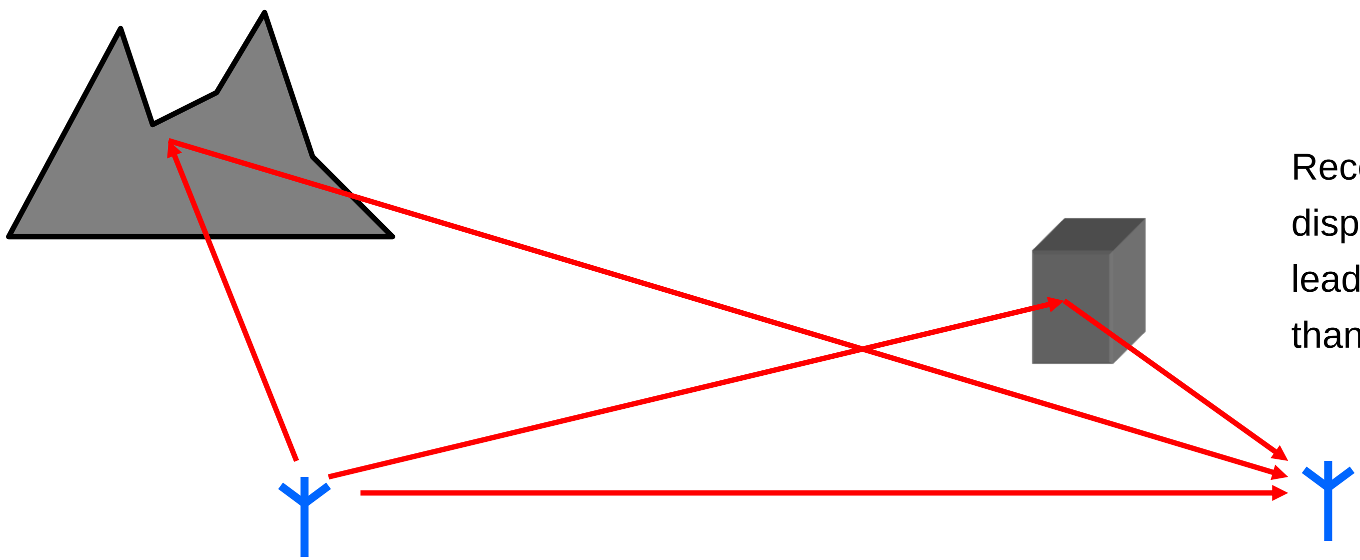
Chapter 6

Chapter 5 covered narrowband channels where the transmit signal was a pure sinusoid.

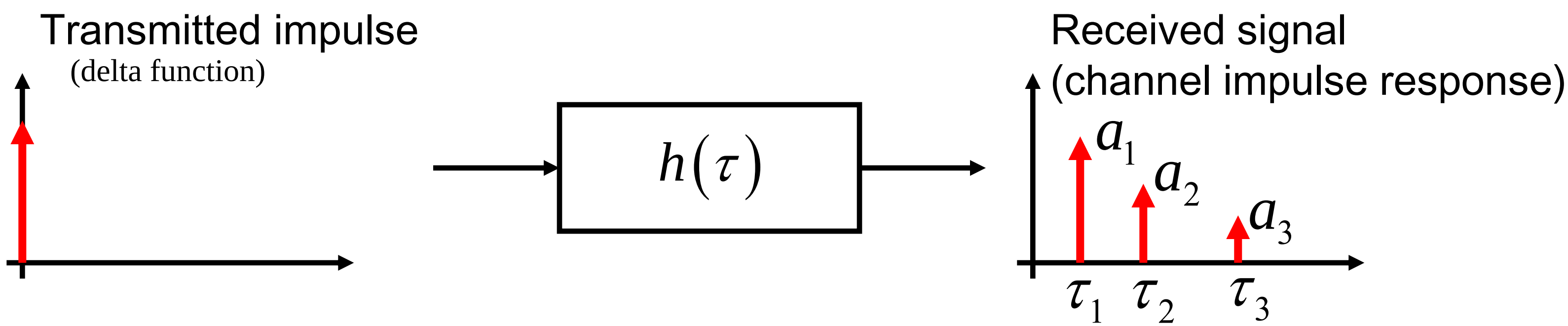
Wideband channels

Delay Dispersion - the arriving signal has a longer duration than the transmitting signal (the impulse response of the channel is not a delta function). This is the same as the channel transfer function changing over the bandwidth of interest (the frequency selectivity of the channel not being constant). Wideband channels are required for multiple access and/or high data rates.

Delay (time) dispersion
A simple case



Received signal shows delay dispersion in the time domain leading to ISI yet less impact than from fading

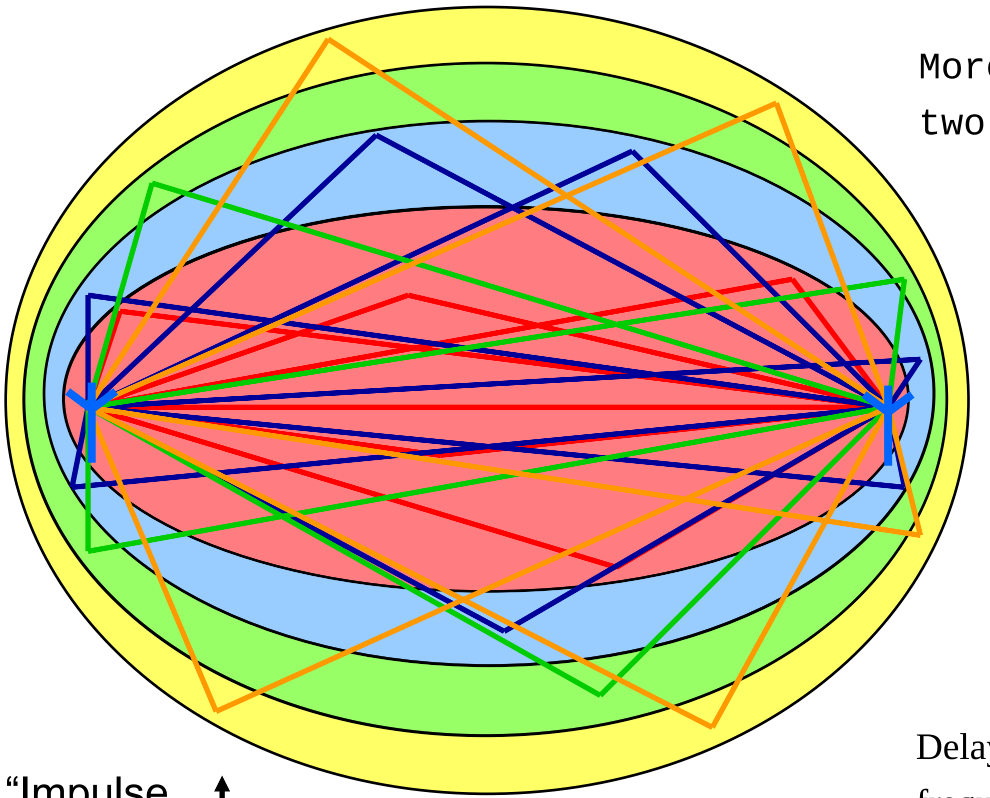


`Impulse Response of the Channel`

$$h(\tau)=a_1\delta(\tau-\tau_1)+a_2\delta(\tau-\tau_2)+a_3\delta(\tau-\tau_3)$$

see Figure 6.1 for transfer function in the Frequency Domain with deep nulls

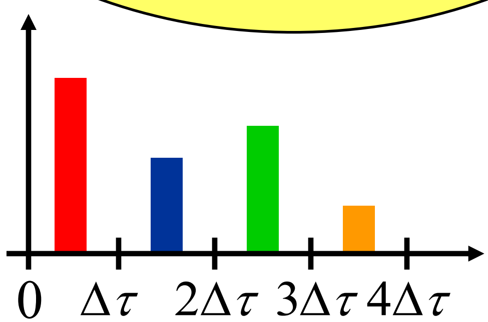
Delay (time) dispersion
One reflection/path, many paths



More generalized case versus the simple two-path model of Chapter 5

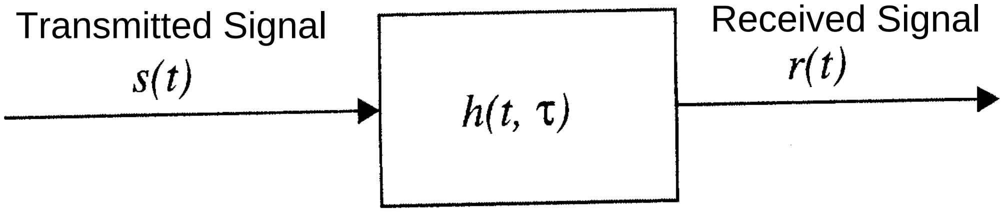
Signals that interact with objects on the same ellipse (red, blue, etc.) arrive at the RX at the same time

“Impulse response”

