DSP First

Second Edition

Modified by TL Harman Spring 2021
For CENG 3315

Review1_Basics.pptx

CENG 3315

Pearson

Signal Processing Ch1

Analog Signals – s(t); t is time

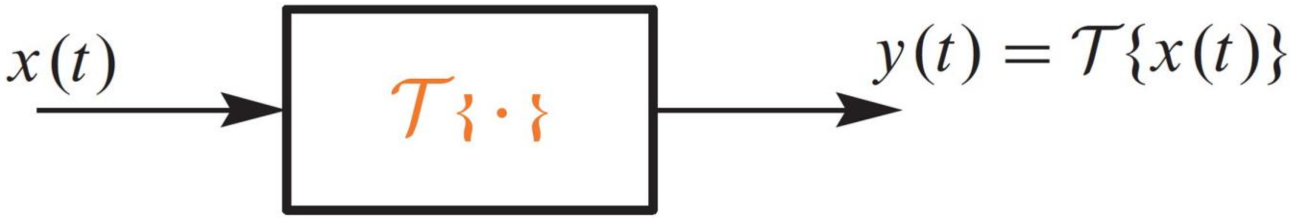
- Analogous to the actual physical signal
- Typically continuous – temperature, etc. Values are real numbers

Discrete-time signals s[n]=s(nTs); n= 1,2,...

- Ts is the sampling period in seconds
- Amplitude of s(nTs) is a real number
- Signal is Quantized in Time!

Mathematical Representation of Systems (2 of 3)

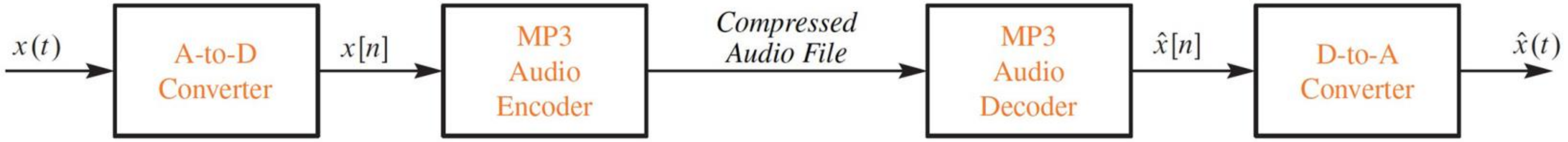
Figure 1-5: Block Diagram Representation of a Continuous-time System



If System is LINEAR, y(a $x1(t)$ $+b$ $x2(t)) = a\ y(x1(t)) + b\ y(x2(t))$

Y = mX Linear Y1=mX+b NOT Linear (Check by doubling X, does Y or Y1 double?)

Figure 1-7: Simplified Block Diagram for MP3 Audio Compression and Playback System



Tuning-Fork Experiment (1 of 2)

SINUSOIDS REALLY EXIST IN NATURE 😊

Figure 2-2: Picture of a Tuning Fork for 440 Hz

TRY IT ON THE PHONE!

