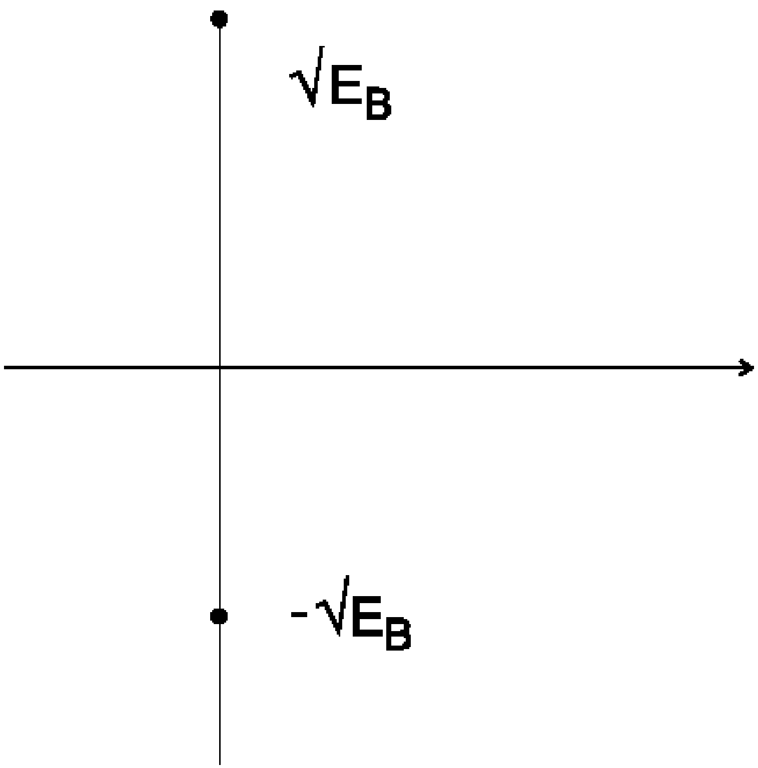


Binary phase-shift keying (BPSK)
Rectangular pulses

Complex representation

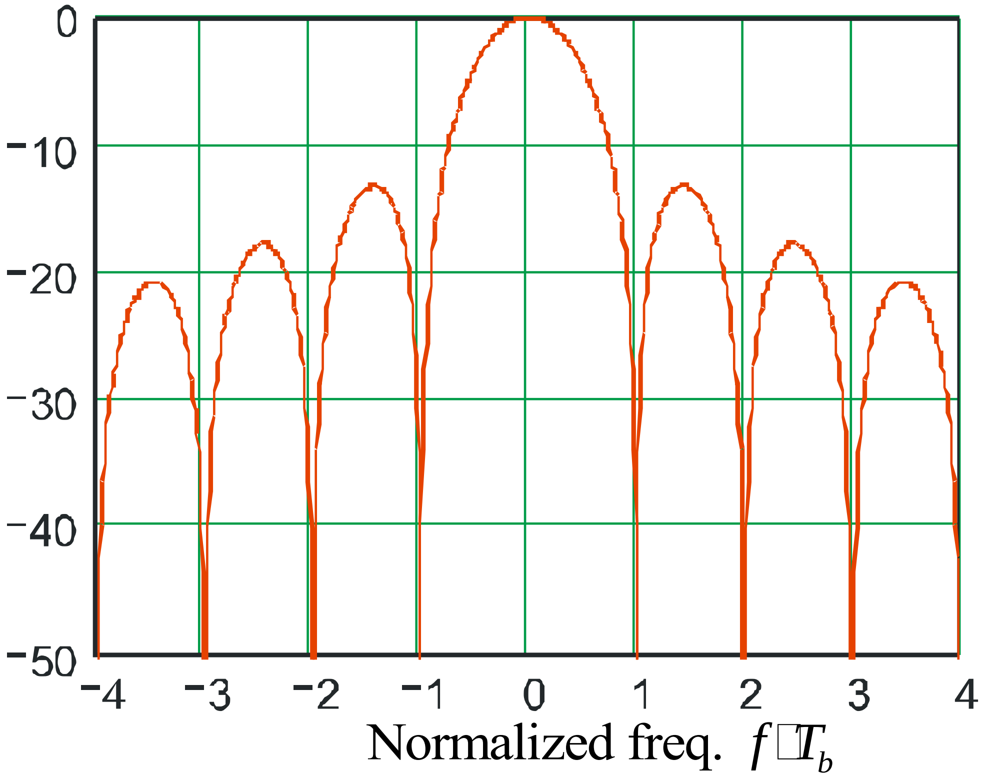


Signal space diagram



Binary phase-shift keying (BPSK)
Rectangular pulses

Power spectral density for BPSK

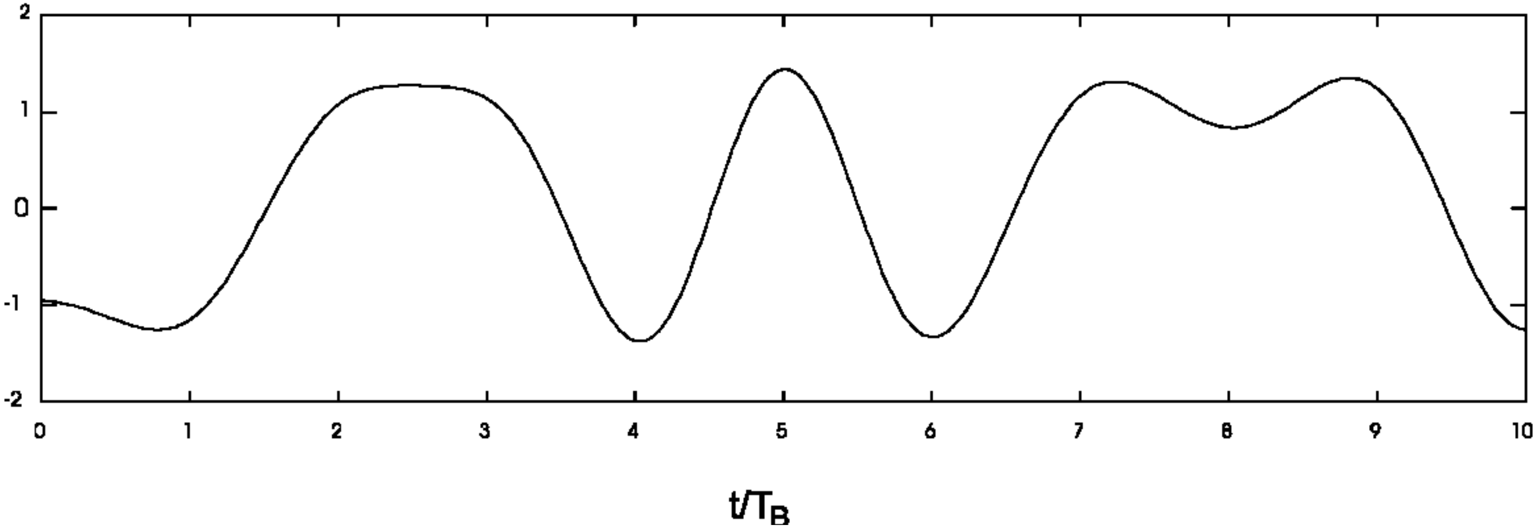


Contained percentage of total energy	spectral efficiency
90%	$0.59 Bit/s/Hz$
99%	$0.05 Bit/s/Hz$

Binary amplitude modulation (BAM) Raised-cosine pulses (roll-off 0.5)

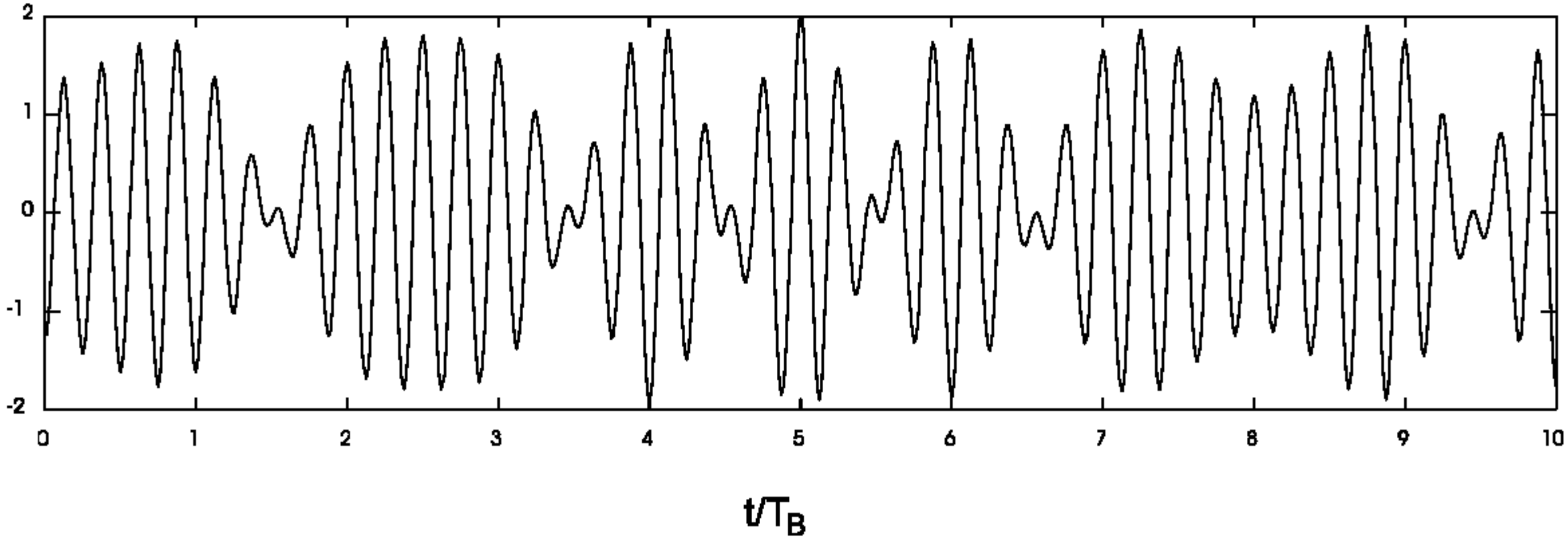
Base-band

Nyquist type pulse instead of square wave (binary) pulses



Radio signal

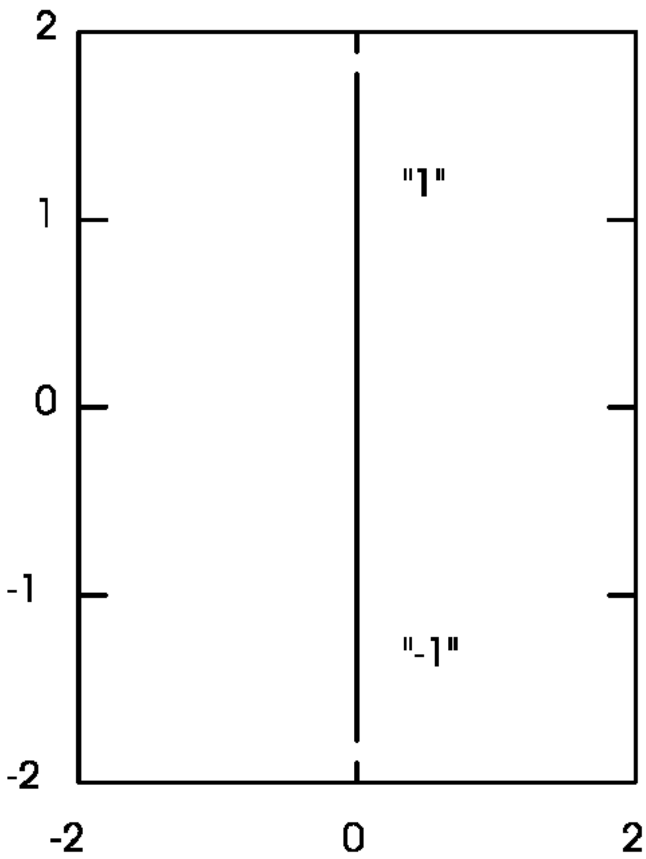
no longer has a constant amplitude



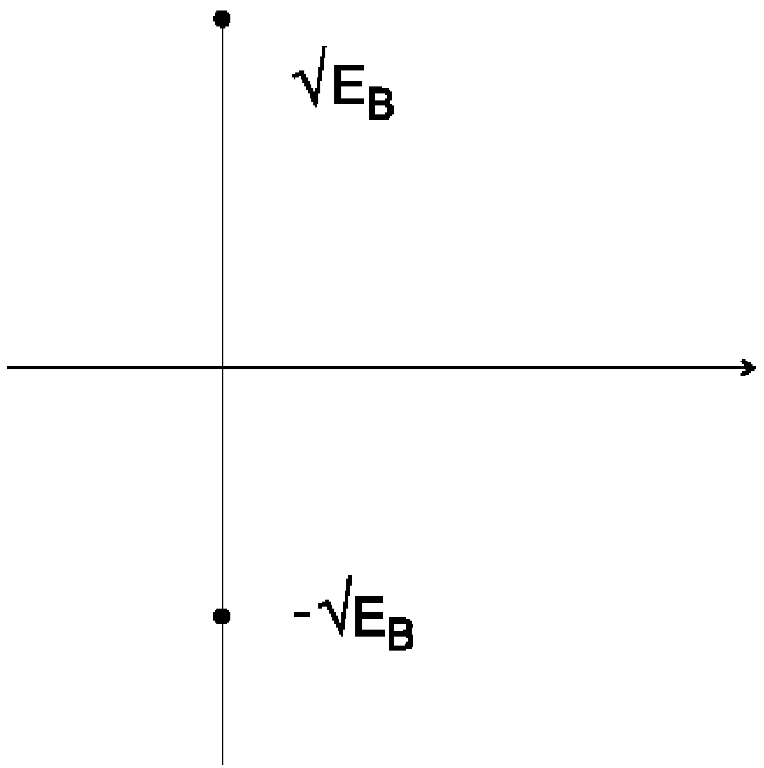
Binary amplitude modulation (BAM) Raised-cosine pulses (roll-off 0.5)

(sigals represented by vectors)

Complex representation

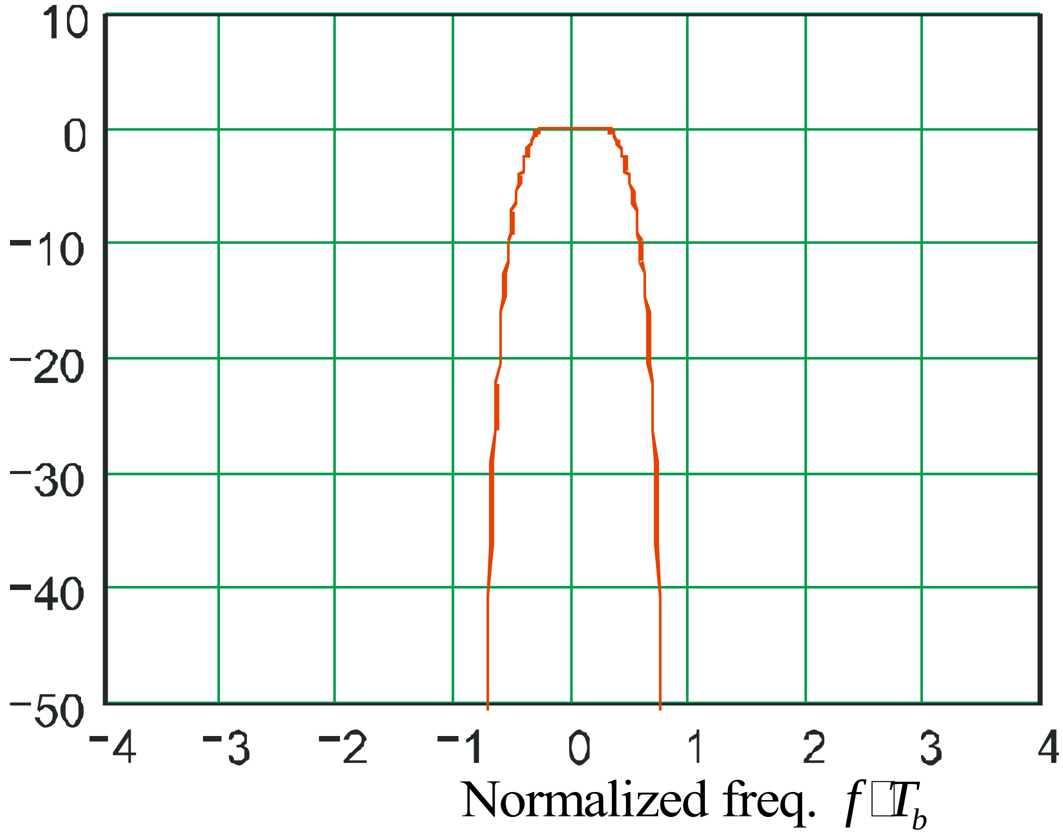


Signal space diagram



Binary amplitude modulation (BAM) Raised-cosine pulses (roll-off 0.5)