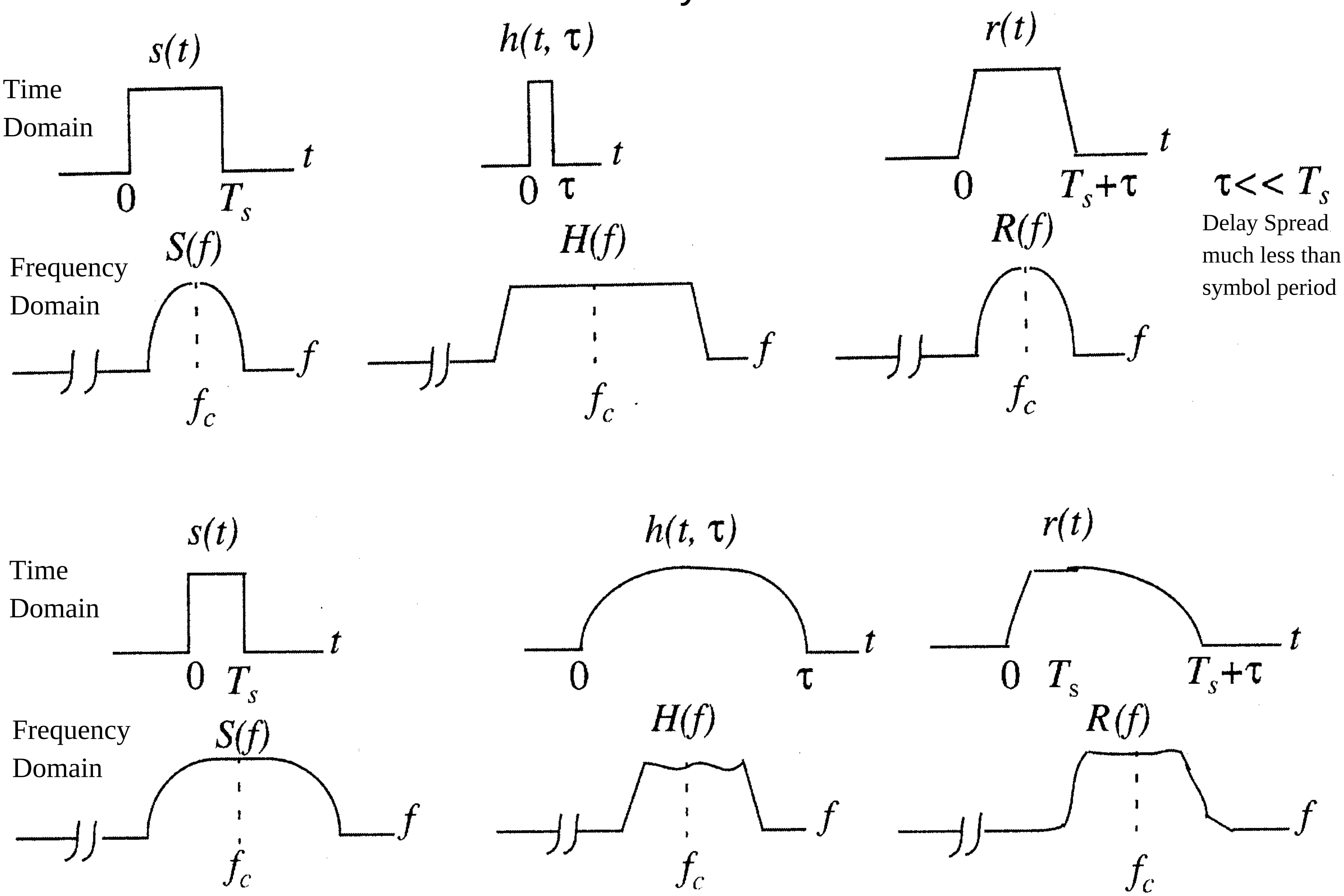


Delay dispersion in the time domain (t) translates into frequency selectivity in the frequency domain (f). Group responses into same 'bin' and use equations from Chapter 5 for each delay bin.

Narrowband versus Wideband Systems



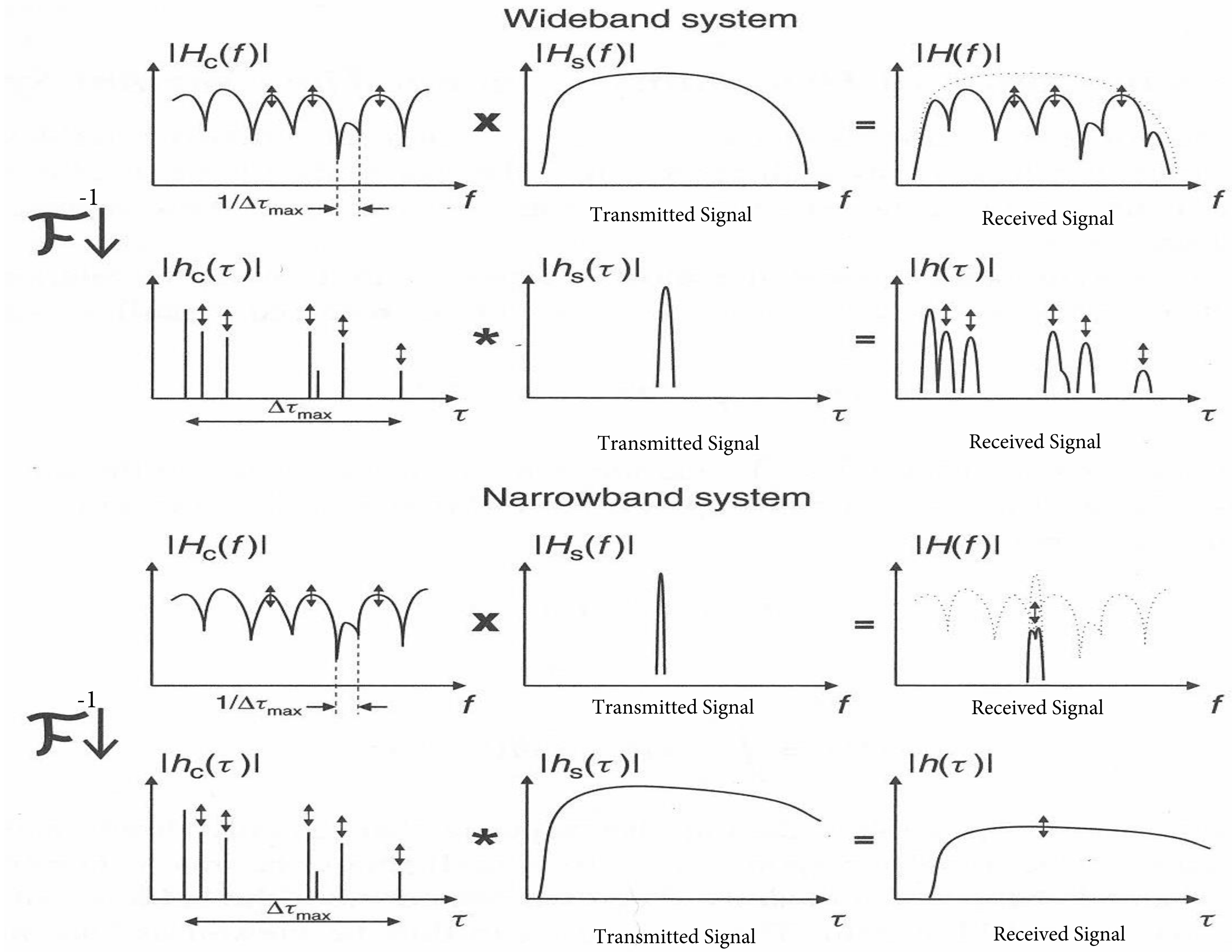
Narrowband System



Wideband System

These diagrams show the time domain and the frequency domain responses of both systems. In the wideband system, the shape and duration of the received signal R(f) in the frequency domain or r(t) in the time domain is different from the shape of the transmitted signal. The channel is frequency selective as shown in H(f) and the channel induces intersymbol interference (ISI). In the narrowband system or flat fading as shown in H(f), the spectral characteristics of transmitted signal are preserved although the gain of the channel gain changes over time caused by multipath and best described by a Rayleigh distribution.

Narrowband versus Wideband Systems



In a wide-band system, the shape and duration of the received signal $| H (f) |$ in the frequency domain or $| h (\tau) |$ in the time domain is different from the shape of the transmitted signal