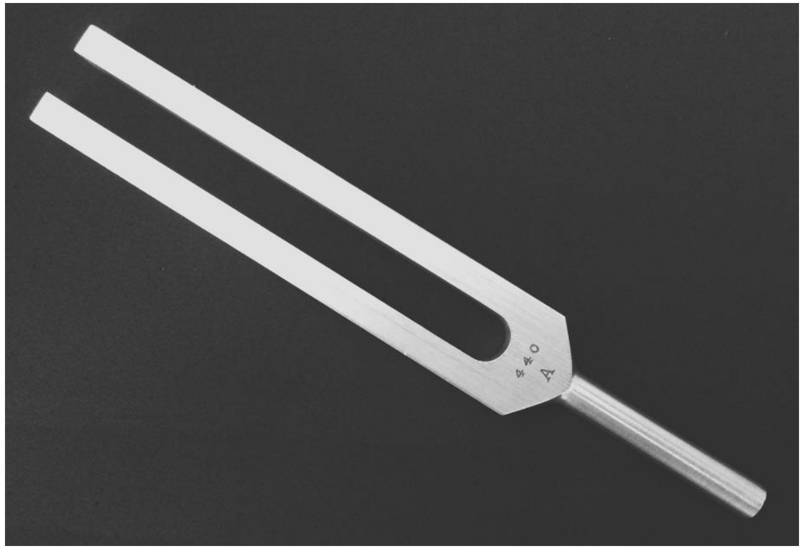
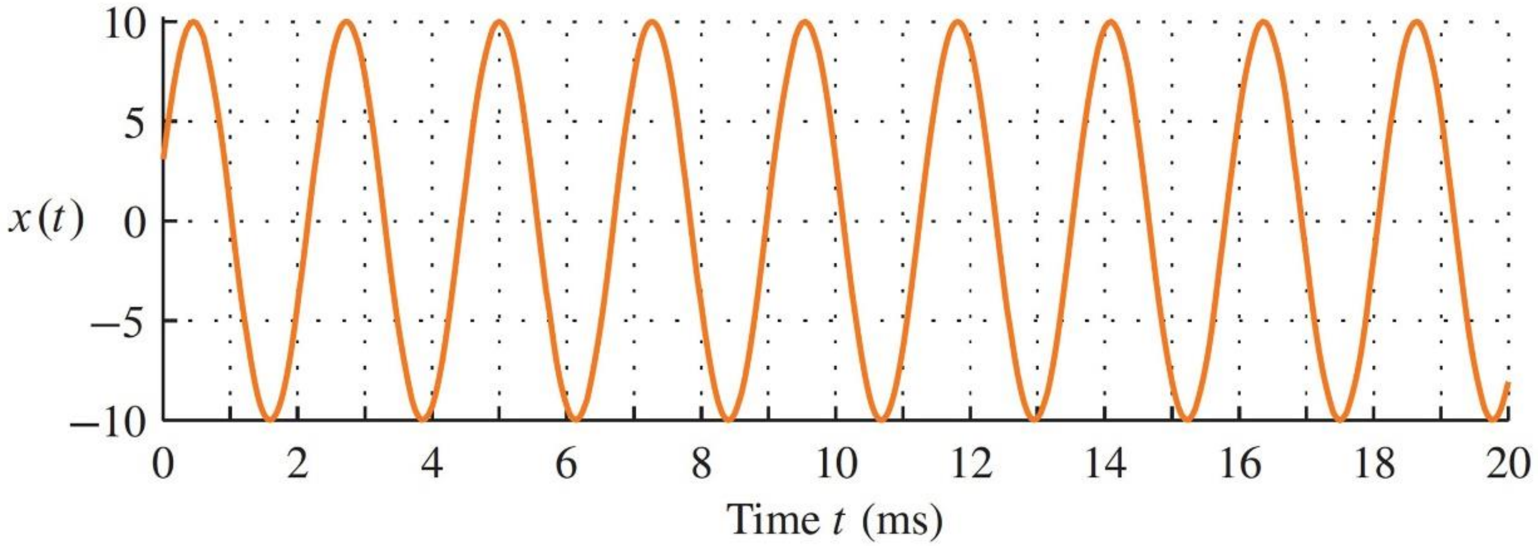


Figure 2-1: Sinusoidal Signal Generated From the Formula:

$$X(t) = 10\cos\left(2\pi(440)t - 0\cdot 4\pi\right)$$



T= (1/440) = .002273 sec
= 2.273 ms
T = Period in Seconds

Cosine wave shifted to the right by 0.4 pi radians (72 degrees) or 0.2x2pi.
(1/440)*(0.2) = .000455 or .455 ms. This is 2/10 of the Perios