Power spectral density for BAM

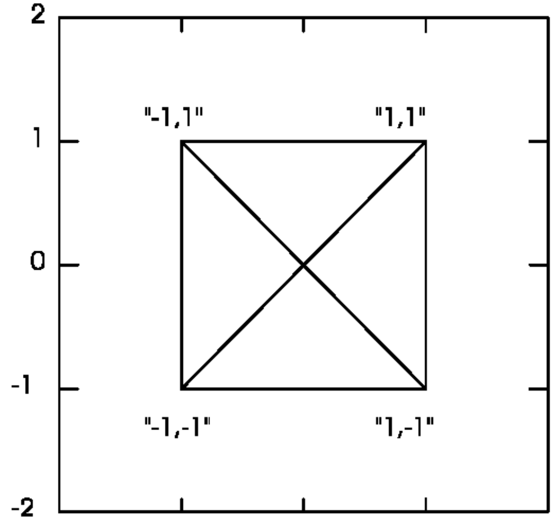


Contained percentage of total energy	spectral efficiency
90%	$1.02 Bit/s/Hz$
99%	$0.79 Bit/s/Hz$

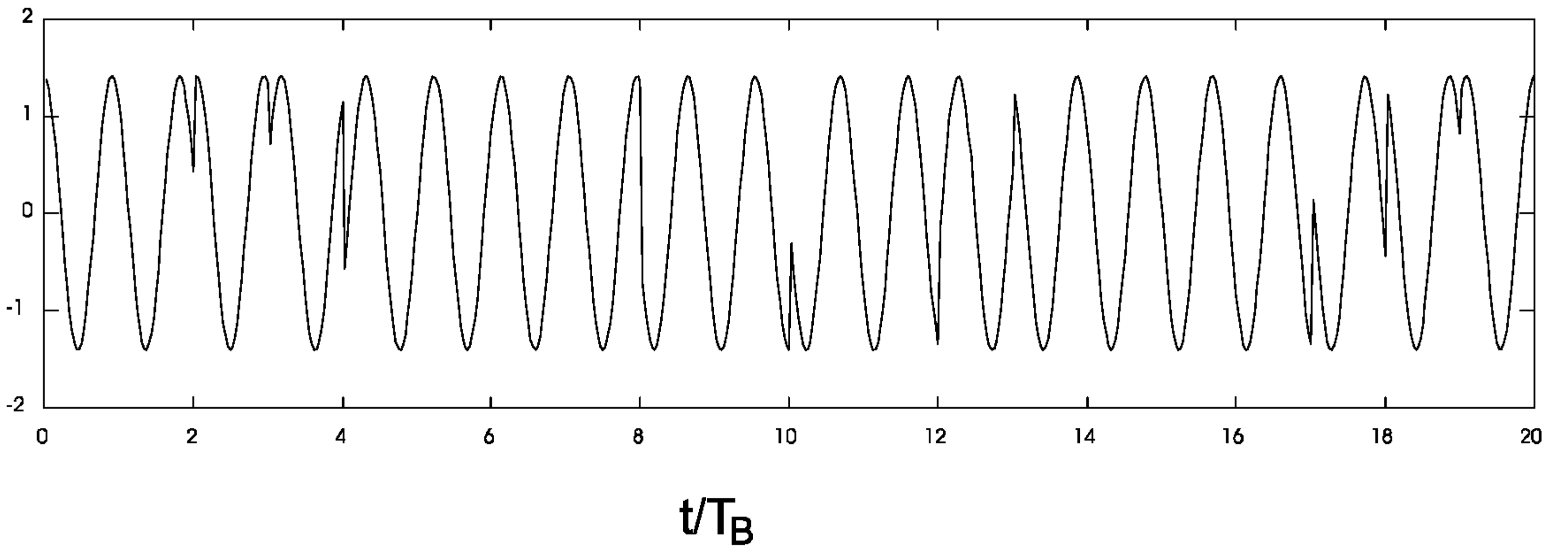
Quaternary PSK (QPSK or 4-PSK) Rectangular pulses

Data stream is split into two data streams where each stream has 1/2 the data rate of the original

Complex representation

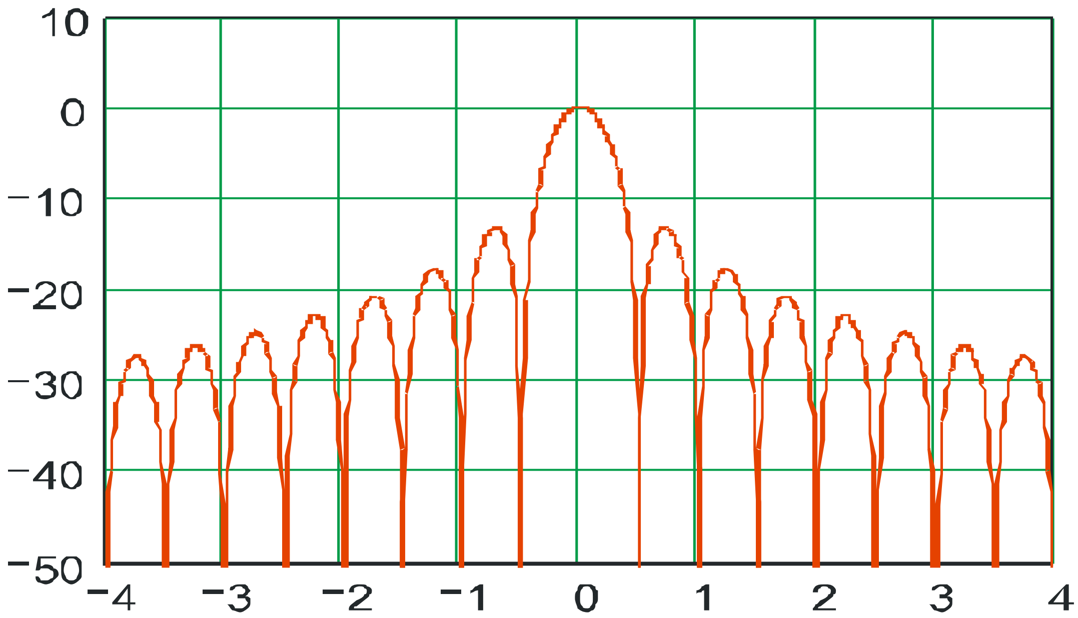


Radio signal



Quaternary PSK (QPSK or 4-PSK) Rectangular pulses

Power spectral density for QPSK

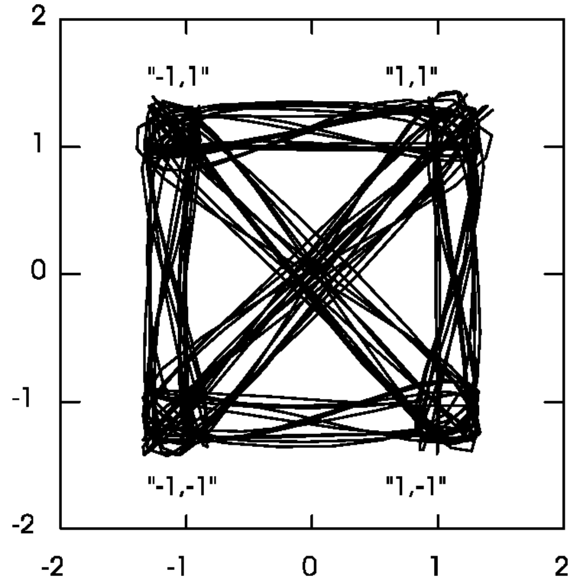


Contained percentage of total energy	spectral efficiency
90%	$1,18 Bit/s/Hz$
99%	$0.10 Bit/s/Hz$

(2X the efficiency of BPSK)

Quadrature ampl.-modulation (QAM) Root raised-cos pulses (roll-off 0.5)

Complex representation, an
I/Q diagram (eye pattern/diagram

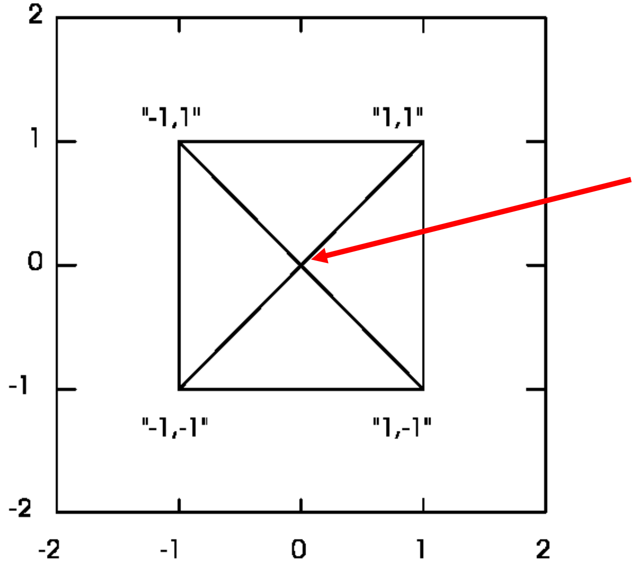


Contained percentage of total energy	spectral efficiency
90%	$2.04 Bit/s/Hz$
99%	$1.58 Bit/s/Hz$

Amplitude variations
The problem

Signals with high amplitude variations leads to less efficient amplifiers.

Complex representation of QPSK



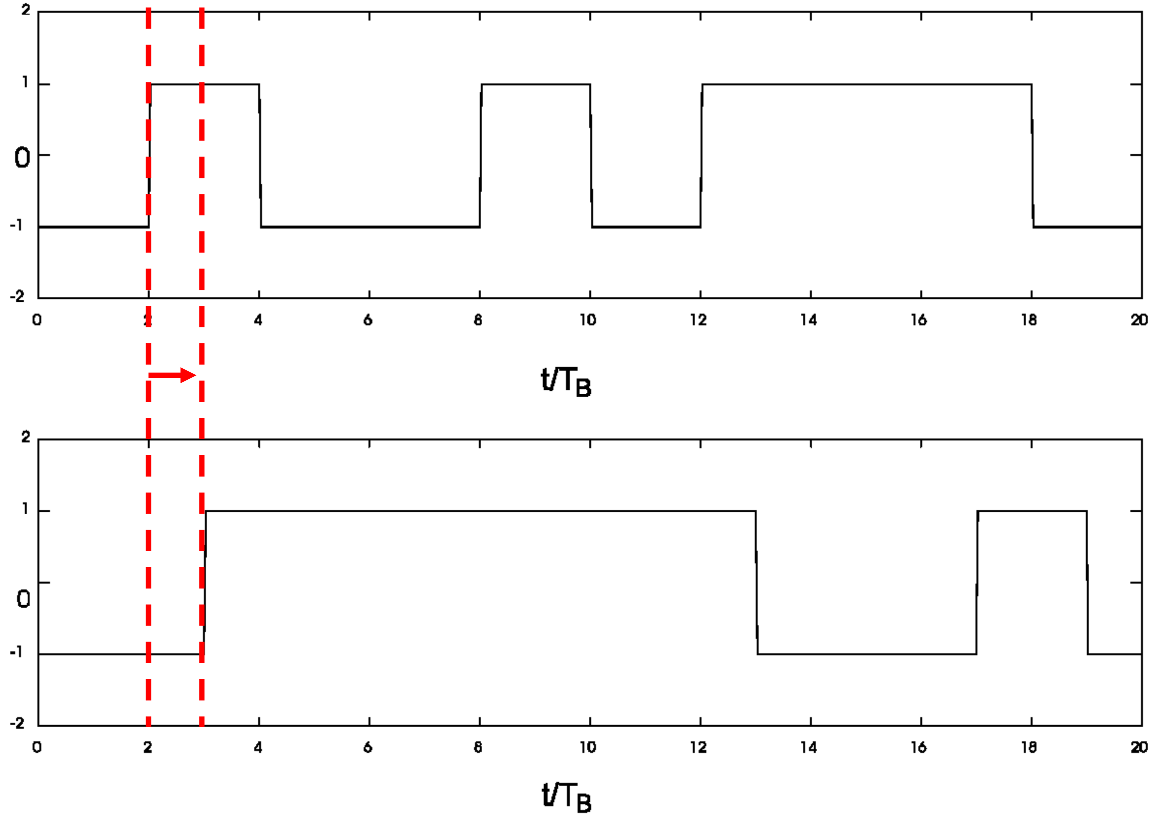
Diagonal State Transitions Possible
(one symbol time to the next a phase
change of as much 180 degrees is possible)

Offset QPSK (OQPSK) Rectangular pulses

In-phase signal

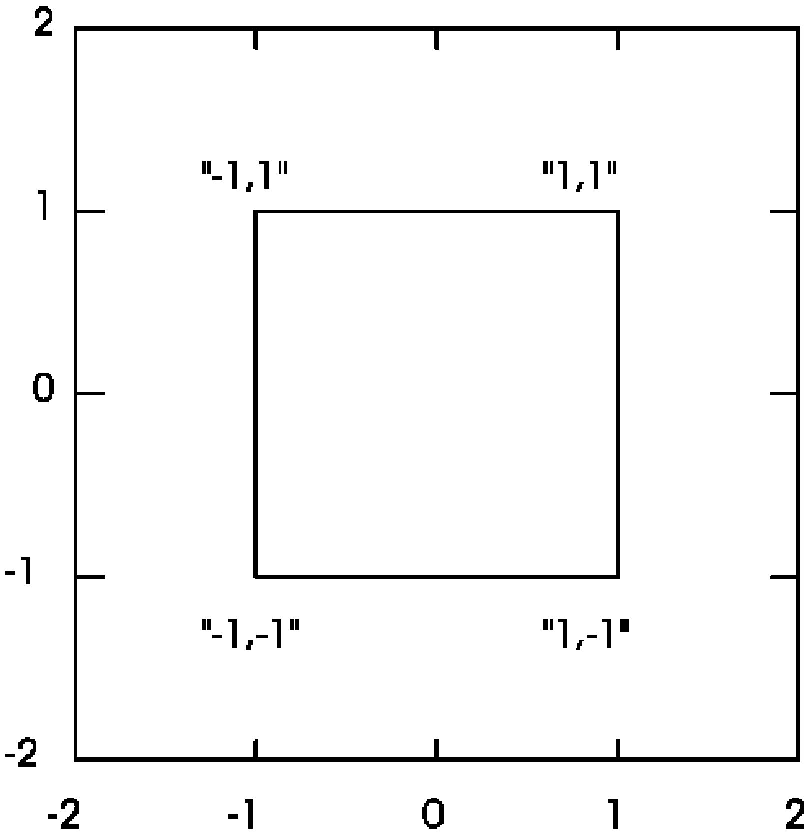
Quadrature signal

Delay Q signal by one bit time



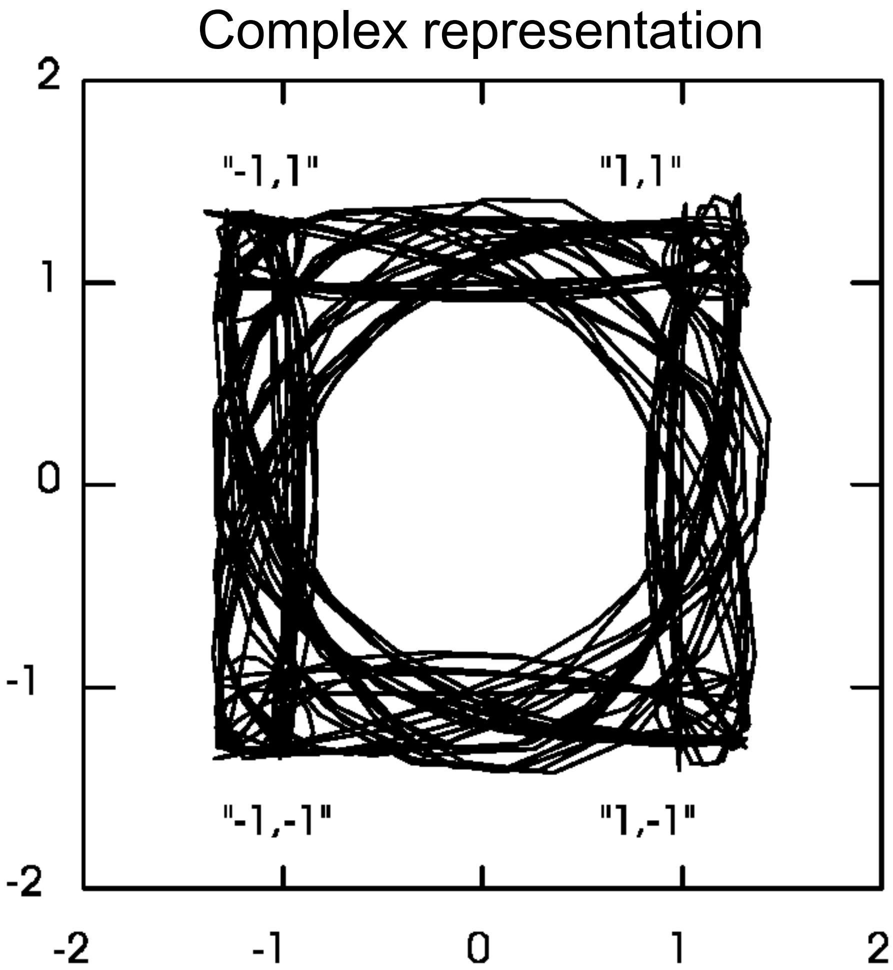
Offset QPSK
Rectangular pulses

Complex representation



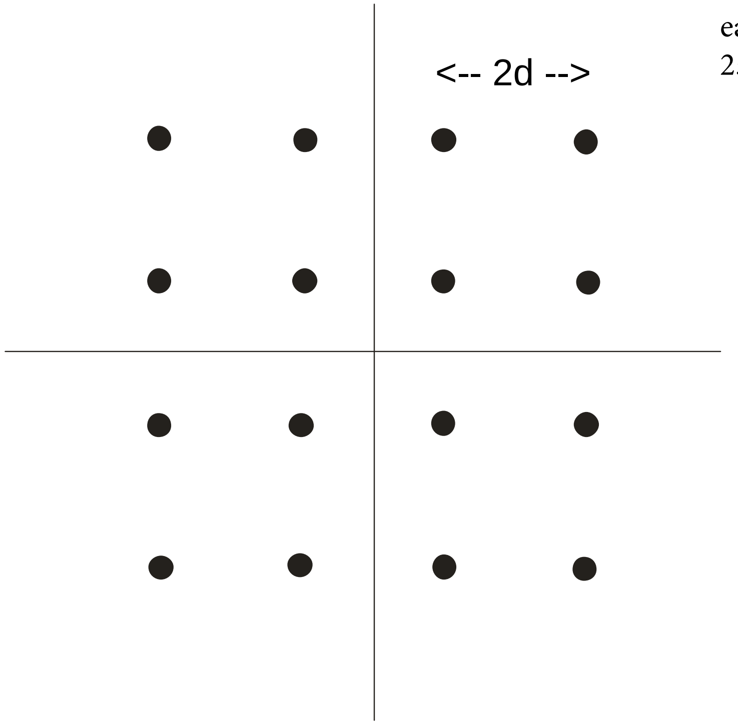
Only one of the two bits I(t) and Q(t) can change sign at anytime and thus the phase change in the combined signal s(t) never exceeds 90 degrees (easier on the transmitter while also limiting spreading/adjacent channel interference because of smaller phase changes)