$|H_s (f)|$ frequency domain $|h_s (\tau)|$ time domain

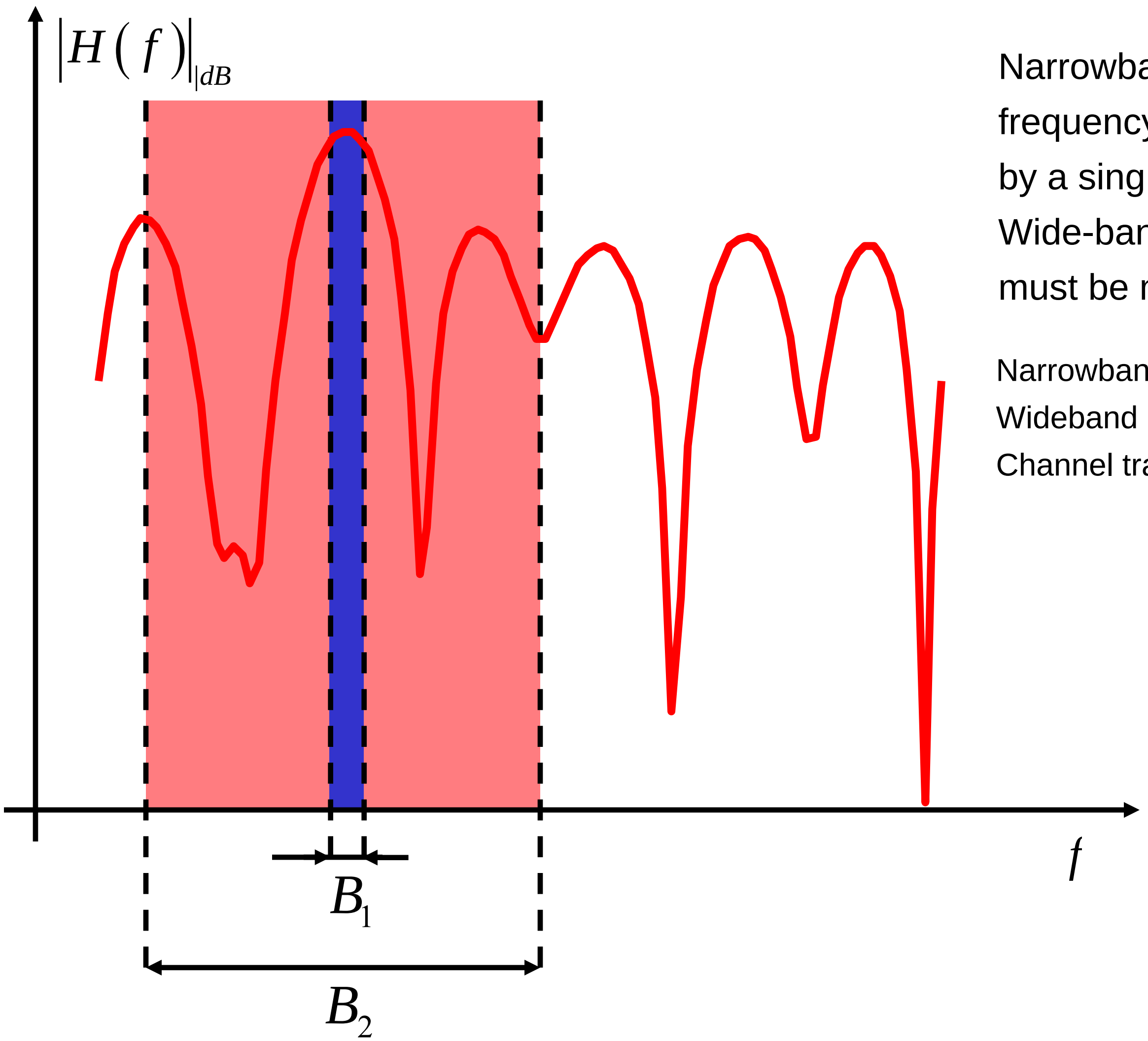
In a narrow-band system, the shape of the received signal is the same as the transmitted signal.

$|H_c (f)|$ = transfer function frequency domain

$|h_c (\tau)|$ = impulse response time domain

These diagrams show the time domain and the frequency domain responses of both systems.

Narrow- versus wide-band Channel frequency response



Narrowband: transfer function is not frequency dependent which can be described by a single attenuation coefficient - a constant
Wide-band: details of the transfer function must be modeled (large variations as shown)

Narrowband - blue signal
Wideband - wide pink area
Channel transfer function in the frequency domain - red

System functions (1)

- Time-variant impulse response $h(t,\tau)$ h (time, delay)
  - Due to movement, impulse response changes with time
  - Input-output relationship:

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(t,\tau)d\tau$$

h( ) completely characterizes the channel and is a function of 2 variables: time and delay --> LTV

- Time-variant transfer function *H(t,f)*
  - Perform Fourier transformation with respect to τ

$$H(t,f) = \int_{-\infty}^{\infty} h(t,\tau)\exp(-j2\pi f\tau)d\tau$$

  - Input-output relationship

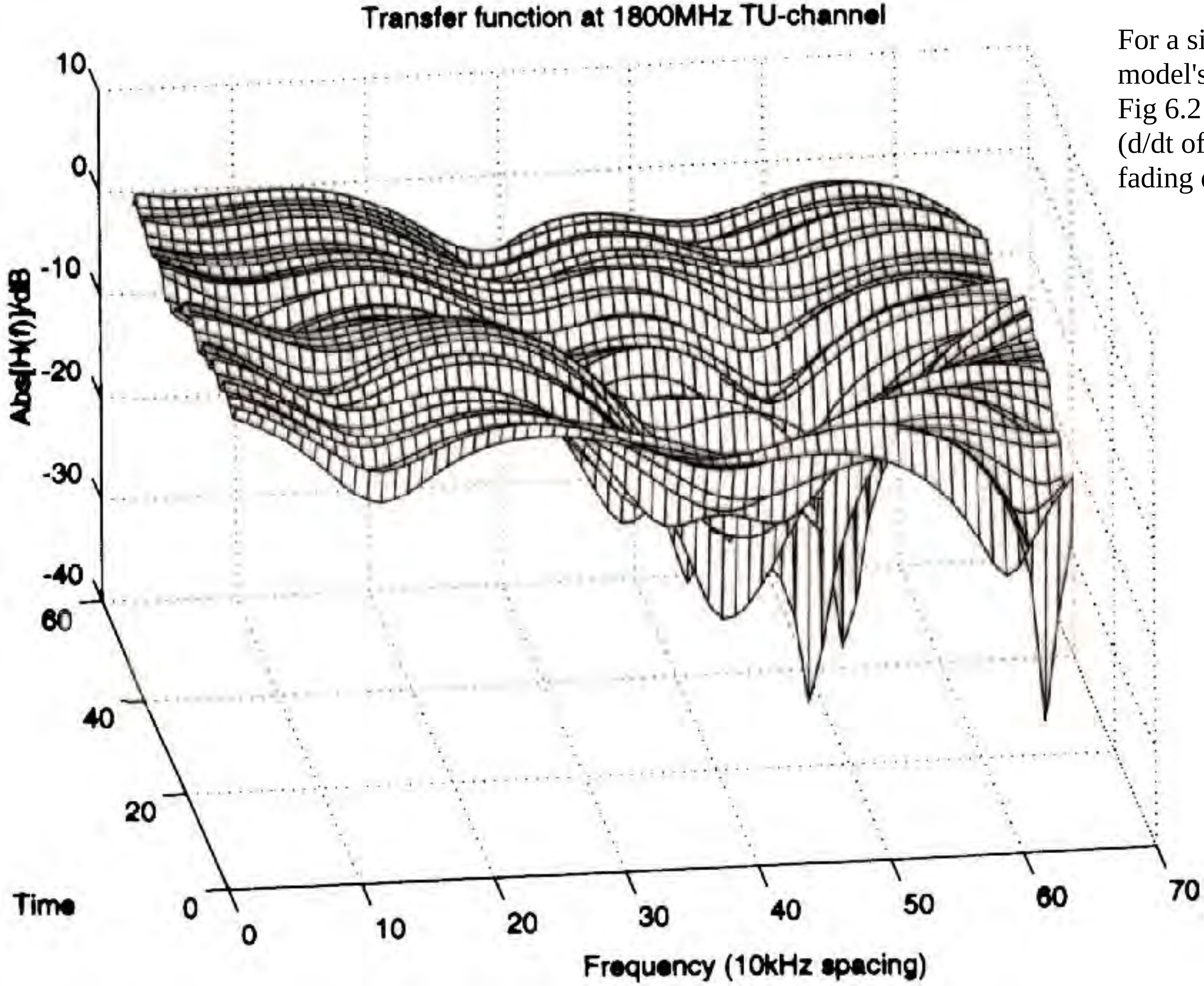
$$Y(\tilde{f}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(f)H(t,f)\exp(j2\pi ft)\exp(-j2\pi\tilde{f}t)dfdt$$

becomes *Y(f)=X(f)H(f)* only in *slowly* time-varying channels

LTI theory applies with minor mods

Output signal = input signal multiplied by the spectrum of the currently valid transfer function (quasi static system)

Transfer function, Typical urban

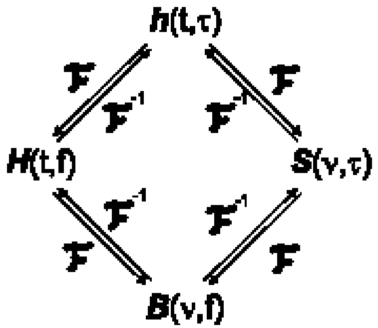


For a simpler representation of a two path model's transfer function - Fig 6.1 pg 103 Fig 6.2 pg 103 shows that the group delay (d/dt of the phase) is very large at the fading dips in the transfer function.

System functions (2)

- Further equivalent system functions:
  - Since impulse response depends on two variables, Fourier transformation can be done w.r.t. each of them

  -> *four* equivalent system descriptions are possible:
    - Impulse response two variables time and tau (delay)
    - Time-variant transfer function H(t,f) - Fourier transform of impulse response wrt tau
    - Spreading function $$S(\nu,\tau) = \int_{-\infty}^{\infty} h(t,\tau)\exp(-j2\pi\nu t)dt$$
    - Doppler-variant spreading function $$B(\nu,f) = \int_{-\infty}^{\infty} S(\nu,\tau)\exp(-j2\pi f\tau)d\tau$$

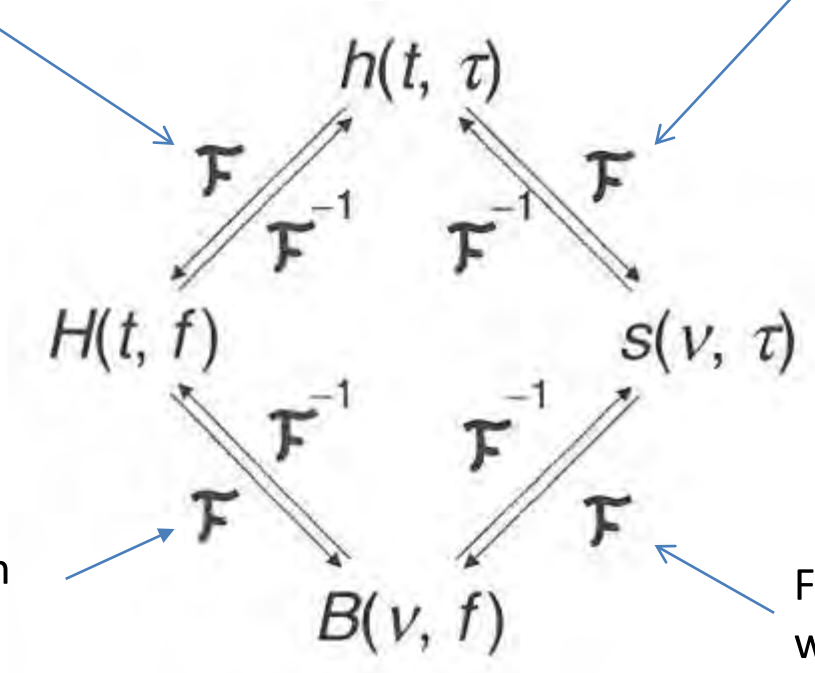


Wireless Channels – Interrelation Between Deterministic System Functions

Fourier Transform
wrt τ (delay)

Time variant
impulse response

Fourier Transform
wrt t (time)



Time variant
Transfer function

Doppler variant
Impulse response -
spreading function
S(Doppler, delay)

Fourier Transform
wrt t (time)

Fourier Transform
wrt τ (delay)

Doppler variant transfer function
B(Doppler shift, frequency)

Stochastic system functions

The autocorrelation function describes the relationship between the second-order moments of the amplitude pdf of the signal y at different times and if the pdf is zero-mean Gaussian, then the 2nd order description contains all the required information of the channel and received signal.

- ACF - autocorrelation function (second-order statistics)

$$R_h(t,t',\tau,\tau') = E\{h^*(t,\tau)h(t',\tau')\}$$

Note: the ACF of the channel depends on 4 variables

- Input-output relationship:

$$R_{yy}(t,t') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{xx}(t-\tau,t'-\tau')R_h(t,t',\tau,\tau')d\tau d\tau'$$

Autocorrelation function (ACF) of the received signal is a combination of the ACF of the transmit signal & the ACF of the channel

Trying to make things simple

Exam physical properties of the channel --> simplify correlation function --> WSS (Wide-Sense Stationary) + US (Uncorrelated scatters --> assumptions lead to WSSUS Model

The WSSUS model: mathematics

- If WSSUS is valid, ACF depends only on two variables (instead of four)

Wide-Sense Stationary (WSS) assumption depends only on the differences in t - t' where the ***statistical properties*** of the channel don't change with time.

- ACF of impulse response becomes

$$R_h(t, t+\Delta t, \tau, \tau') = \delta(\tau - \tau') P_h(\Delta t, \tau)$$

$P_h(\Delta t, \tau)$*....Delay cross power spectral density*

- ACF of transfer function

$$R_H(t+\Delta t, f+\Delta f) = R_H(\Delta t, \Delta f)$$

- ACF of spreading function

$$R_s(\nu, \nu', \tau, \tau') = \delta(\nu - \nu')\delta(\tau - \tau') P_s(\nu, \tau)$$

$P_s(\nu, \tau)$*.......Scattering function*

Fading still a dynamic factor, just the statistics are stationary which leads to a quasi-stationary environment over a time interval (movement of less than 10 λ).

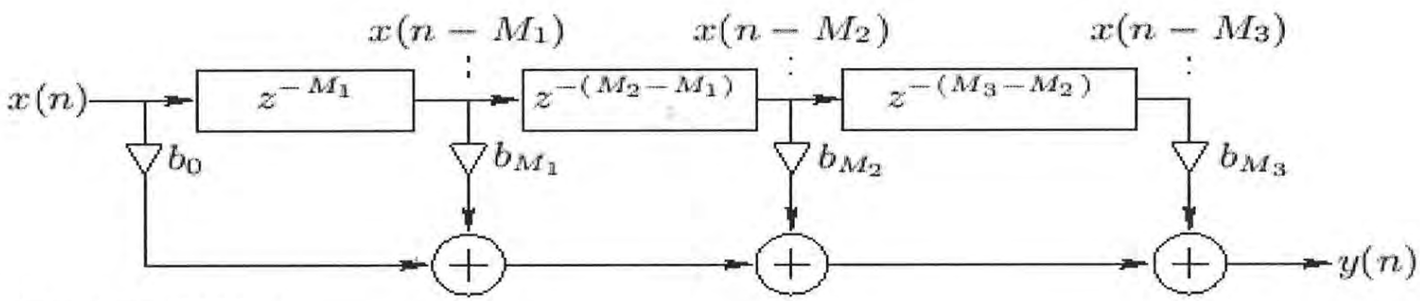
Uncorrelated Scatters (US) assumption depends on differences in frequency. Not truly valid in an indoor environment where for example scatters off a wall are correlated.

Thus WSSUS assumptions more applicable to the outdoors . Popular model (WSSUS) assumptions but not necessarily valid.

Digital Representation – WSSUS Model

Example Tapped Delay Line

An example of a TDL with two internal taps is shown below. The total delay line length is M_3 samples, and the internal taps are located at delays of M_1 and M_2 samples, respectively. The output signal is a linear combination of the input signal $x(n)$, the delay-line output $x(n - M_3)$, and the two tap signals $x(n - M_1)$ and $x(n - M_2)$.



Tapped Delay Line (TDL).

The difference equation of the TDL ABOVE is, by inspection,

$$y(n) = b_0 x(n) + b_{M_1} x(n - M_1) + b_{M_2} x(n - M_2) + b_{M_3} x(n - M_3)$$