Offset QAM (OQAM)
Raised-cosine pulses



Higher-order modulation

16-QAM signal space diagram

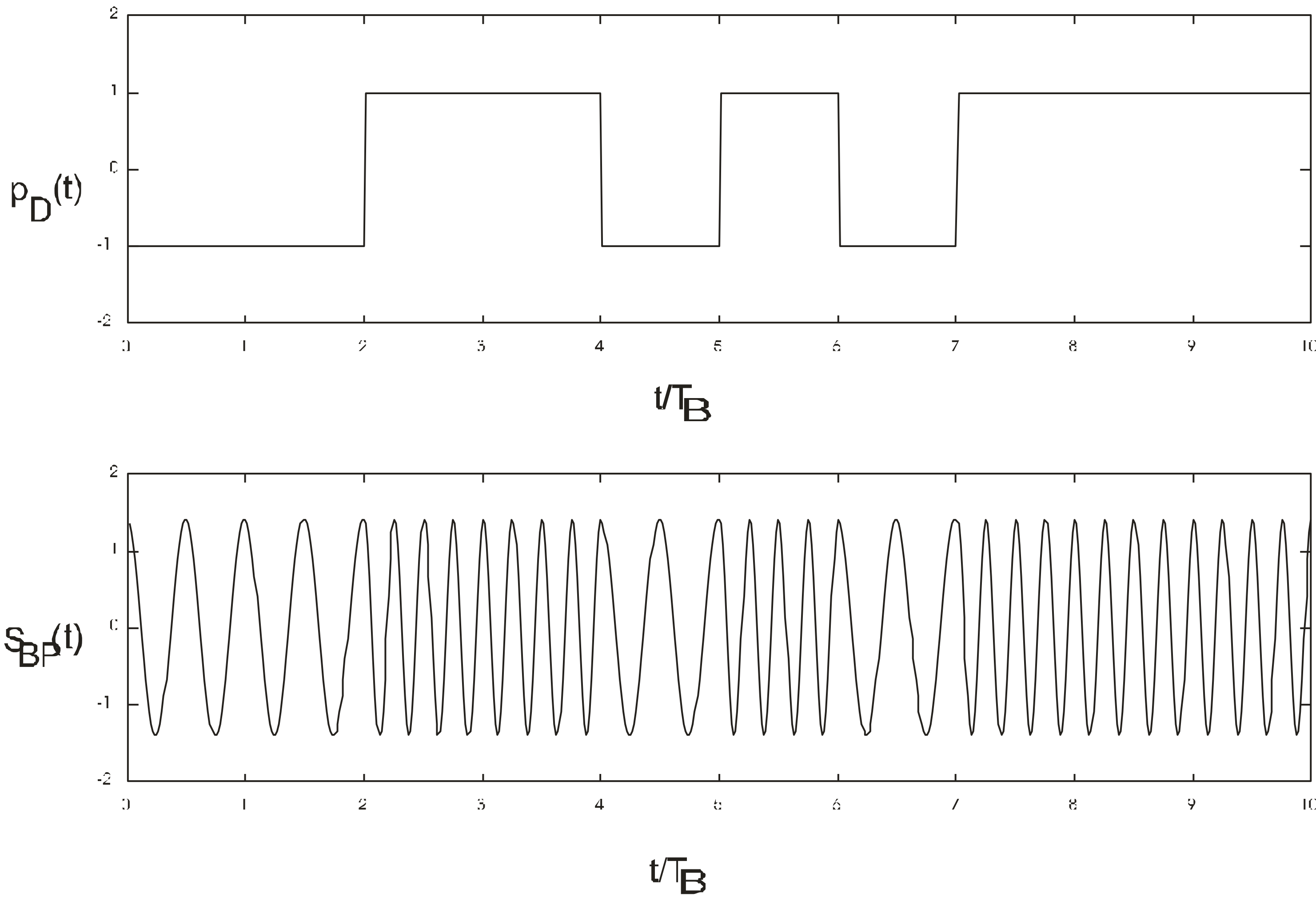


Transmits multiple bits in both the in-phase and the quadrature-phase component - a signal with positive or negative polarity as well as multiple amplitude levels on each component. Further advances with 64-QAM and 256-QAM

Binary frequency-shift keying (BFSK) Rectangular pulses

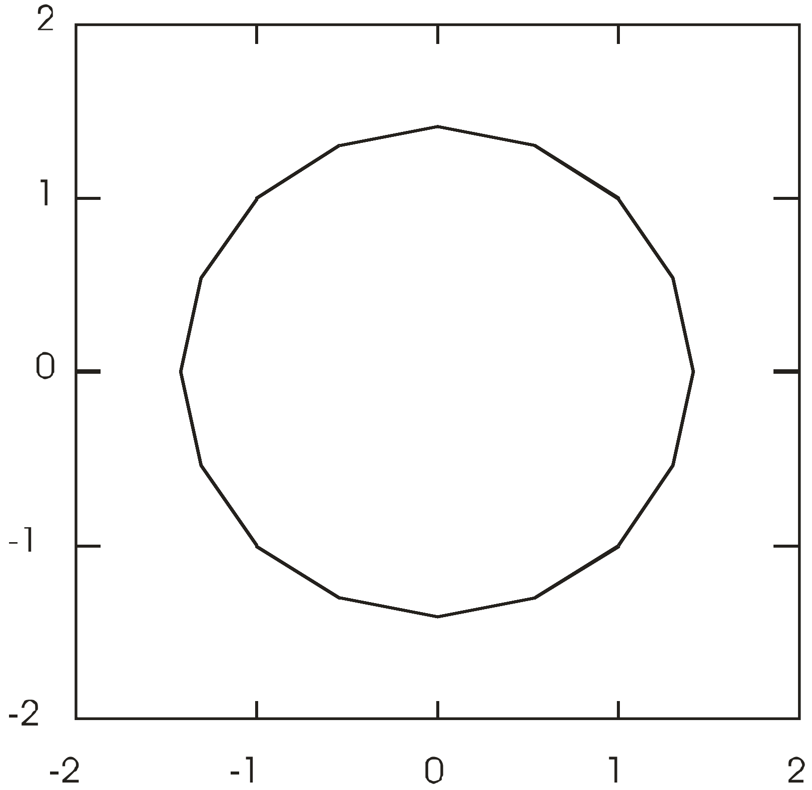
Base-band

Radio signal

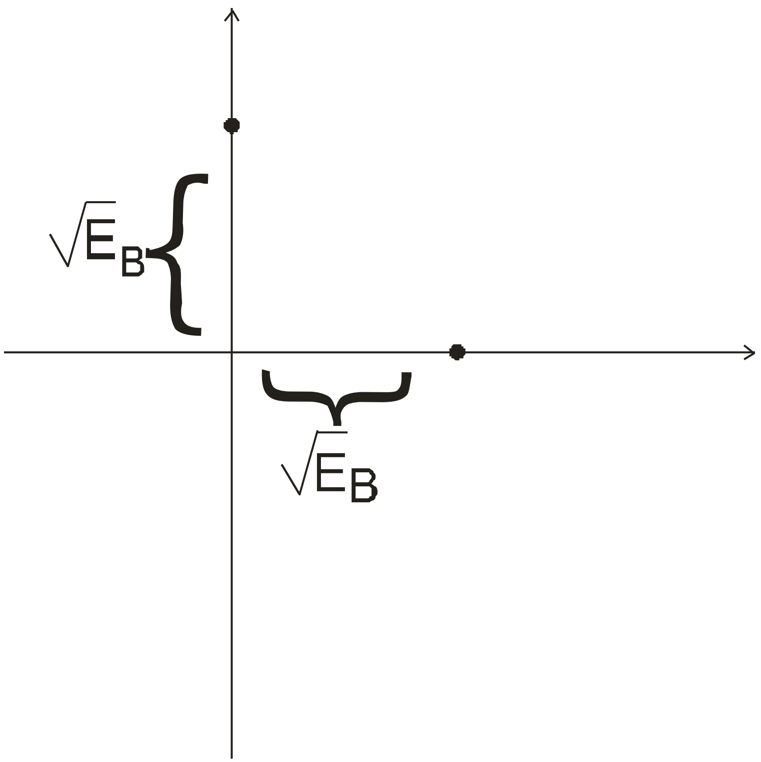


Binary frequency-shift keying (BFSK) Rectangular pulses

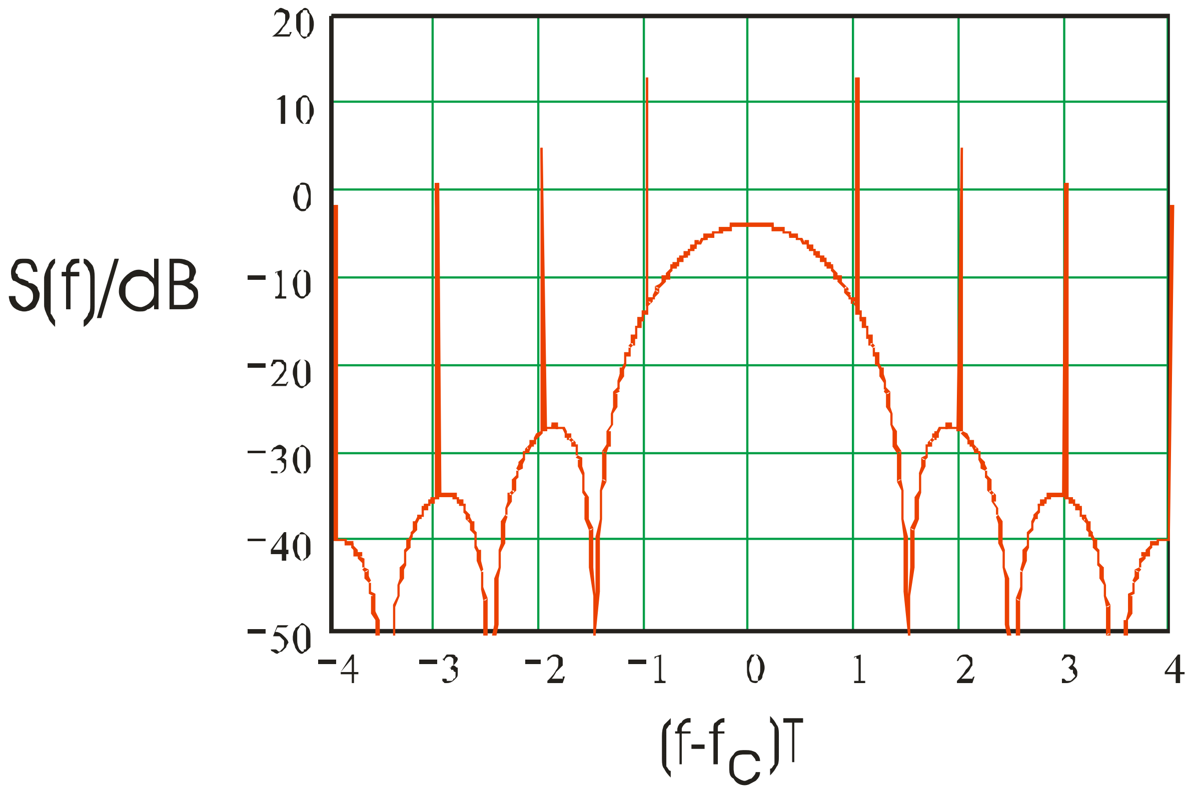
Complex representation



Signal space diagram



Binary frequency-shift keying (BFSK) Rectangular pulses



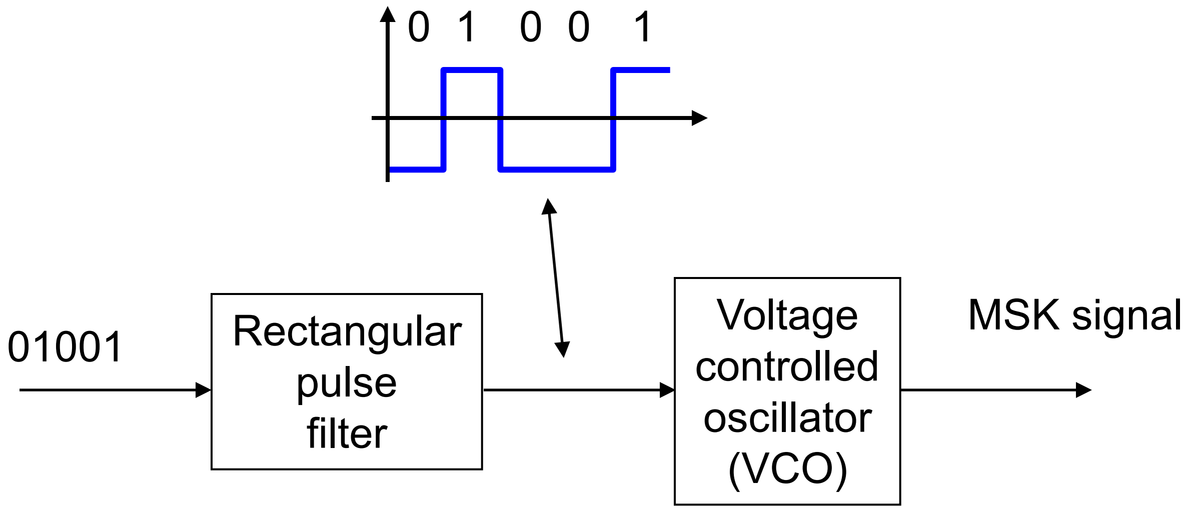
spikes occur at bit transitions resulting in undesirable spectral properties

Contained percentage of total energy	spectral efficiency
90%	$0.59 Bit/s/Hz$
99%	$0.05 Bit/s/Hz$

Minimum shift keying (MSK)