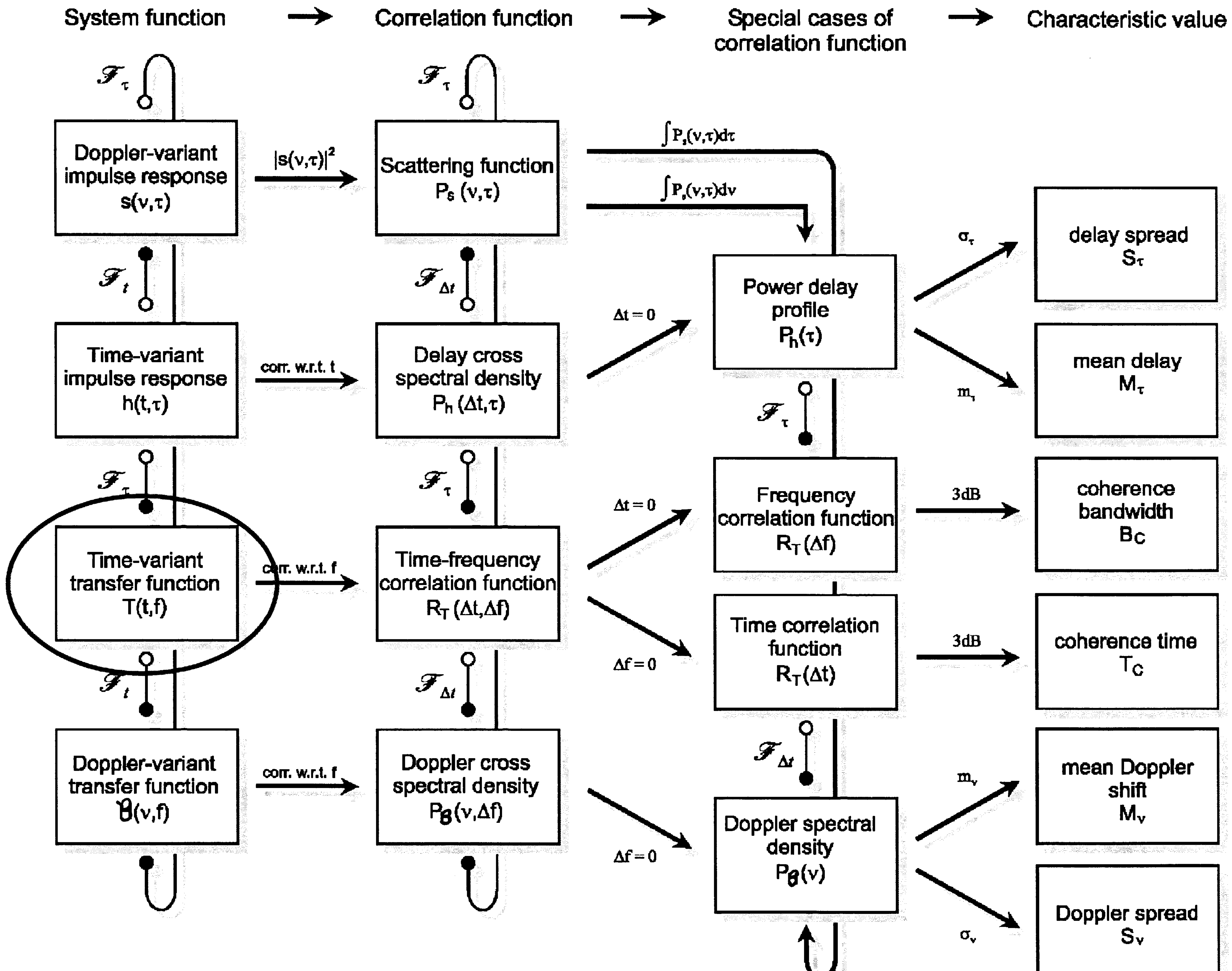
corresponding to the transfer function

$$H(z) = b_0 + b_{M_1} z^{-M_1} + b_{M_2} z^{-M_2} + b_{M_3} z^{-M_3}$$

Condensed parameters

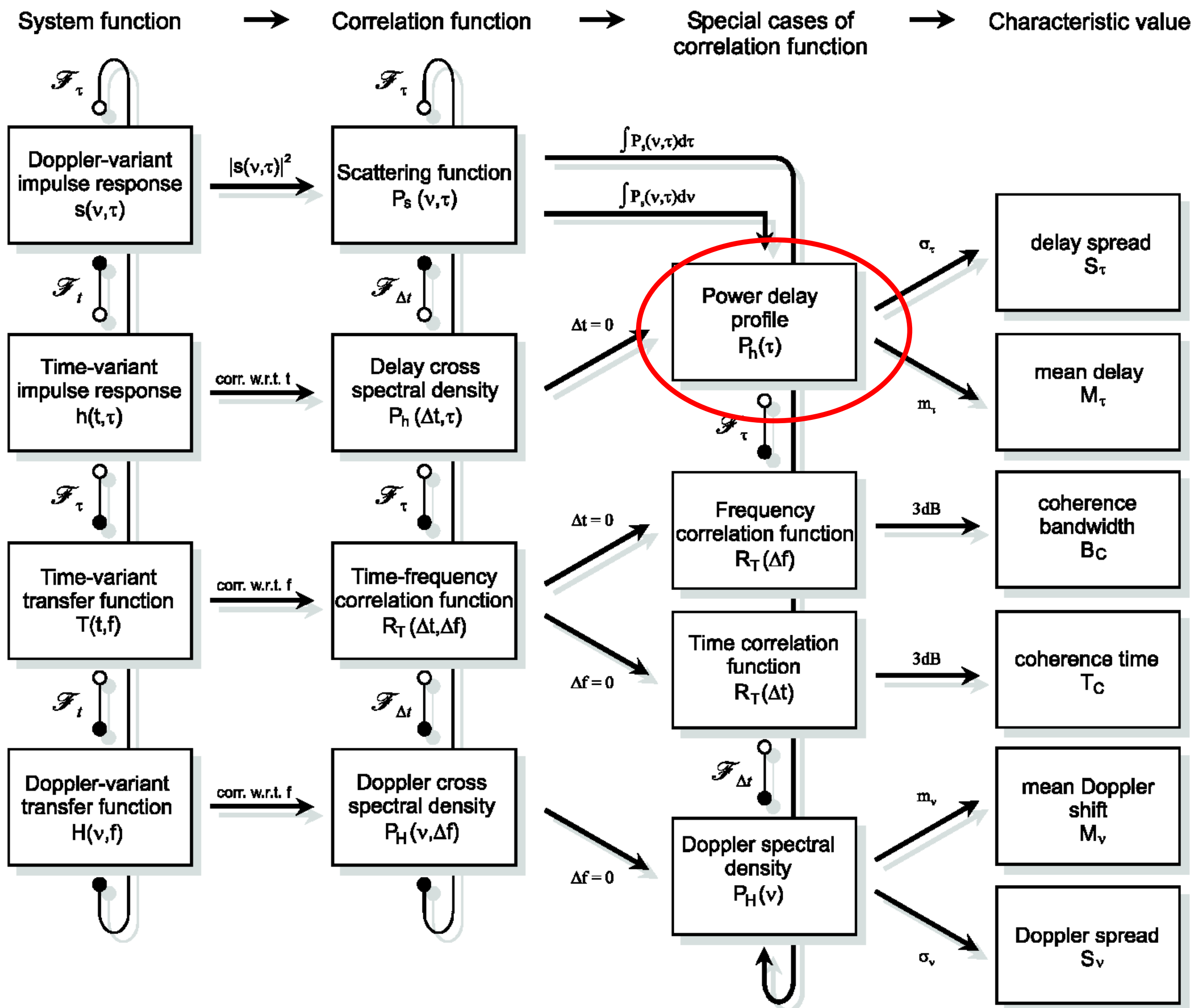
- Correlation functions depend on two variables (still a cumbersome representation)
- For concise characterization of channel, we desire
  - A function depending on one variable or `even better`
  - A single (scalar) parameter How much info is lost? Acceptable?
- Most common condensed parameters integrate over one variable --> single variable
  - Power delay profile integrate the scattering function over the Doppler shift --> how much power arrives at the RX within a bounded delay
  - Rms delay spread
  - Coherence bandwidth Bcoh A better measure for OFDM
  - Doppler spread
  - Coherence time Tcoh How fast a channel changes

Channel measures



CORRECT PG 117

Channel measures



Condensed parameters
Power-delay profile

One interesting channel property is the **power-delay profile** (PDP), which is the expected value of the received power at a certain delay:

$$P(\tau) = \mathrm{E}_t\left[\left|h(t,\tau)\right|^2\right]$$

E_t denotes expectation over time.

For our tapped-delay line we get:

$$P(\tau) = \mathrm{E}_t\left[\left|\sum_{i=1}^{N}\alpha_i(t)\exp\left(j\theta_i(t)\right)\delta(\tau-\tau_i)\right|^2\right]$$

$$= \sum_{i=1}^{N}\mathrm{E}_t\left[\alpha_i^{\,2}(t)\right]\delta(\tau-\tau_i) = \sum_{i=1}^{N}2\sigma_i^{\,2}\delta(\tau-\tau_i)$$

Average power of tap i.

Condensed parameters Power-delay profile (cont.)

We can “reduce” the PDP into more compact descriptions of the channel:

Total power (time integrated):

$$P_m = \int_{-\infty}^{\infty} P(\tau)\, d\tau$$

Average mean delay:

$$T_m = \frac{\int_{-\infty}^{\infty} \tau P(\tau)\, d\tau}{P_m}$$

Average rms delay spread:

Good parameter for FDMA and TDMA systems

$$S = \sqrt{\frac{\int_{-\infty}^{\infty} \tau^2 P(\tau)\, d\tau}{P_m} - T_m}$$

For our tapped-delay line channel:

$$P_m = \sum_{i=1}^{N} 2\sigma_i^2$$

$$T_m = \frac{\sum_{i=1}^{N} \tau_i 2\sigma_i^2}{P_m}$$

$$S = \sqrt{\frac{\sum_{i=1}^{N} \tau_i^2 2\sigma_i^2}{P_m} - T_m}$$

Condensed parameters Frequency correlation

A property closely related to the power-delay profile (PDP) is the **frequency correlation** of the channel. It is in fact the Fourier transform of the PDP:

$$\rho_f(\Delta f) = \int_{-\infty}^{\infty} P(\tau)\exp(-j2\pi\Delta f\,\tau)\,d\tau$$

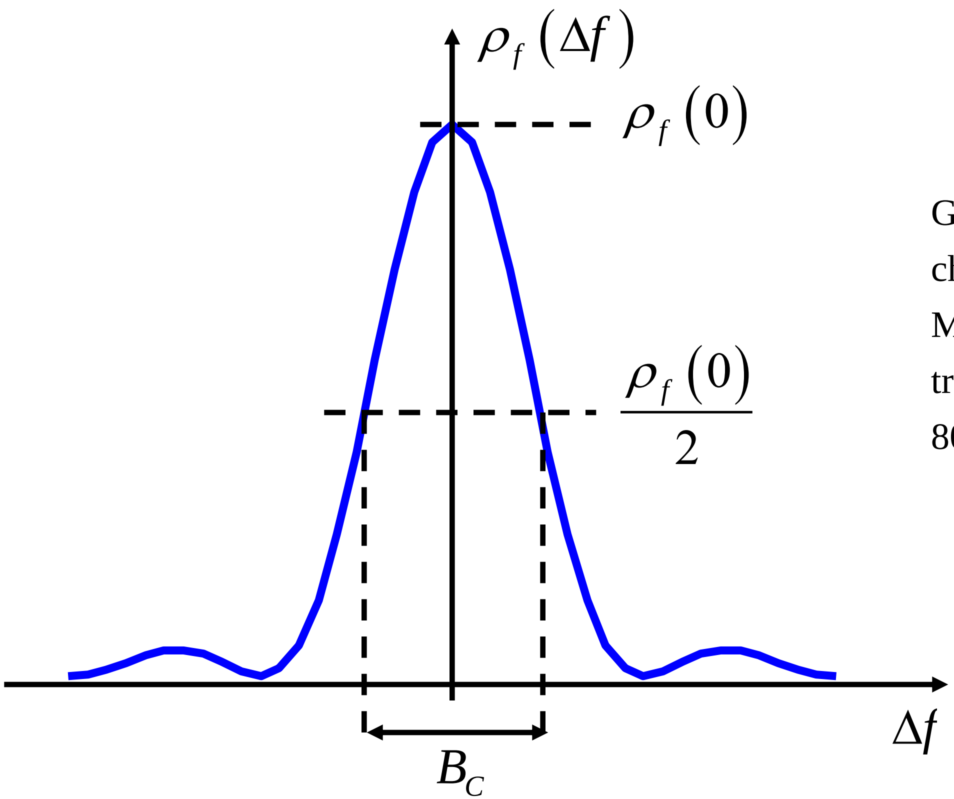
For our tapped delay-line channel we get:

$$\rho_f(\Delta f) = \int_{-\infty}^{\infty} \left(\sum_{i=1}^{N} 2\sigma_i^{\,2}\delta(\tau-\tau_i) \right) \exp(-j2\pi\Delta f\,\tau)\,d\tau$$