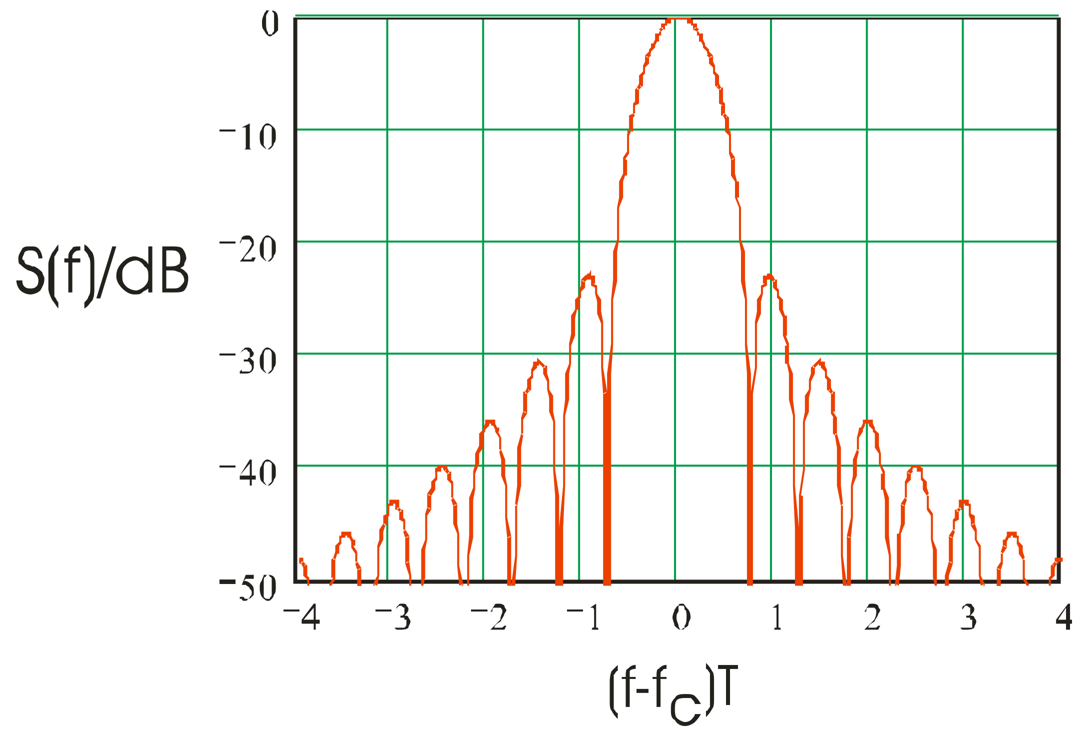
Important in wireless communications
best viewed as offset QAM
or OQAM

Simple MSK implementation



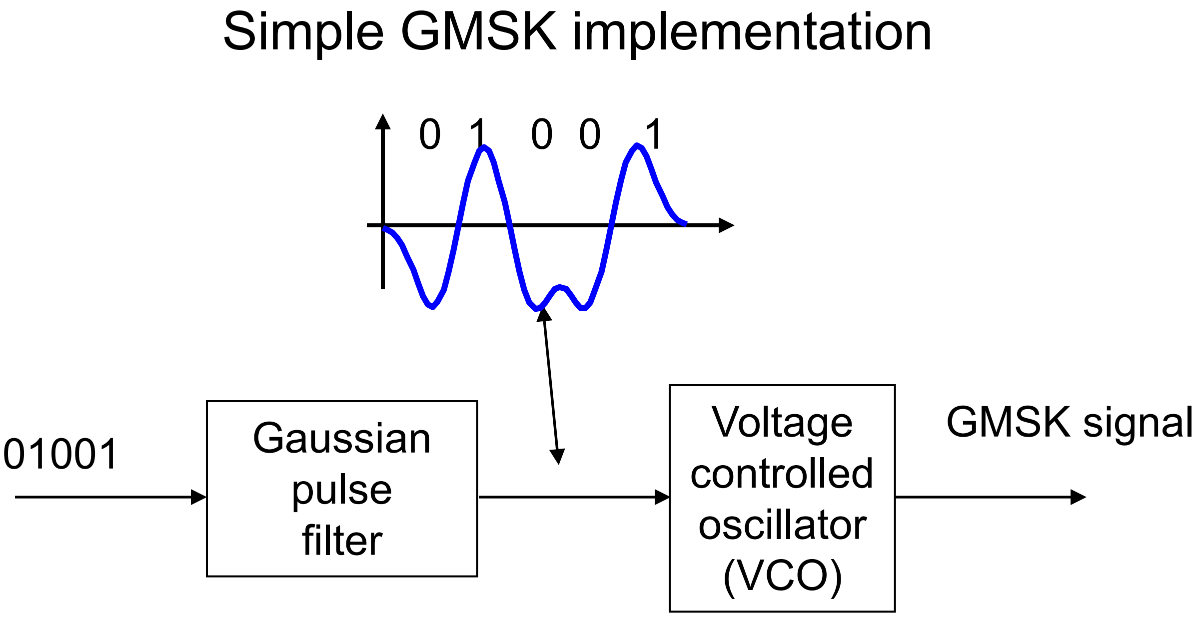
Minimum shift keying (MSK)

Power spectral density of MSK



Contained percentage of total energy	spectral efficiency
90 %	1,29 Bit / s / Hz
99 %	0,85 Bit / s / Hz

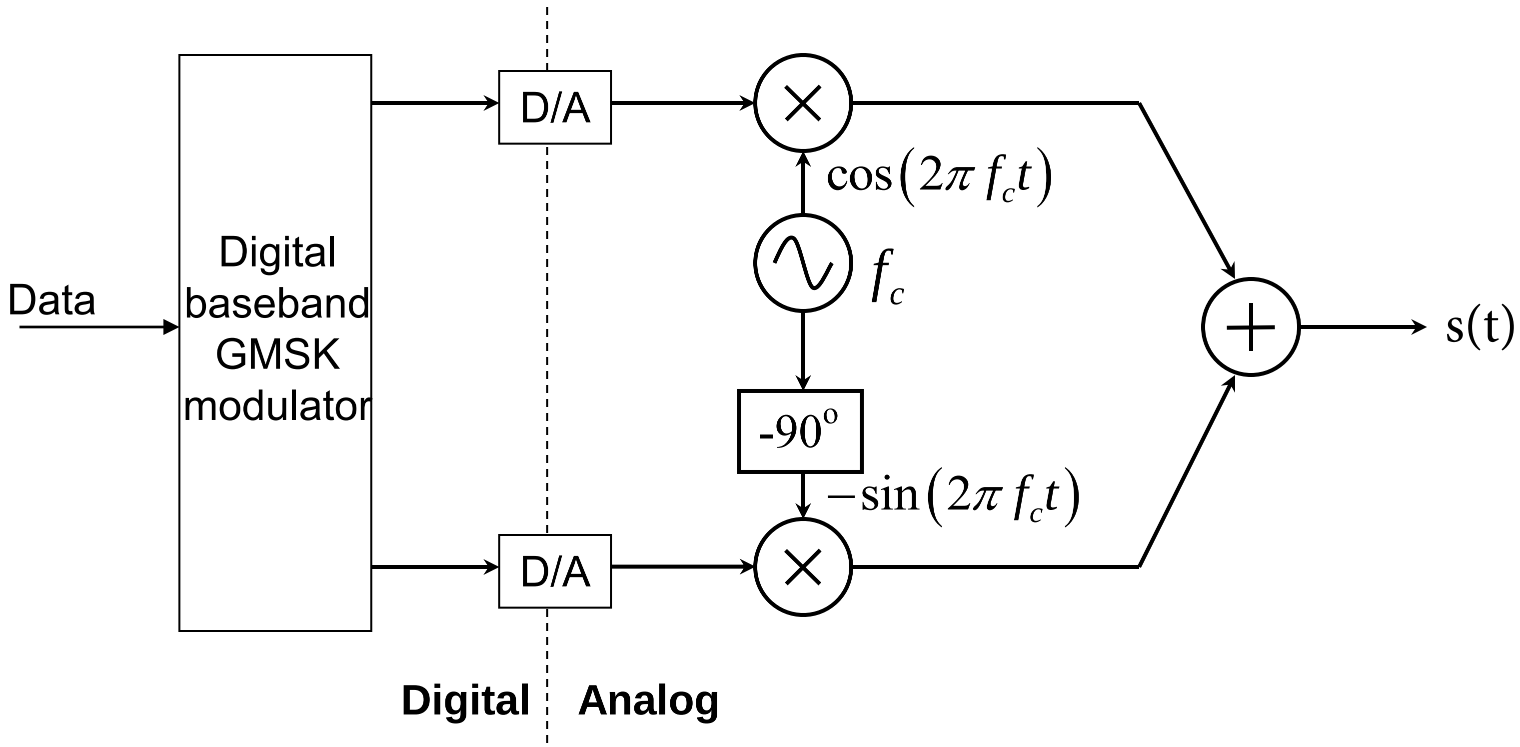
Gaussian filtered MSK (GMSK)



GMSK is used in Bluetooth and cellular GSM
(Global System for Mobile communications)

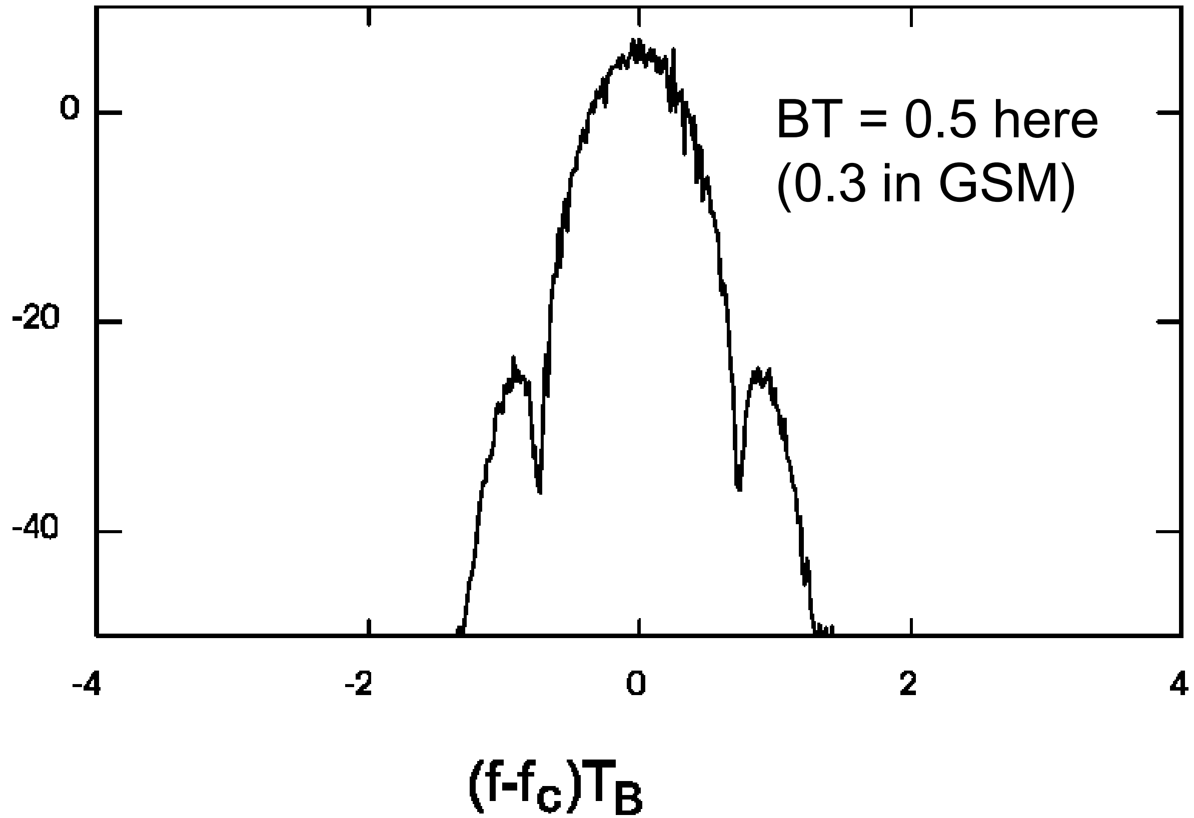
Gaussian filtered MSK (GMSK)

Digital GMSK implementation



Gaussian filtered MSK (GMSK)

Power spectral density of GMSK.



Contained percentage of total energy	spectral efficiency
90 %	1,45 Bit / s / Hz
99 %	0,97 Bit / s / Hz

How do we use all these spectral efficiencies?

Example: Assume that we want to use MSK to transmit 50 kbit/sec, and want to know the required transmission bandwidth.

Take a look at the spectral efficiency table:

Contained percentage of total energy	spectral efficiency
90 %	1,29 Bit / s / Hz
99 %	0,85 Bit / s / Hz

The 90% and 99% bandwidths become:

$$B_{90\%} = 50000 / 1.29 = 38.8 \text{ kHz}$$

$$B_{99\%} = 50000 / 0.85 = 58.8 \text{ kHz}$$

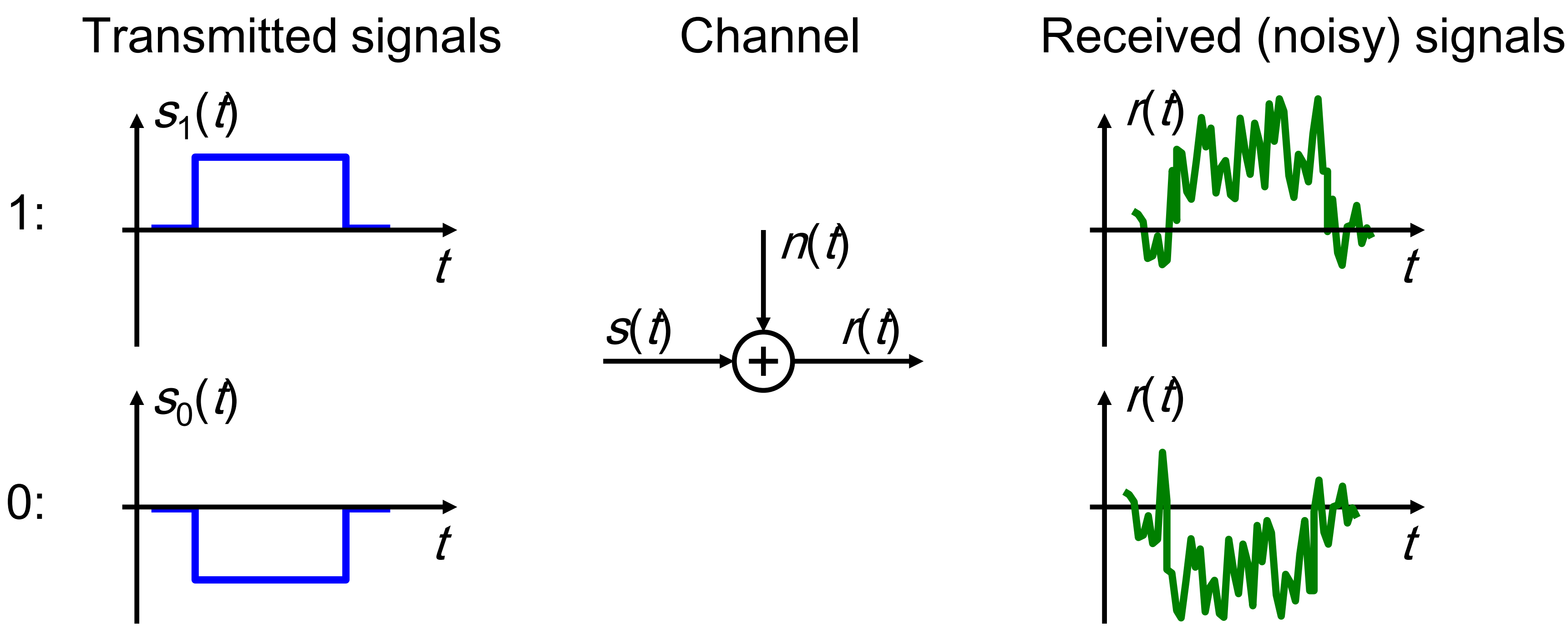
Summary

Modulation method	spectral efficiency for 90 % of total energy Bit / s / Hz	spectral efficiency for 99 % of total energy Bit / s / Hz	envelope variations w ratio of maximum and minimum amplitude
BPSK	0,59	0,05	1
BAM (α=0.5)	1,02	0,79	∞
QPSK, OQPSK, $\pi/4$-QPSK	1,18	0,10	1
MSK	1,29	0,85	1
GMSK (B_G T= 0.5)	1,45	0,97	1
QAM ($\alpha = 0.5$)	2,04	1,58	∞
OQAM ($\alpha = 0.5$)	2,04	1,58	2.6
FSK		$< 1/(2 f_D T_B)$	1

Demodulation and BER computation

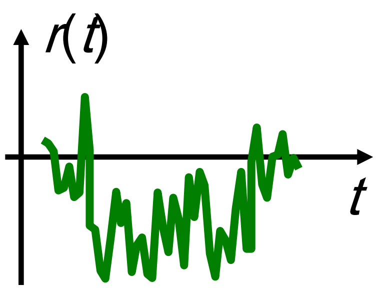
OPTIMAL RECEIVER AND BIT ERROR PROBABILITY IN AWGN CHANNELS

Optimal receiver
Transmitted and received signal



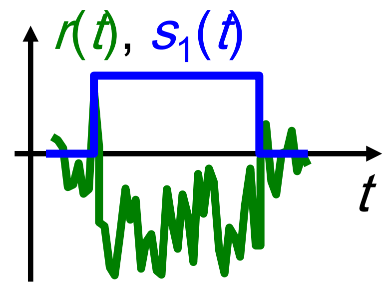
Optimal receiver
A first "intuitive" approach

Assume that the following signal is received:

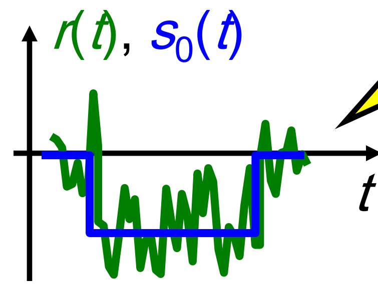


Comparing it to the two possible **noise free** received signals:

1:



0:



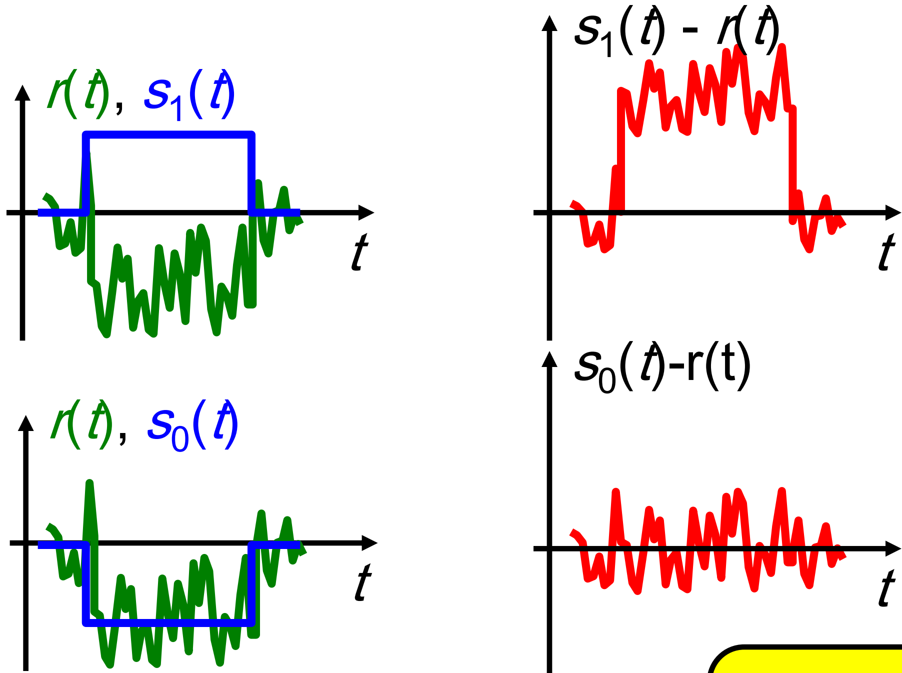
This seems to be the best "fit". We assume that "0" was the transmitted bit.

Optimal receiver
Let's make it more measurable

To be able to better measure the "fit" we look at the **energy** of the **residual** (difference) between received and the possible noise free signals:

1:

0:



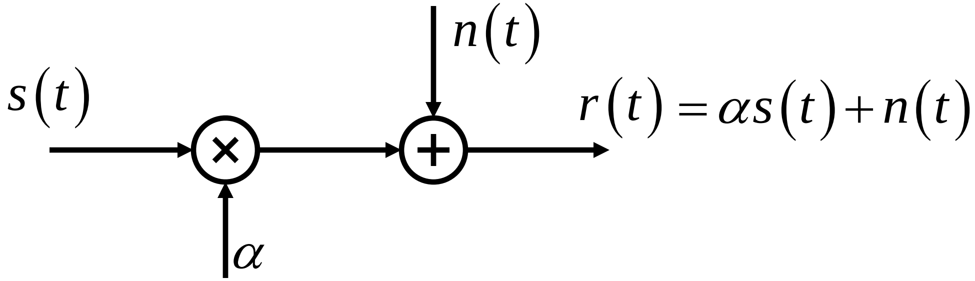
$$\int \left| s_1(t) - r(t) \right|^2 dt$$

$$\int \left| s_0(t) - r(t) \right|^2 dt$$

This residual energy is much smaller. We assume that "0" was transmitted.

Optimal receiver
The AWGN channel

The additive white Gaussian noise (AWGN) channel



$s(t)$ - transmitted signal

α - channel attenuation

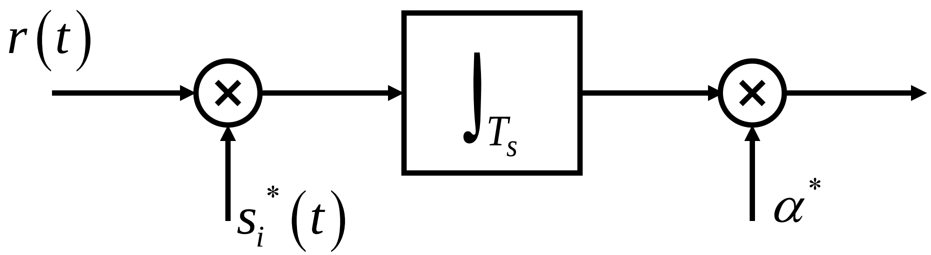
$n(t)$ - white Gaussian noise

$r(t)$ - received signal

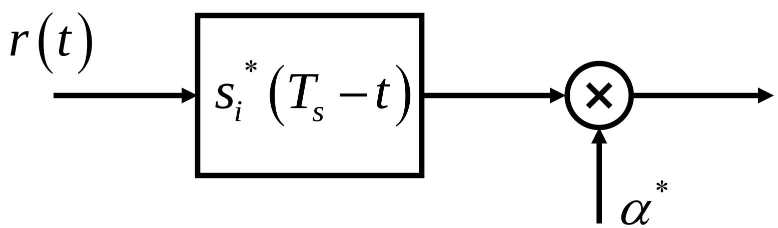
In our digital transmission system, the transmitted signal $s(t)$ would be one of, let's say M, different alternatives $s_0(t)$, $s_1(t)$, ... , $s_{M-1}(t)$.

Optimal receiver
The AWGN channel, cont.

The central part of the comparison of different signal alternatives is a correlation, that can be implemented as a correlator:



or a matched filter (matched to the possible transmit waveforms)



The real part of the output from either of these is sampled at $t = T_s$

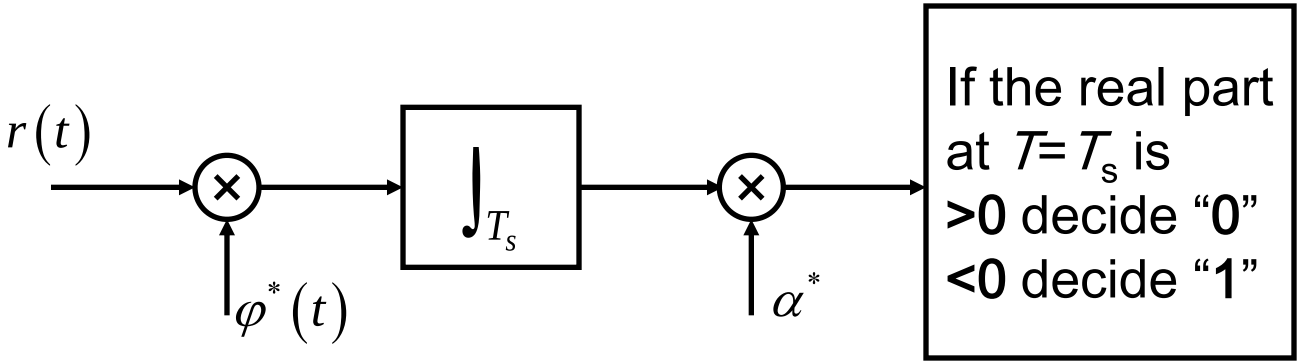
where T_s is the symbol time (duration).

Optimal receiver
Antipodal signals

In antipodal signaling (BPSK), the alternatives (for "0" and "1") are

$$s_0(t) = \varphi(t)$$
$$s_1(t) = -\varphi(t)$$

This means that we only need ONE correlation in the receiver for simplicity:

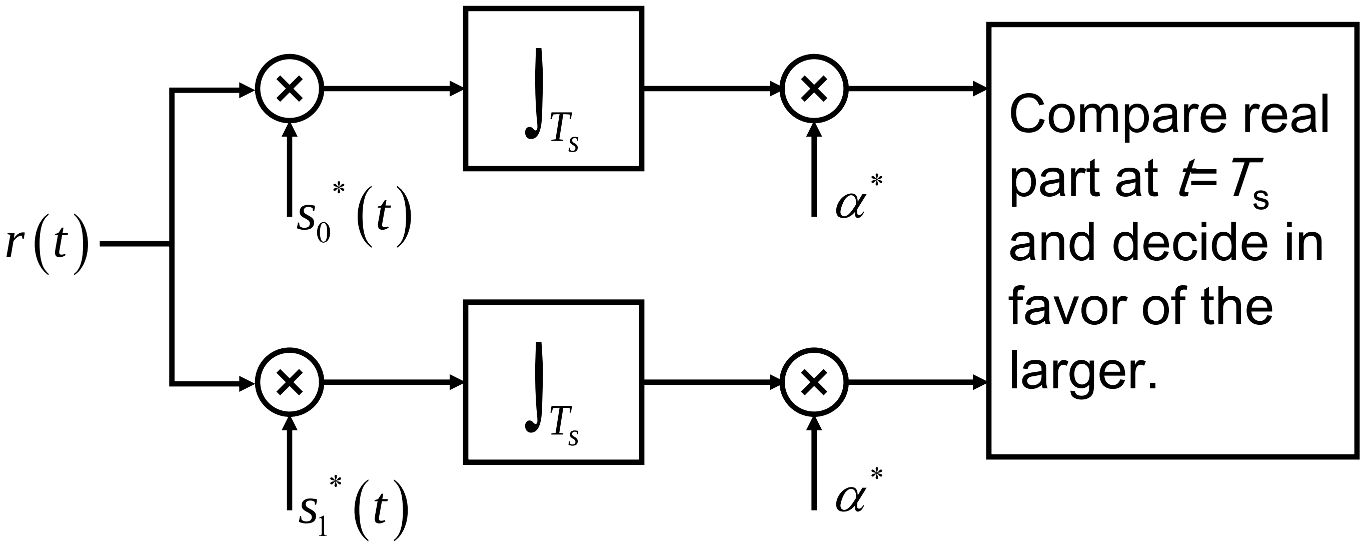


Optimal receiver
Orthogonal signals

(BFSK)

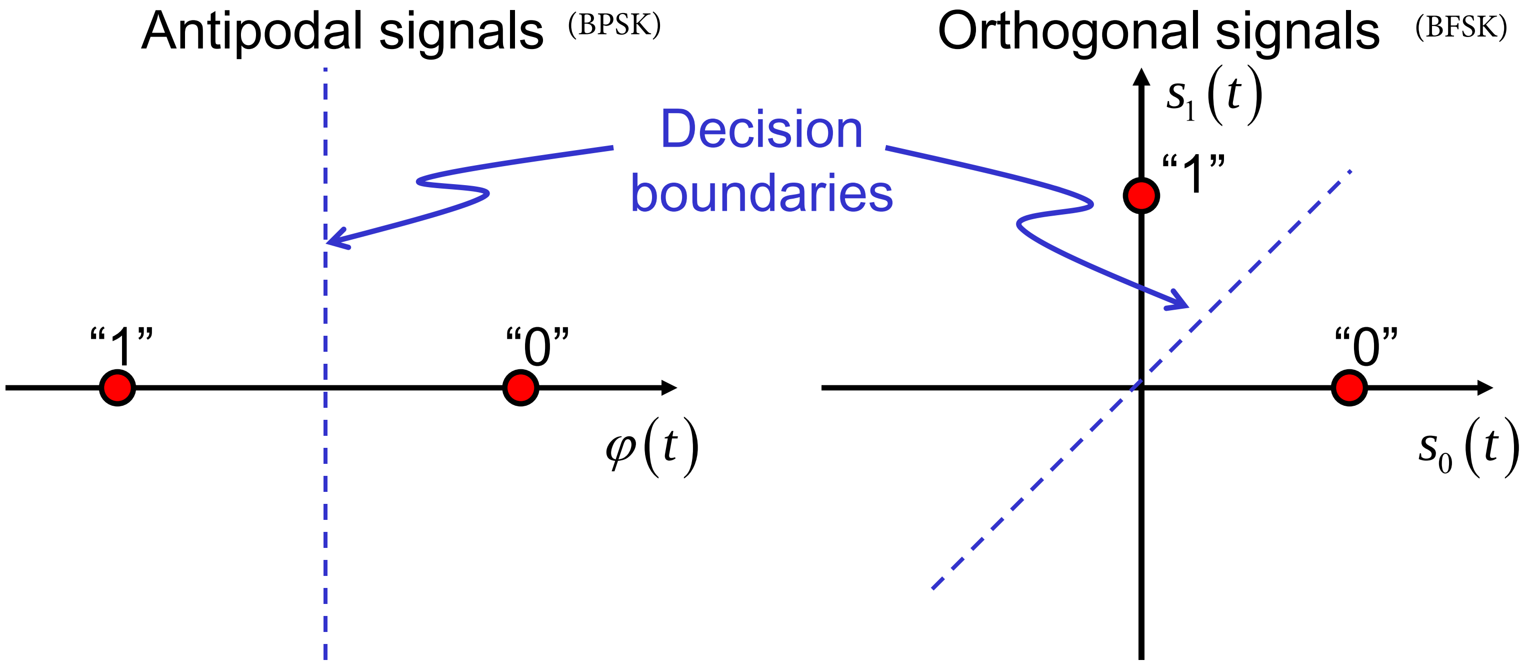
In binary orthogonal signaling, with equal energy alternatives $s_0(t)$ and $s_1(t)$ (for “0” and “1”) we require the property:

$$\langle s_0(t), s_1(t) \rangle = \int s_0(t) s_1^*(t) dt = 0$$



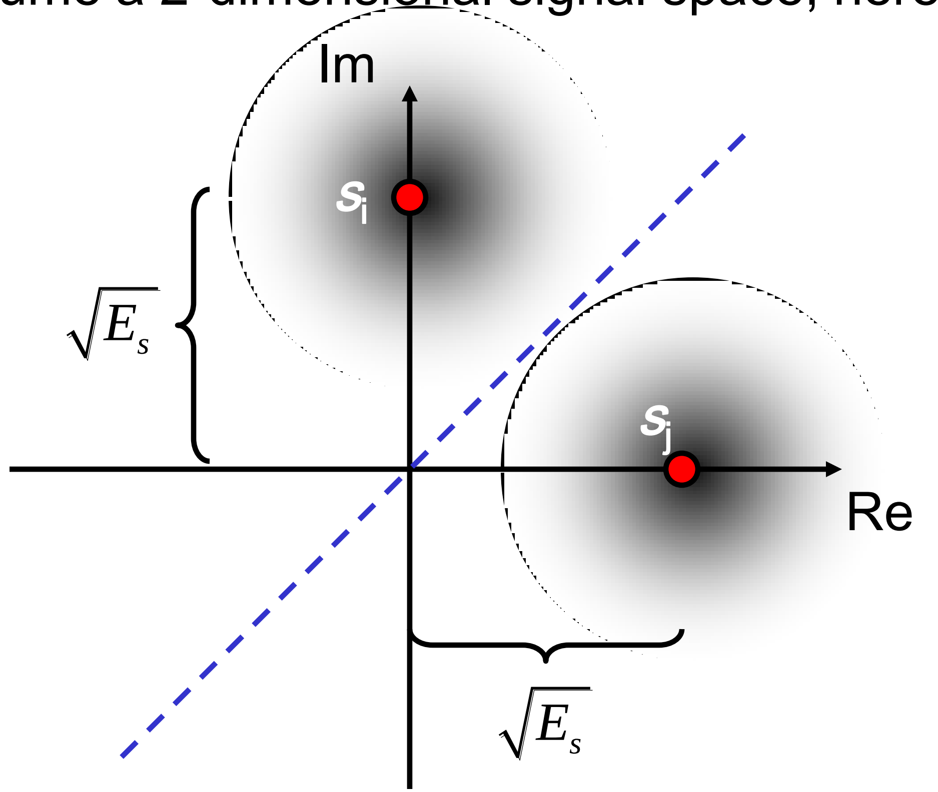
Optimal receiver
Interpretation in signal space

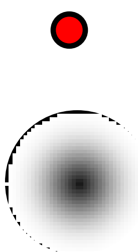
Antipodal Signals are negatives of each other



Optimal receiver
The noise contribution

Assume a 2-dimensional signal space, here viewed as the complex plane





This normalization of axes implies that the noise centered around each alternative is complex Gaussian

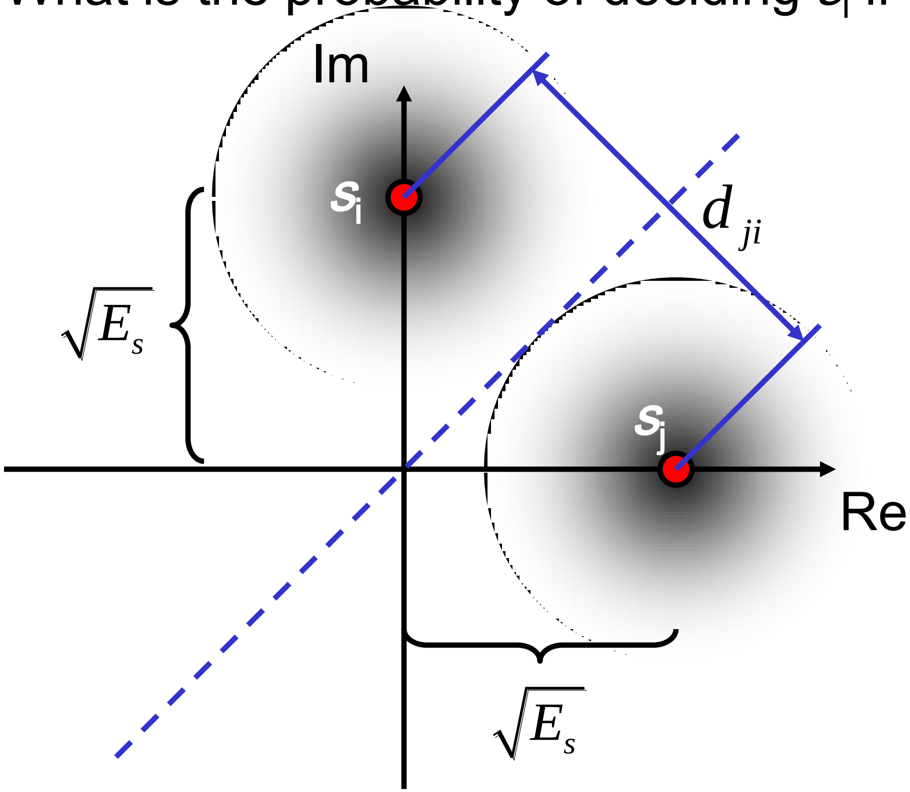
$$N\left(0,\sigma^2\right)+jN\left(0,\sigma^2\right)$$

with variance $\sigma^2 = N_0/2$ in each direction.