$$= \sum_{i=1}^{N} 2\sigma_i^{\,2} \exp(-j2\pi\Delta f\,\tau_i)$$

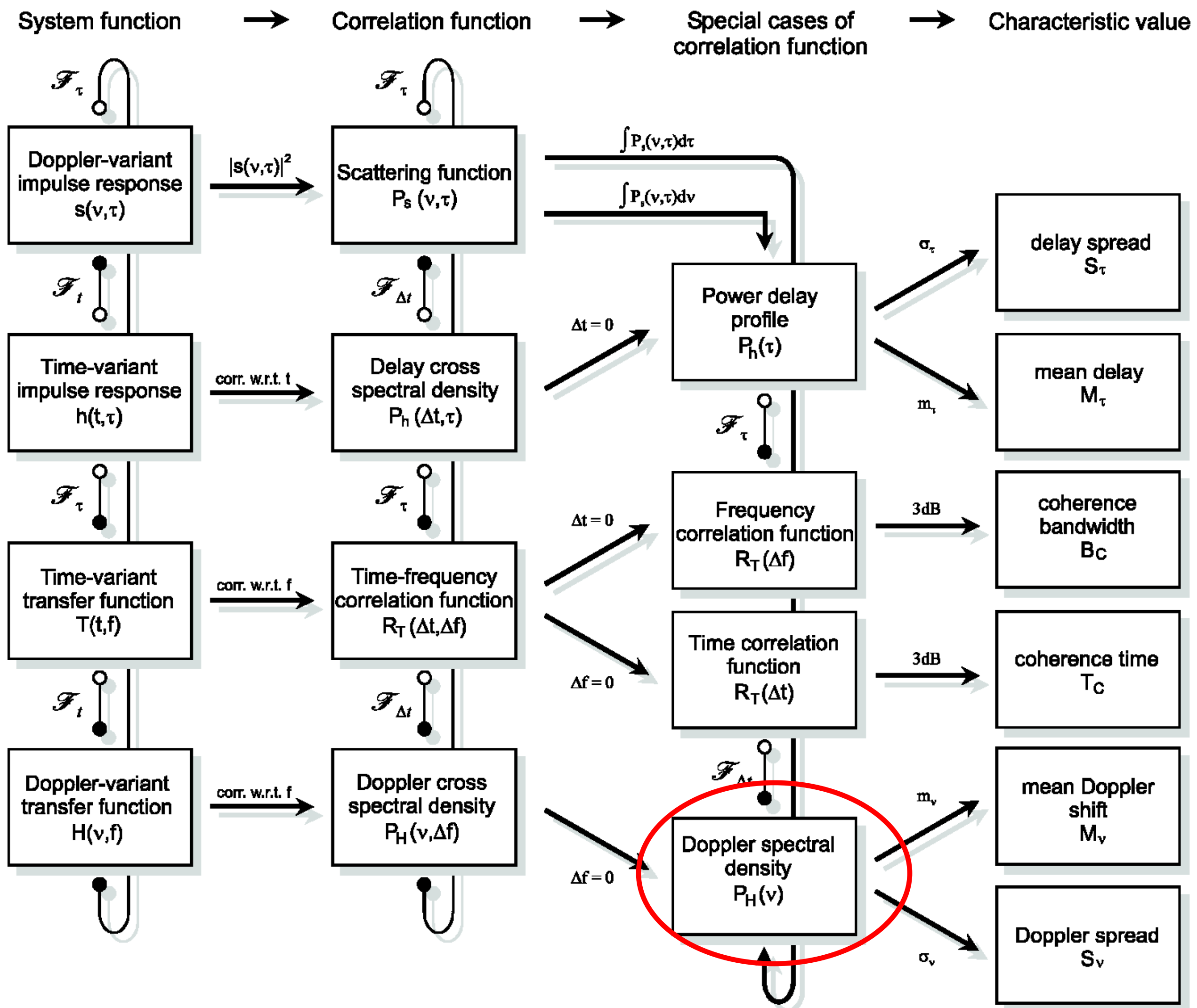
Condensed parameters Coherence bandwidth

Defines the frequency difference that is required so that the frequency correlation coefficient is smaller than some threshold



Good condensed (single value) parameter to reflect the channel properties of Orthogonal Frequency Division Multiplexing (OFDM) systems where the information is transmitted on many parallel carriers. Originated in 802.11a and used in 4G (LTE) cellular today

Channel measures



Condensed parameters The Doppler spectrum

Given the scattering function P_s (doppler spectrum as function of delay) we can calculate a total **Doppler spectrum** of the channel as:

$$P_B(\nu) = \int P_S(\nu,\tau)\,d\tau$$

For our tapped delay-line channel, we have:

Doppler spectrum of tap *i*.

$$P_S(\nu,\tau) = \frac{2\sigma_i^2}{\pi\sqrt{\nu_{i,\max}^2 - \nu^2}}\delta(\tau-\tau_i)$$

$$P_B(\nu) = \int_{-\infty}^{\infty} \frac{2\sigma_i^2}{\pi\sqrt{\nu_{i,\max}^2 - \nu^2}}\delta(\tau-\tau_i)\,d\tau$$

$$= \sum_{i=1}^{N} \frac{2\sigma_i^2}{\pi\sqrt{\nu_{i,\max}^2 - \nu^2}}$$

Condensed parameters
The Doppler spectrum (cont.)

We can "reduce" the Doppler spectrum into more compact descriptions of the channel:

Total power (frequency integrated):

$$P_{B,m} = \int_{-\infty}^{\infty} P_B(\nu)\, d\nu$$

Average mean Doppler shift:

$$T_{B,m} = \frac{\int_{-\infty}^{\infty} \nu P_B(\nu)\, d\nu}{P_{B,m}}$$

Average rms Doppler spread:

$$S_B = \sqrt{\frac{\int_{-\infty}^{\infty} \nu^2 P(\nu)\, d\nu}{P_{B,m}} - T_{B,m}}$$

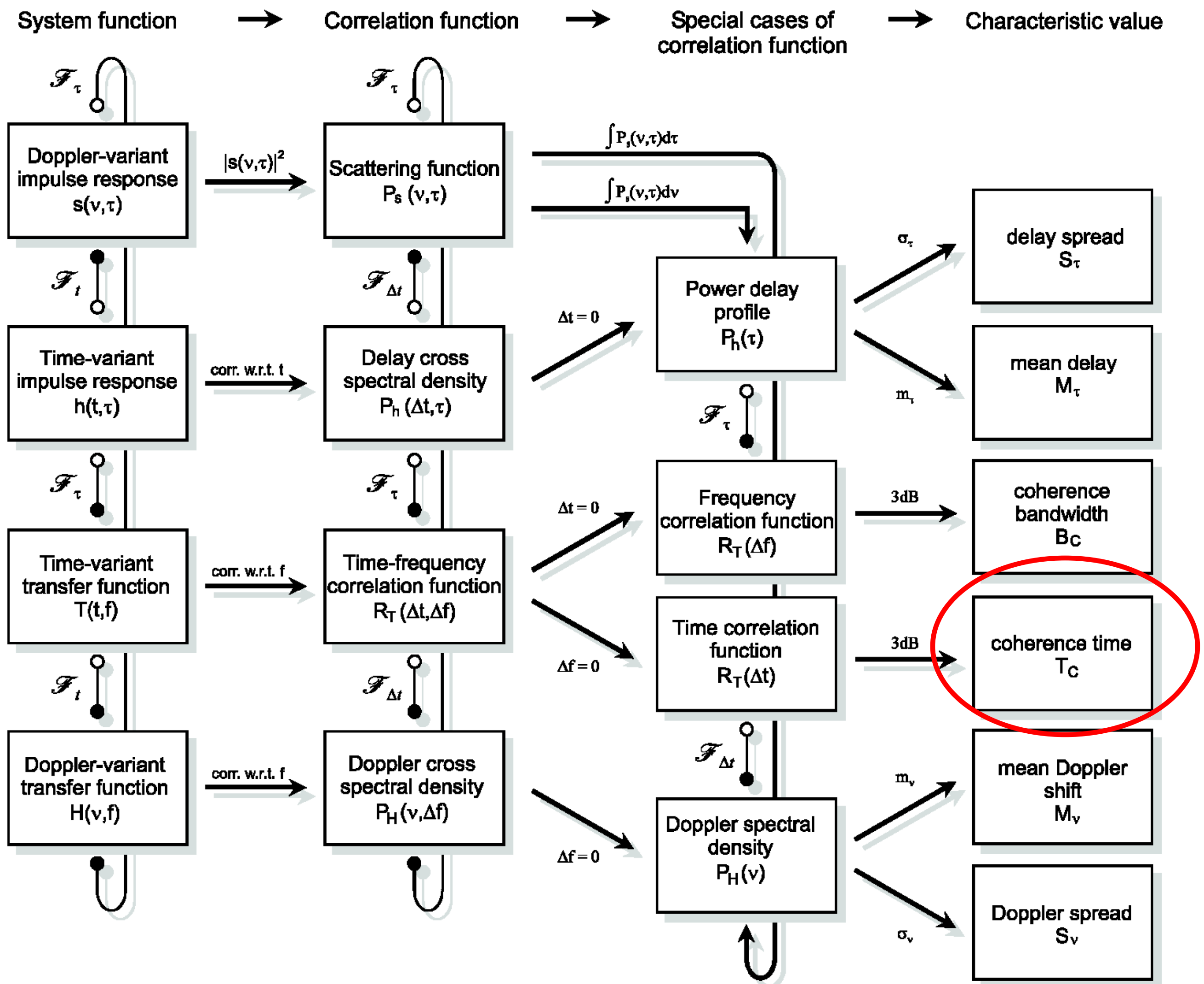
For our tapped-delay line channel:

$$P_{B,m} = \sum_{i=1}^{N} 2\sigma_i^{\,2}$$

$$T_{B,m} = 0$$

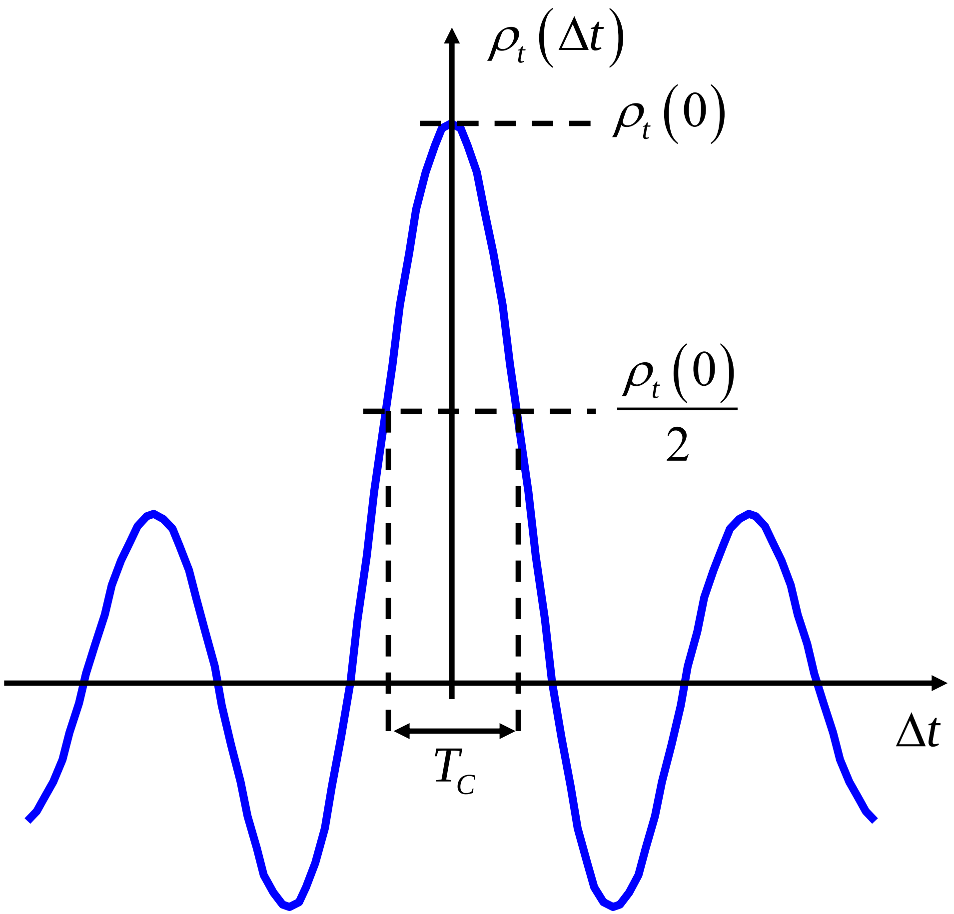
$$S_B = \sqrt{\frac{\sum_{i=1}^{N} \sigma_i^{\,2} \nu_{i,\max}^{\;\;2}}{P_{B,m}}}$$

Channel measures



Condensed parameters Coherence time

Given the time correlation of a channel, we can define the **coherence time** T_C:



A measure of how fast a channel is changing

Fast Fading - Coherence time is much less than symbol duration whereas slow fading is just the opposite.

Fast fading only deals with the rate of change of the channel due to motion (user, IO's, etc.)

Condensed parameters
The time correlation

A property closely related to the Doppler spectrun is the **time correlation** of the channel. It is in fact the inverse Fourier transform of the Doppler spectrum:

$$\rho_t(\Delta t) = \int_{-\infty}^{\infty} P_B(\nu)\exp(j2\pi\nu\Delta t)\,d\nu$$

For our tapped-delay line channel we get

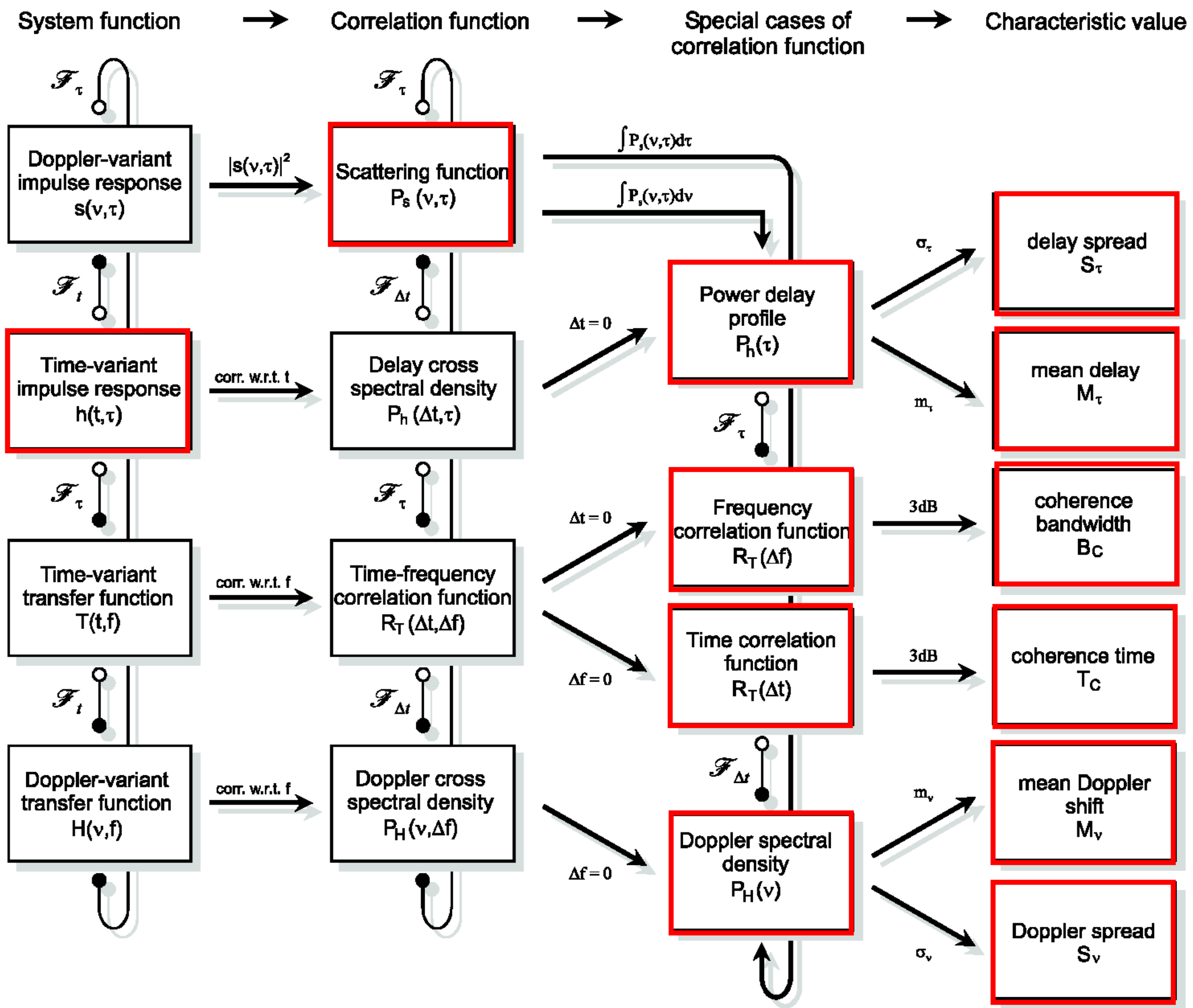
$$\rho_t(\Delta t) = \int_{-\infty}^{\infty} \sum_{i=1}^{N} \frac{2\sigma_i^2}{\pi\sqrt{\nu_{i,\max}^2 - \nu^2}} \exp(j2\pi\nu\Delta t)\,d\nu$$

$$= \sum_{i=1}^{N} \int_{-\infty}^{\infty} \frac{2\sigma_i^2}{\pi\sqrt{\nu_{i,\max}^2 - \nu^2}} \exp(j2\pi\nu\Delta t)\,d\nu$$

$$= \sum_{i=1}^{N} 2\sigma_i^2 J_0(2\pi\nu_{i,\max}\Delta t)$$

Sum of time correlations for each tap.

It's much more complicated than what we have discussed!



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Double directional impulse response DDIR

TX position

RX position

number of multipath components for these positions

$$h(t,\vec{r}_{\mathrm{TX}},\vec{r}_{\mathrm{RX}},\tau,\Omega,\Psi) = \sum_{\ell=1}^{N(\vec{r})} h_\ell(t,\vec{r}_{\mathrm{TX}},\vec{r}_{\mathrm{RX}},\tau,\Omega,\Psi)$$

direction-of-arrival

delay

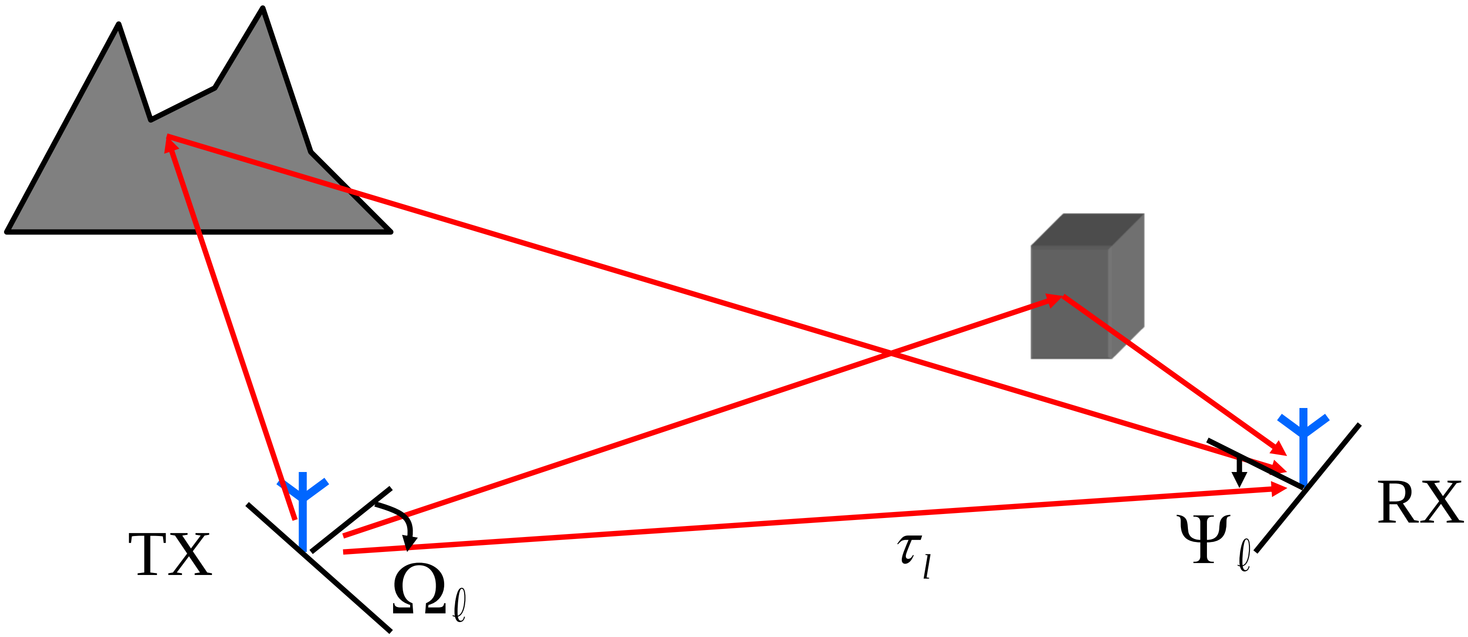
direction-of-departure

$$h_\ell(t,\vec{r}_{\mathrm{TX}},\vec{r}_{\mathrm{RX}},\tau,\Omega,\Psi) = |a_\ell| e^{j\varphi_\ell}\delta(\tau-\tau_\ell)\delta(\Omega-\Omega_\ell)\delta(\Psi-\Psi_\ell)$$

Directional properties important for spatial diversity and multiple/directional antennas, e.g., MIMO and 802.11n

The impact of the multipath components (MPCs) depends on the antennas used - changing antennas changes the impulse response

Physical interpretation



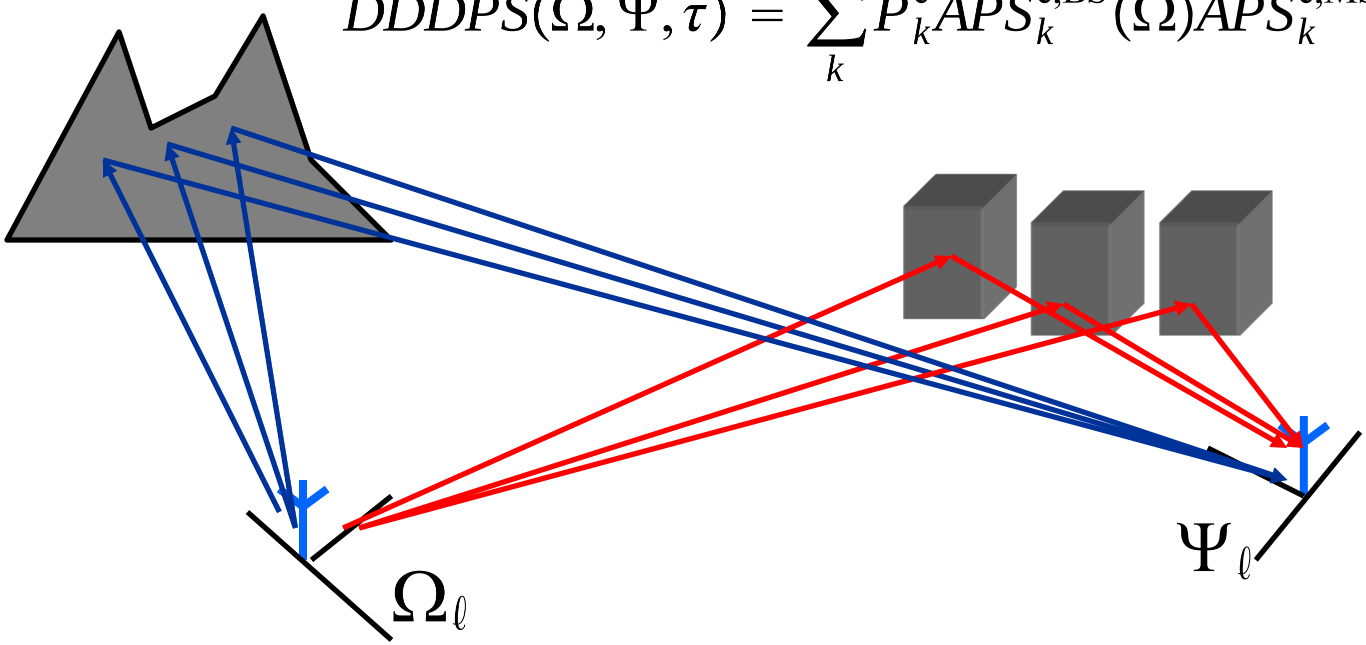
Directional models

- The double directional delay power spectrum is sometimes factorized w.r.t. DoD, DoA and delay.

$$DDDPS(\Omega, \Psi, \tau) = APS^{\mathrm{BS}}(\Omega) APS^{\mathrm{MS}}(\Psi) PDP(\tau)$$

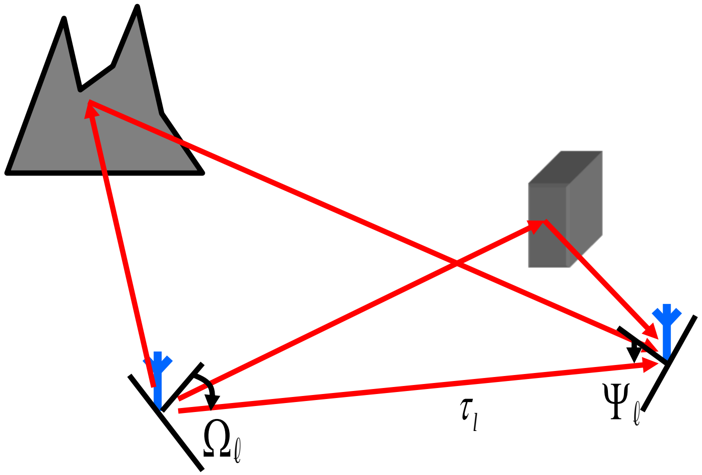
- Often in reality there are groups of scatterers with similar DoD and DoA – clusters

$$DDDPS(\Omega, \Psi, \tau) = \sum_k P_k^{\mathrm{c}} APS_k^{\mathrm{c,BS}}(\Omega) APS_k^{\mathrm{c,MS}}(\Psi) PDP_k^{\mathrm{c}}(\tau)$$



Angular spread

$$E\{s^*(\Omega,\Psi,\tau,v)s(\Omega',\Psi',\tau',v')\} \;=\; P_s(\Omega,\Psi,\tau,v)\delta(\Omega-\Omega')\delta(\Psi-\Psi')\delta(\tau-\tau')\delta(v-v')$$



double directional delay power spectrum

$$DDDPS(\Omega,\Psi,\tau) \;=\; \int \mathrm{P_s}(\Psi,\Omega,\tau,v)\mathrm{d}v$$

angular delay power spectrum

$$ADPS(\Omega,\tau) \;=\; \int DDDPS(\Psi,\Omega,\tau)G_{\mathrm{MS}}(\Psi)d\Psi$$

angular power spectrum

$$APS(\Omega) \;=\; \int \mathrm{APDS}(\Omega,\tau)\mathrm{d}\tau$$

power

$$P \;=\; \int \mathrm{APS}(\Omega)\mathrm{d}\Omega$$

Matlab/Simulink - Communications System Toolbox

- Key Features
- System Design, Characterization, and Visualization
- Analog and Digital Modulation
- Source and Channel Coding
- Channel Modeling and RF Impairments
- Equalization and Synchronization
- Stream Processing in MATLAB and Simulink
- Implementing a Communications System

Key Features

- Algorithms for designing the physical layer of communications systems, including source coding, channel coding, interleaving, modulation, channel models, MIMO, equalization, and synchronization
- GPU-enabled System objects for computationally intensive algorithms such as Turbo, LDPC, and Viterbi decoders
- Interactive visualization tools, including eye diagrams, constellations, and channel scattering functions
- Graphical tool for comparing the simulated bit error rate of a system with analytical results
- Channel models, including AWGN, Multipath Rayleigh Fading, Rician Fading, MIMO Multipath Fading, and LTE MIMO Multipath Fading
- Basic RF impairments, including nonlinearity, phase noise, thermal noise, and phase and frequency offsets
- Algorithms available as MATLAB functions, MATLAB System objects, and Simulink blocks
- Support for fixed-point modeling and C and HDL code generation