Fundamental question: What is the probability that we end up on the wrong side of the decision boundary?

Optimal receiver
Pair-wise symbol error probability

What is the probability of deciding s_i if s_j was transmitted?



We need the distance between the two symbols. In this orthogonal case:

$$d_{ji} = \sqrt{\sqrt{E_s}^2 + \sqrt{E_s}^2} = \sqrt{2E_s}$$

The Modulation method doesn't impact the decision

The probability of the noise pushing us across the boundary at distance $d_{ji}/2$ is

$$P(s_j \to s_i) = Q\left(\frac{d_{ji}/2}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

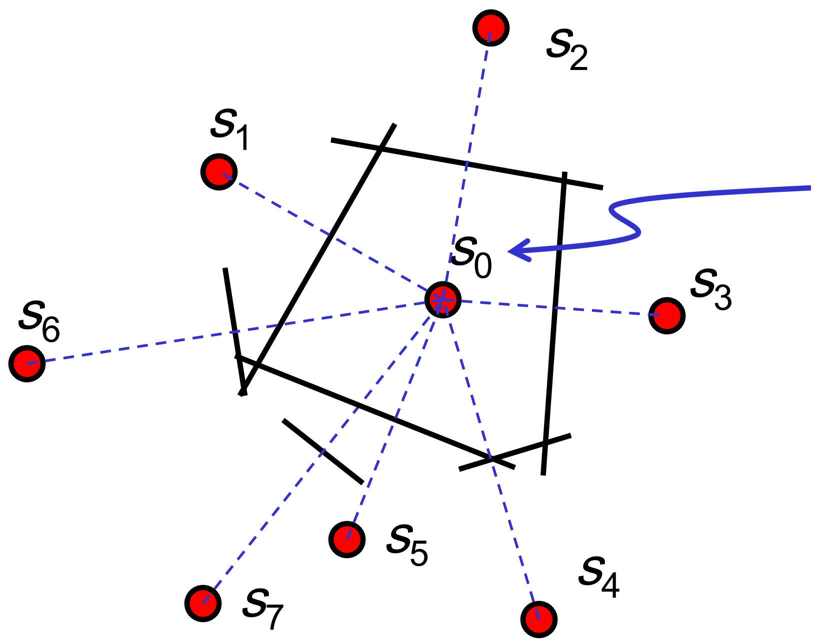
$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right)$$

Optimal Receiver

Calculation of symbol error probability is simple for two signals!

(M-ary modulation methods)

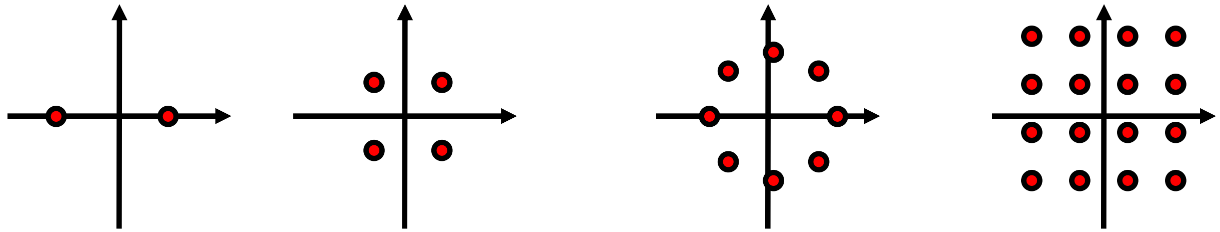
When we have many signal alternatives, it may be impossible to calculate an exact symbol error rate.



When s_0 is the transmitted signal, an error occurs when the received signal is outside this polygon.

Note relantionships of BER, bit error probability and symbol error probability

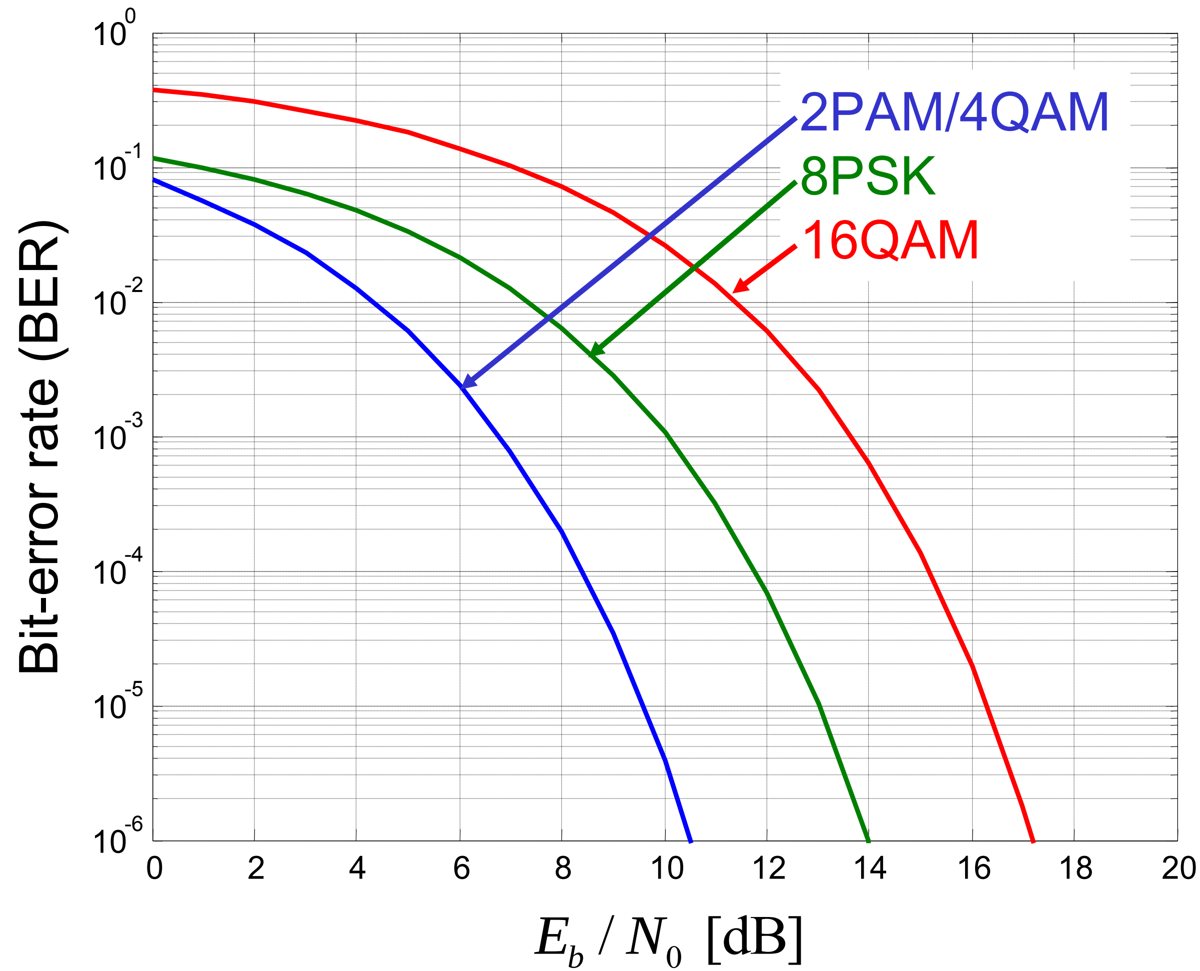
Optimal receiver
Bit-error rates (BER)



EXAMPLES:	2PAM	4QAM	8PSK	16QAM
Bits/symbol	1	2	3	4
Symbol energy	E_b	$2E_b$	$3E_b$	$4E_b$
BER	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$\approx \frac{2}{3} Q\left(\sqrt{0.87 \frac{E_b}{N_0}}\right)$	$\approx \frac{3}{2} Q\left(\sqrt{\frac{E_{b,\max}}{2.25 N_0}}\right)$

Gray coding is used when calculating these BER.

Optimal receiver Bit-error rates (BER)



Summary: the higher the spectral efficiency, the higher the bit energy to noise ratio has to be for the same BER

Optimal receiver
Where do we get E_b and N_0?

Where do those magic numbers E_b and N_0 come from?

The noise power spectral density N_0 is calculated according to

$$N_0 = kT_0F_0 \Leftrightarrow N_{0|dB} = -204 + F_{0|dB}$$

where F_0 is the noise factor of the "equivalent" receiver noise source.

The bit energy E_b can be calculated from the received power C (at the same reference point as N_0). Given a certain data-rate d_b [bits per second], we have the relation

$$E_b = C / d_b \Leftrightarrow E_{b|dB} = C_{|dB} - d_{b|dB}$$

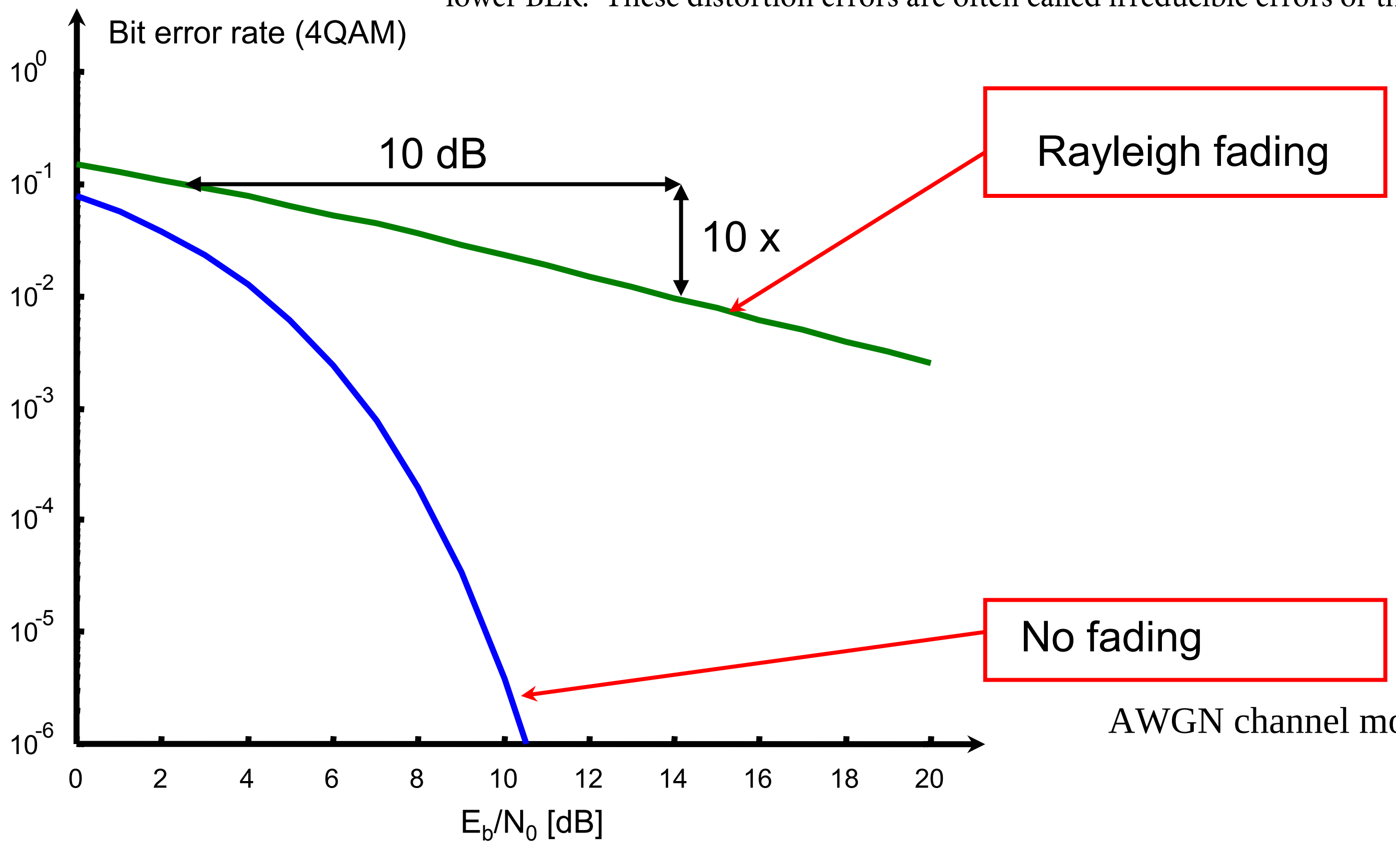
THESE ARE THE EQUATIONS THAT RELATE DETECTOR PERFORMANCE ANALYSIS TO LINK BUDGET CALCULATIONS!

BER IN FADING CHANNELS AND DISPERSION-INDUCED ERRORS

BER in fading channels

THIS IS A SERIOUS PROBLEM!

For high data rates, delay dispersion (multipath --> ISI) is the main transmission error source while at low data rates frequency dispersion (Doppler effect) is the main signal distortion error source. For both, an increase in the transmitter power doesn't lead to a lower BER. These distortion errors are often called irreducible errors or the error floor.

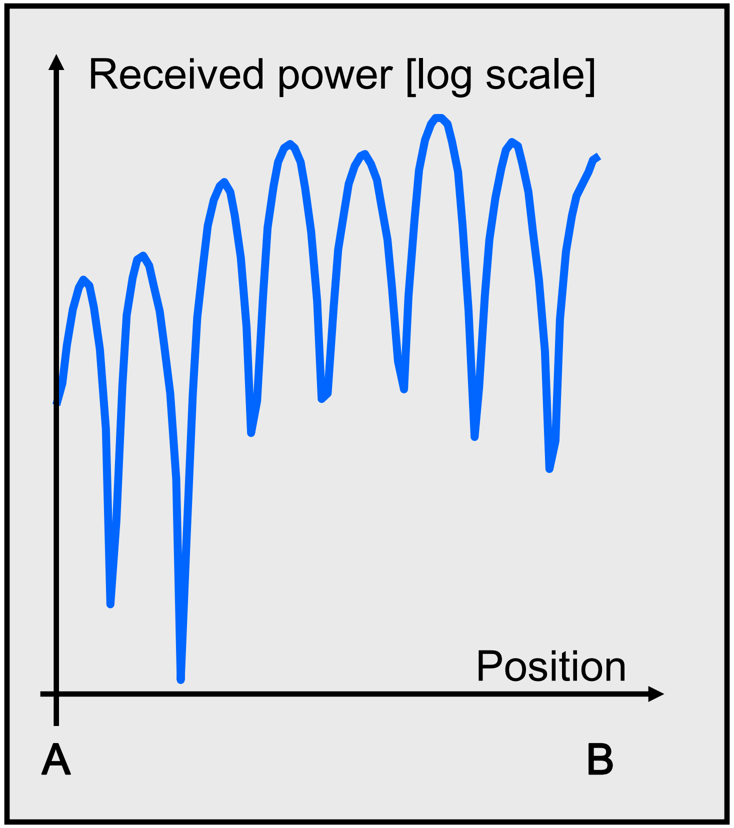


AWGN channel model

Diversity

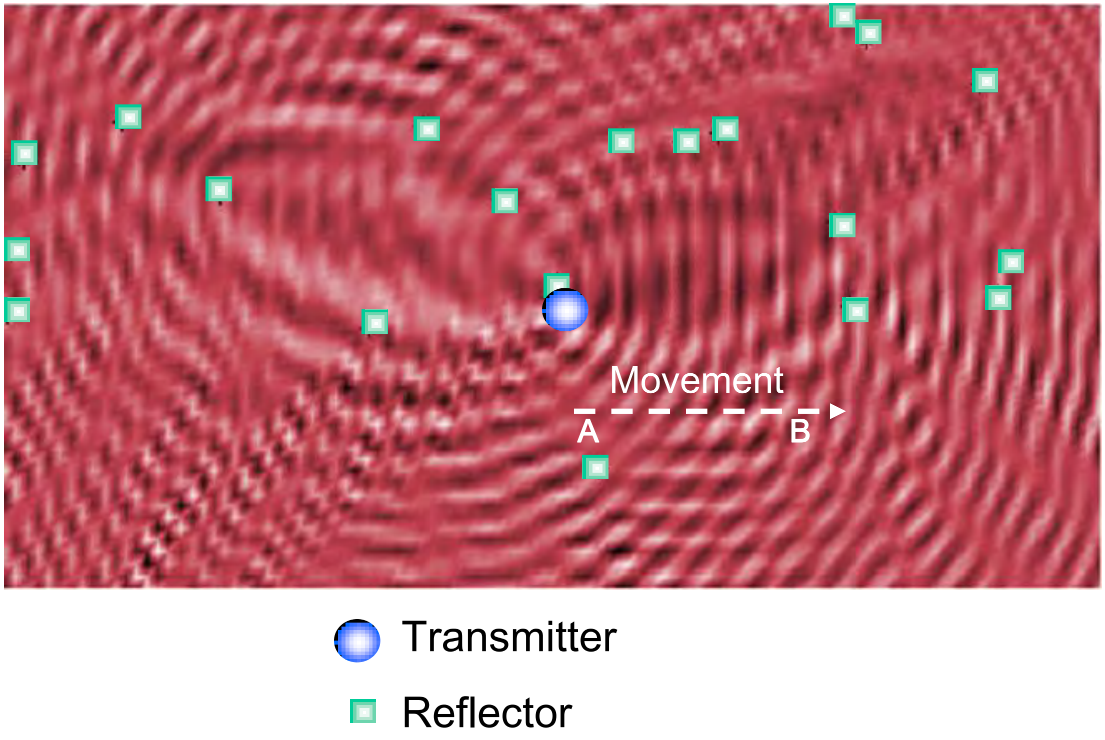
If one is good, two must be better

Diversity arrangements
Let's have a another look at fading again



Channel Transfer Function

Illustration of interference pattern from above



Diversity arrangements
The diversity principle

For AWGN (additive white Gaussian Noise) channels the BER decreases exponentially as the SNR (signal-to-noise) ratio increases (stronger signal, more transmitter power, etc.); however, in Rayleigh fading dominated channels the BER only decreases linearly with the SNR. Thus one would need a 40 dB increase (read as a very big number) in the SNR in order to achieve a 10^{-4} BER

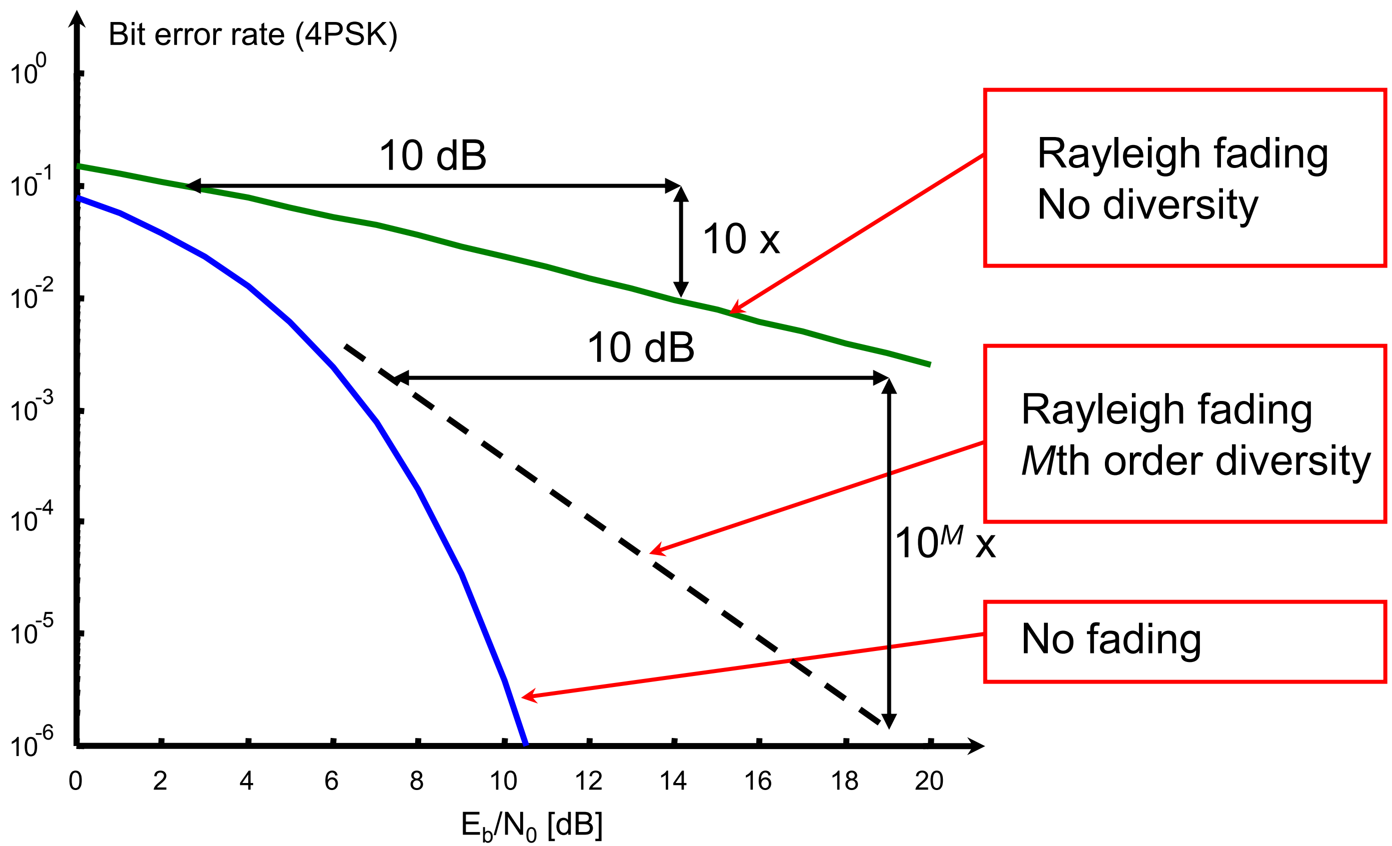
One must be able to change the channel characteristics to solve this problem. Diversity is a method to achieve the desired SNR improvements which will apply to small-scale fading (no way to solve large-scale/shadowing effects).

The principle of diversity is to transmit the same information on *M* statistically independent channels.

By doing this, we increase the chance that the information will be received properly (lower BER, higher SNR).

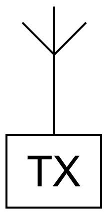
Advantage: **Diversity gain** - improbable that several antennas are in a fading dip simultaneously.
Beamforming gain - even if signal levels at all antennas are the same, the combiner output SNR is larger that the SNR at a single antenna

Diversity arrangements
General improvement trend

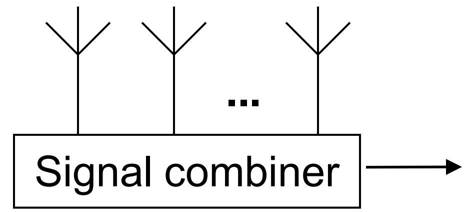


Microdiversity techniques

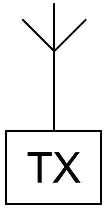
Spatial (antenna) diversity foundation of MIMO (multiple-in multiple-out)

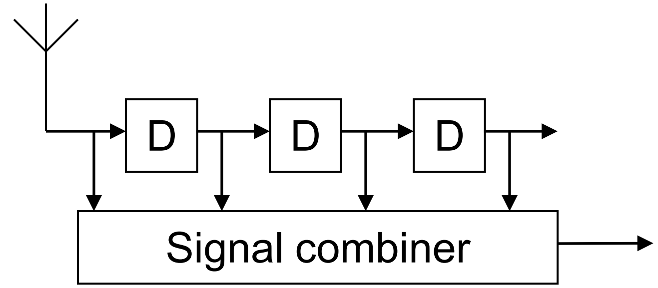


We will focus on this one today!



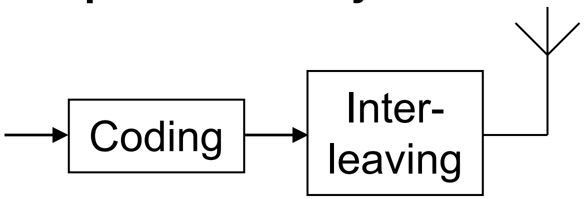
Frequency diversity channels on different frequencies, different by more than the coherence bandwidth of the channel

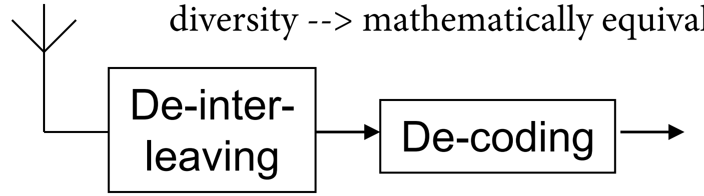




Temporal diversity (TIME)

For moving transmitters, temporal and spatial diversity --> mathematically equivalent





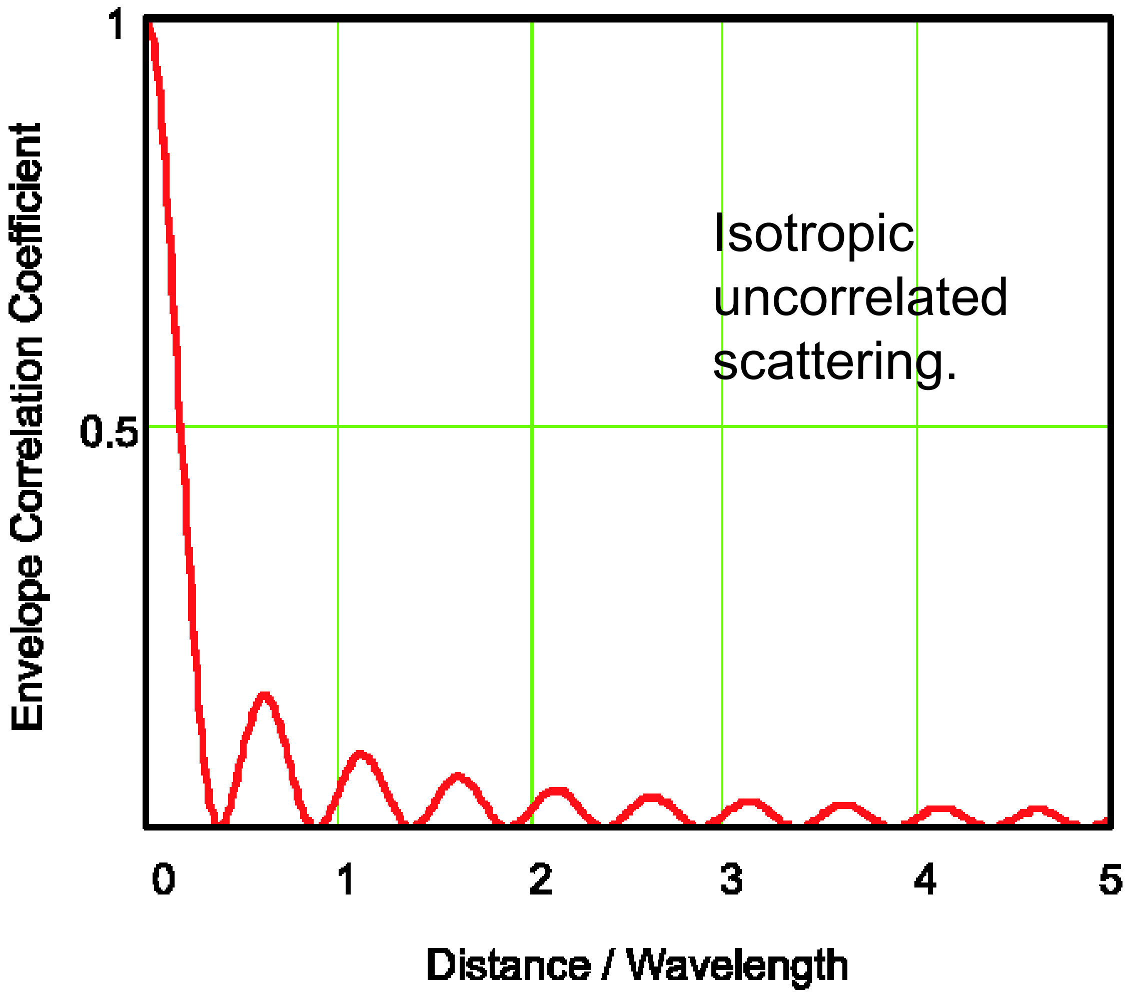
We also have angular (different antenna patterns) and vertical/horizontal polarization diversity

Spatial (antenna) diversity
Fading correlation on antennas

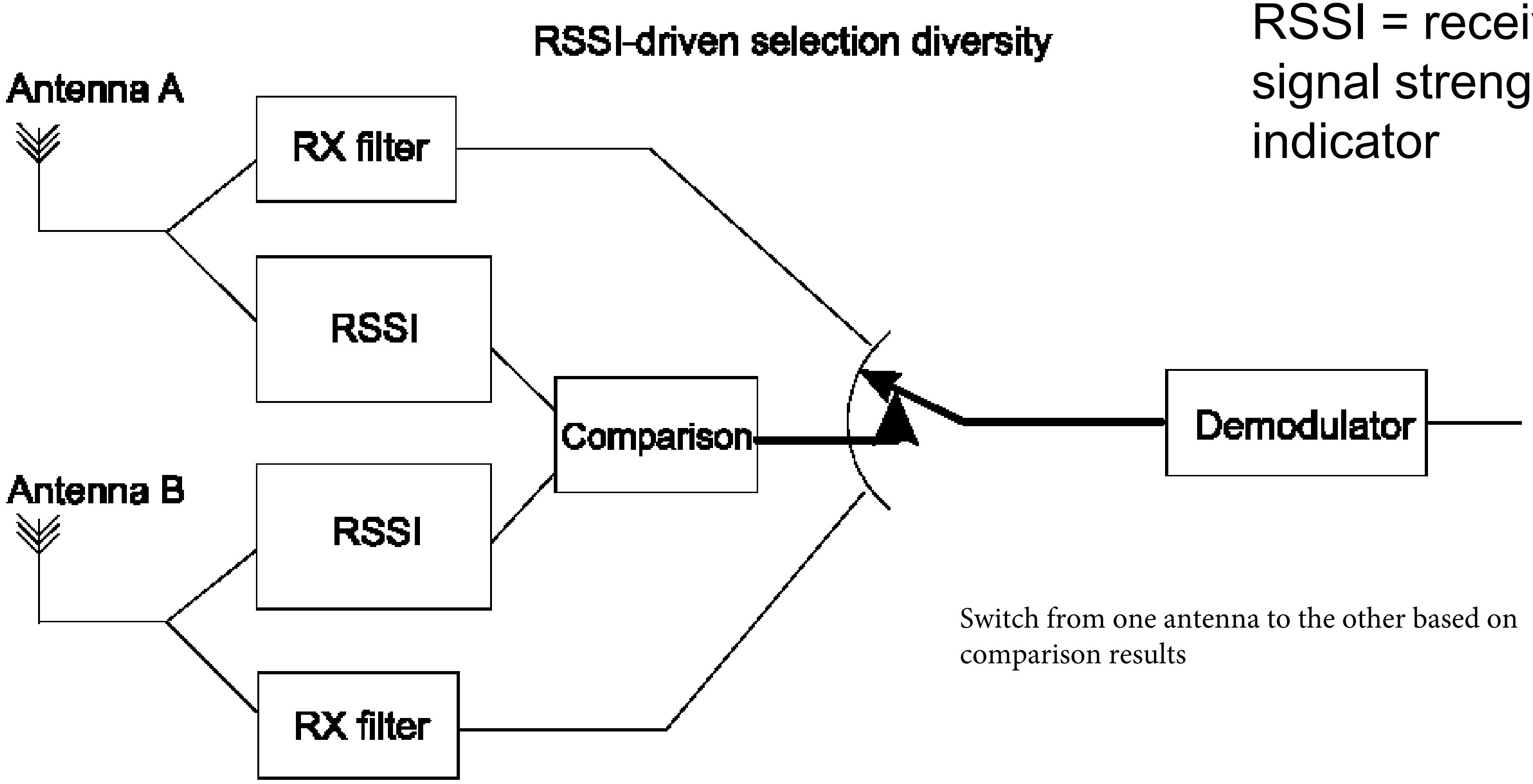
Goal: Statistically independent signals

You don't want your antennas close to each other --> low correlation desired

luckily we're operating at very high frequencies so a wavelength isn't a big distance as shown in the adjacent figure (Figure 13.1 page 253) Textbook suggests 8 cm for the GSM 900 MHz band, obviously less for higher frequencies like 2.4 GHz / 5 GHz WiFi, etc.



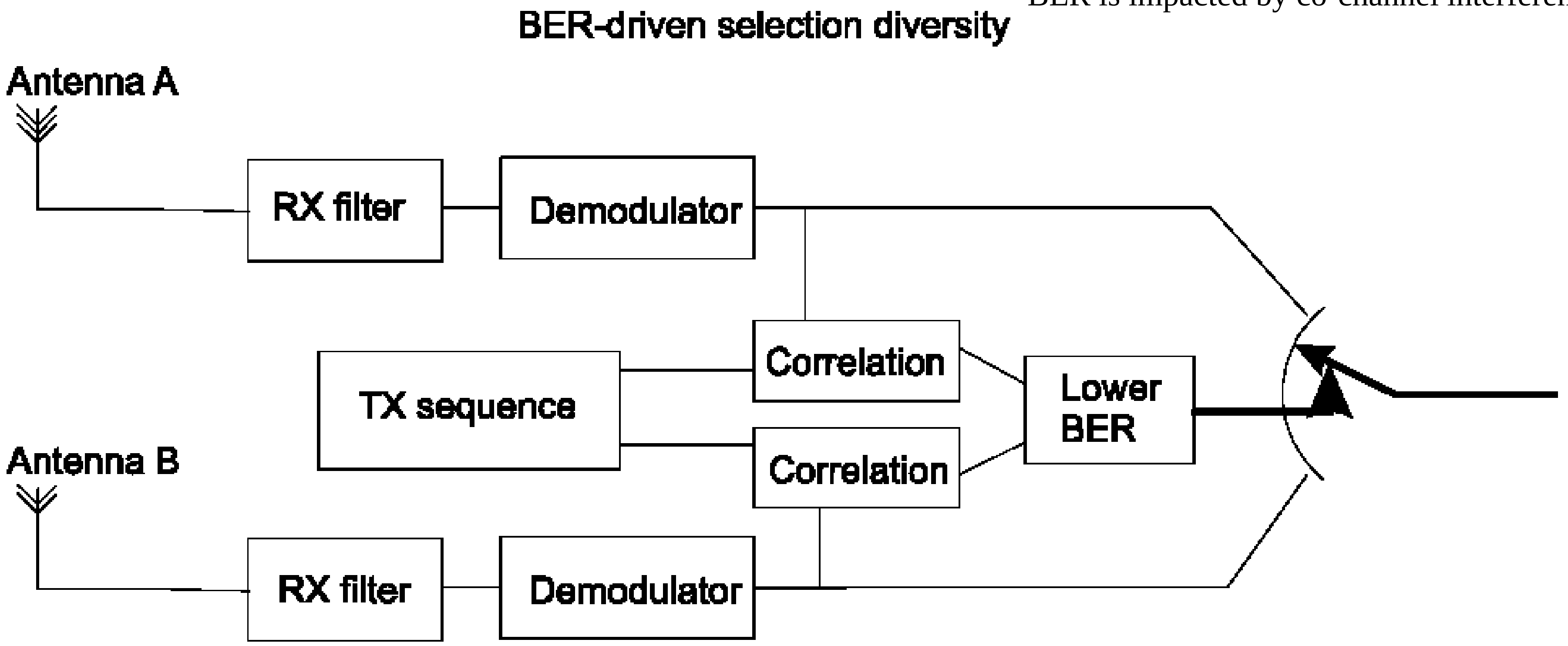
Spatial (antenna) Diversity
Selection Diversity versus Combining Diversity



RSSI = received signal strength indicator

Spatial (antenna) diversity
Selection diversity, cont.

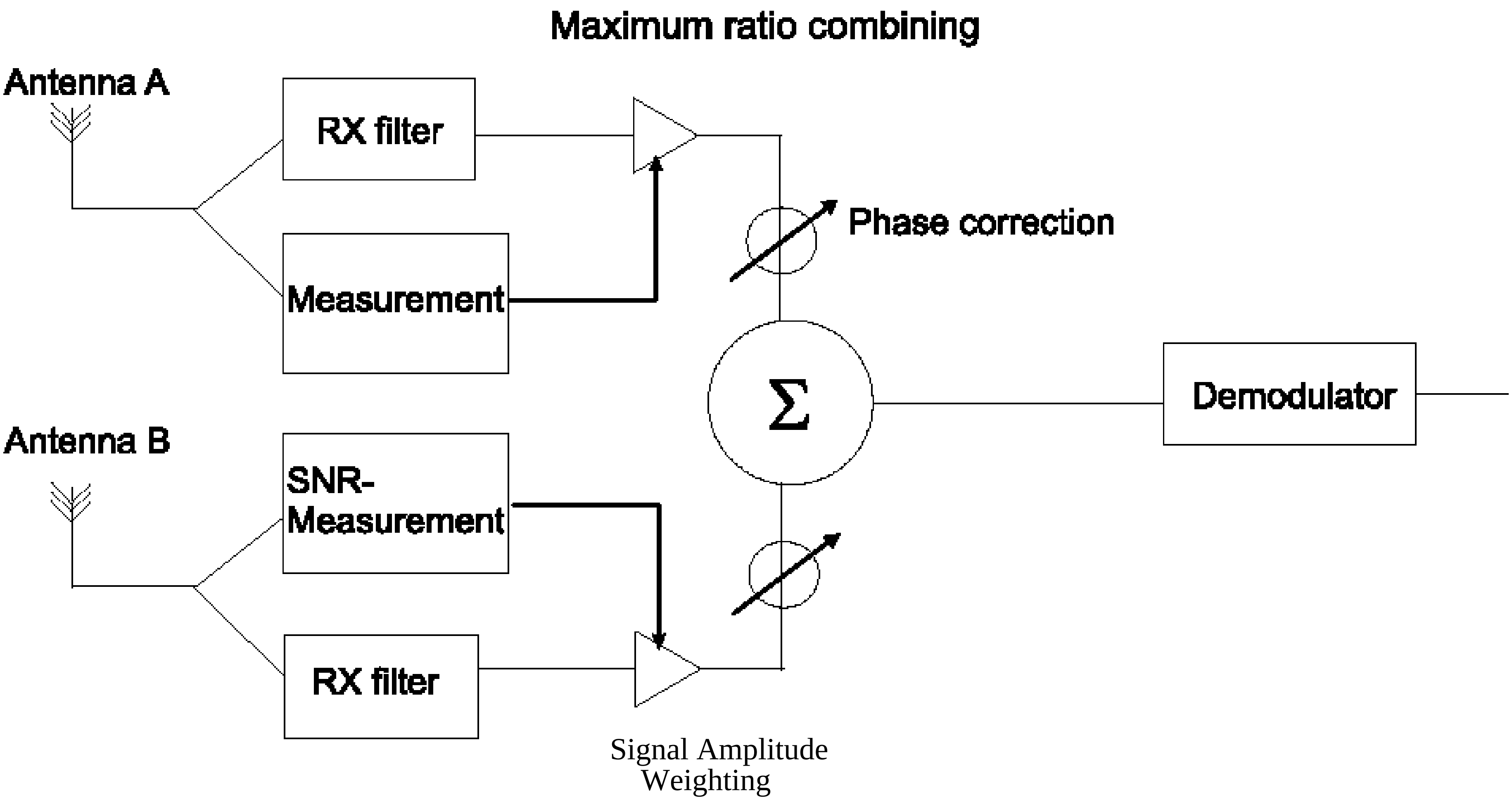
This is the best selection method if BER is impacted by noise, not so good if BER is impacted by co-channel interference.



To reduce hardware complexity, **SWITCHED Diversity**.
Just stay on one antenna until signals falls below some threshold and then switch to the other antenna

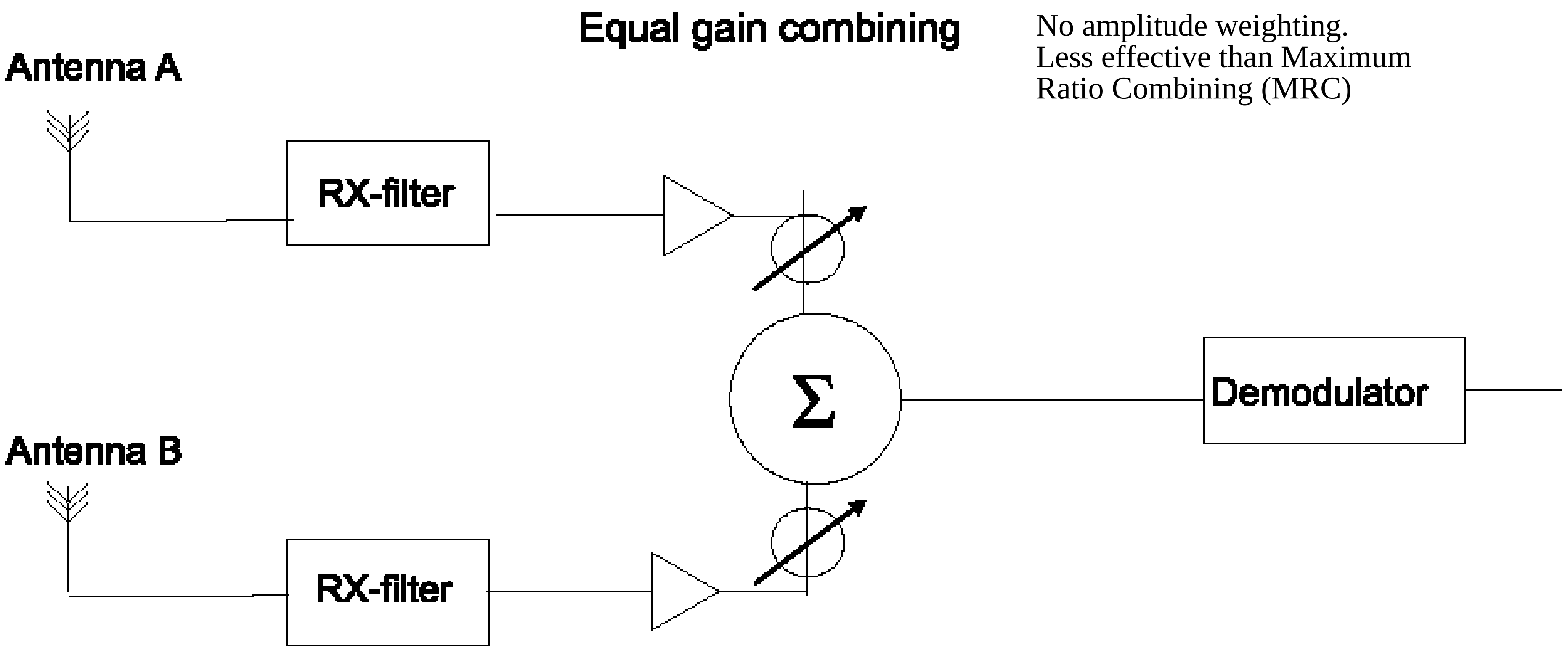
Spatial (antenna) diversity
Maximum ratio combining

To use ALL available signals (antennas A + B)
COMBINING DIVERSITY



Spatial (antenna) diversity
Equal Gain Combining

No amplitude weighting.
Less effective than Maximum
Ratio Combining (MRC)



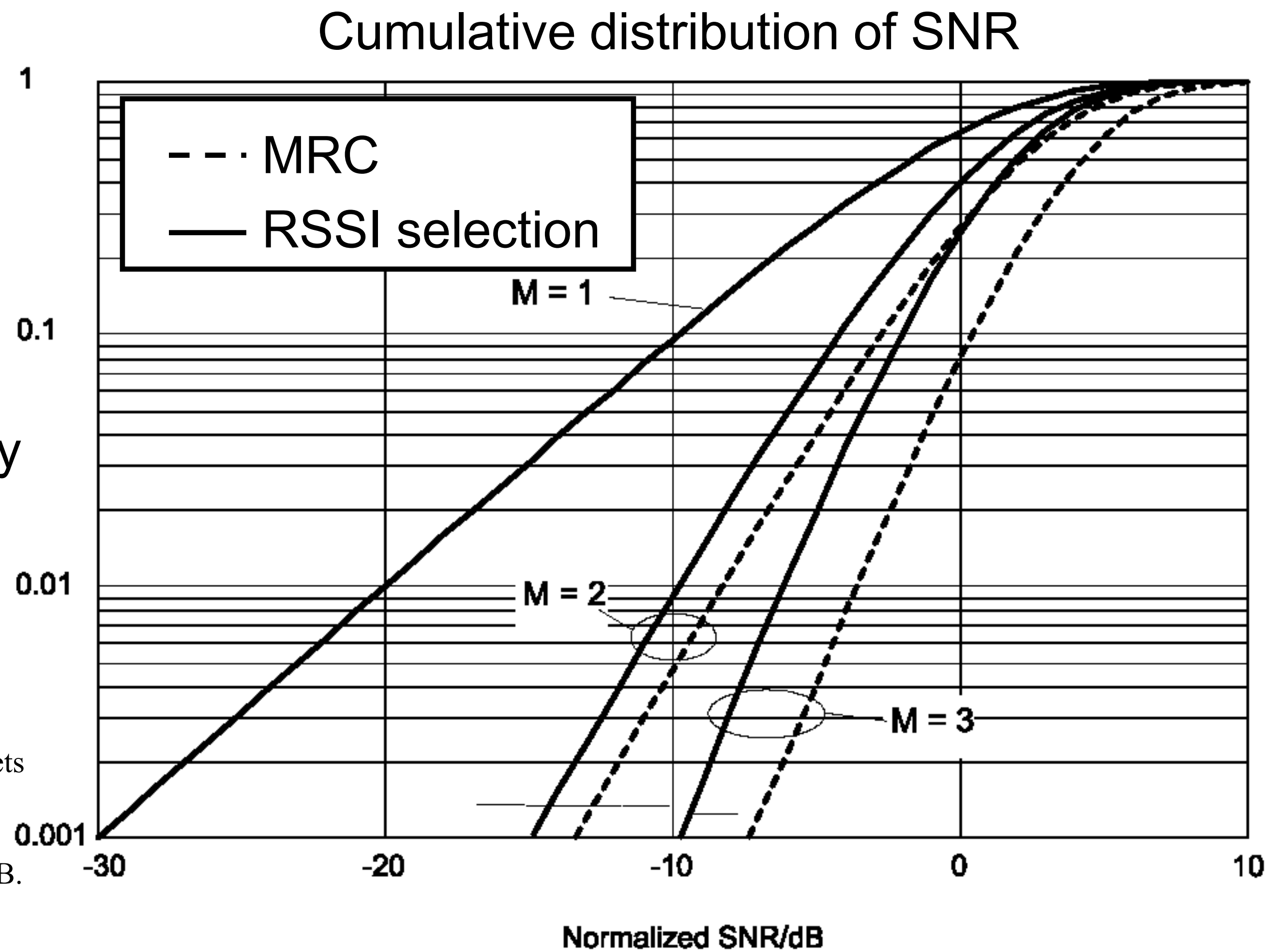
Spatial (antenna) diversity Performance comparison

Figure 13.10

Comparison of SNR distribution for different number of antennas *M* and two different diversity techniques.

Showing MRC > RSSI for more than one antenna (M > 1)

The vertical axis is the ratio of the instant Eb/No to the mean Eb/No. As this ratio gets smaller, the mean power is increasing. The y-axis is thus the % outage versus the x-axis which is the fading margin in dB.



Optimum combining in flat-fading channel

- Most systems interference limited (not noise limited)
- Opt. Comb. reduces not only fading but also interference
- Each antenna can eliminate one interferer or give one diversity degree for fading reduction: ("zero-forcing").
- MMSE or decision-feedback gives even better results

MMSE- min mean square error

- Computation of weights for combining

$$\mathbf{w}_{\mathrm{opt}} = \mathbf{R}^{-1}\mathbf{h}_d$$

Vector of optimum weights

$$\mathbf{R} = \sigma_n^2 \mathbf{I} + \sum_{k=1}^{K} E\{\mathbf{r}_k \mathbf{r}_k^{\dagger}\}$$

Correlation Matrix of noise and interference

For large-scale fading (shadowing effects caused by buildings or mountains in the path), none of these diversity techniques will work so **macrodiversity** (repeaters, simulcast) is the only possible solution.

Transmit Diversity

so far we've only considered multiple receive antennas

- Don't forget the possibility of using more than one transmit antenna
- For noise limited situations, transmit diversity is equal to receive diversity
- Since the state of the communications channel is not available at the TX site, the RX has to have a means of distinguishing between the different TX antenna signals (MIMO)
- One means is DELAY DIVERSITY where in a flat fading channel, the transmitted data is delayed by 1 symbol duration at the other antenna. With variable weighting receivers (Rake RX), the diversity order is equal to the number of antenna elements. And even if the signal is delay dispersive, the scheme still works.
- So for a good channel (flat fading) we make the channel WORSE by adding signal delay dispersion in order to make it BETTER at the RX
- An alternative method is phase-sweeping diversity which introduces temporal variations into the channel such that the RX signal is less likely to remain stuck in a fading dip.