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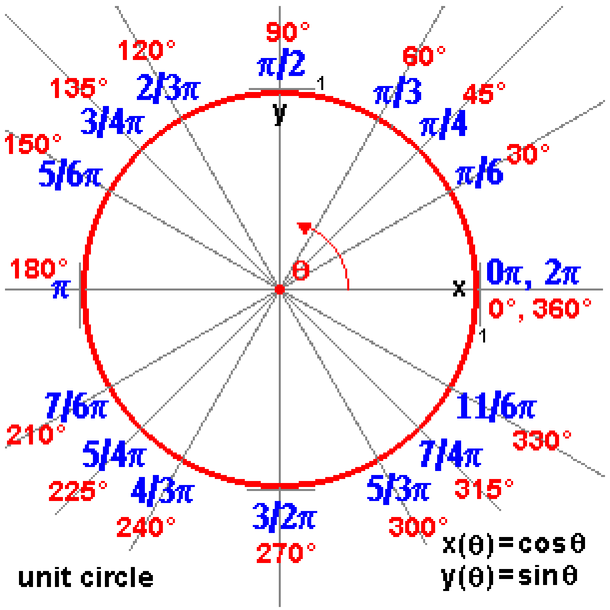
Chapter 2 TLH Sinusoids
KNOW THIS!



Angle θ				
Degrees	Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	undefined
180	π	0	-1	0
270	$\frac{3\pi}{2}$	-1	0	undefined
360	2π	0	1	0

Table 2-1: Basic Properties of the Sine and Cosine Functions

Property	Equation
Equivalence	$\sin\theta = \cos(\theta - \pi/2)$ or $\cos(\theta) = \sin(\theta + \pi/2)$
Periodicity	$\cos(\theta + 2\pi k) = \cos\theta$, when k is an integer
Evenness of cosine	$\cos(-\theta) = \cos\theta$
Oddness of sine	$\sin(-\theta) = -\sin\theta$
Zeros of sine	$\sin(\pi k) = 0$, when k is an integer
Ones of cosine	$\cos(2\pi k) = 1$, when k is an integer
Minus ones of cosine	$\cos[2\pi(k + \frac{1}{2})] = -1$, when k is an integer

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Table 2-2: Some Basic Trigonometric Identities

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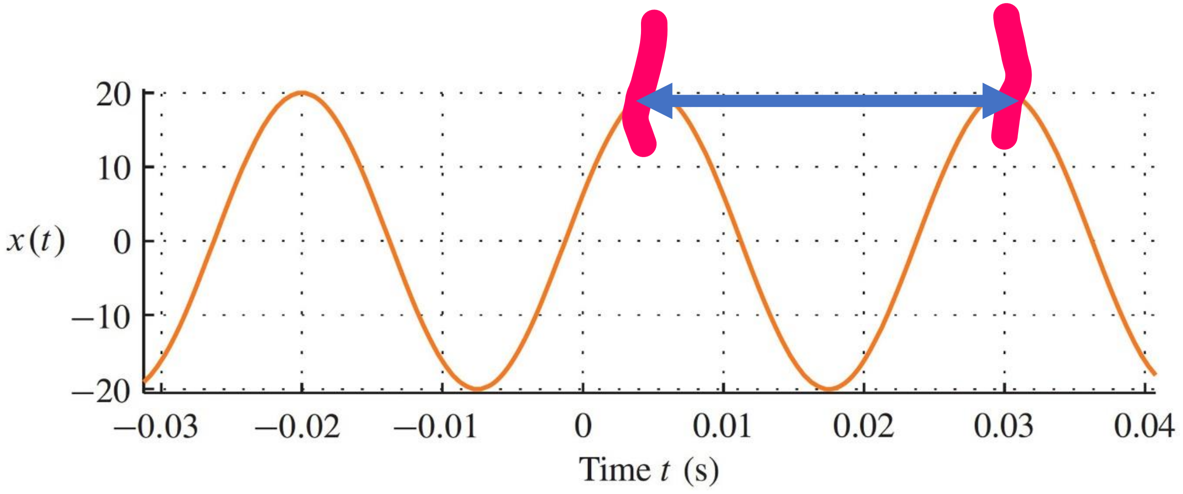
Number	Equation
1	$\sin^2\theta + \cos^2\theta = 1$
2	$\cos 2\theta = \cos^2\theta - \sin^2\theta$
3	$\sin 2\theta = 2\sin\theta\cos\theta$
4	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
5	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Relation of Frequency to Period (1 of 2)

Time-Domain versus Frequency-Domain

Figure 2-6: Sinusoidal signal with parameters *A* = 20, $\Omega_0 = 2\pi(40), F_0 = 40 \text{ Hz, and}$
$\phi = -0.4\pi rad.$



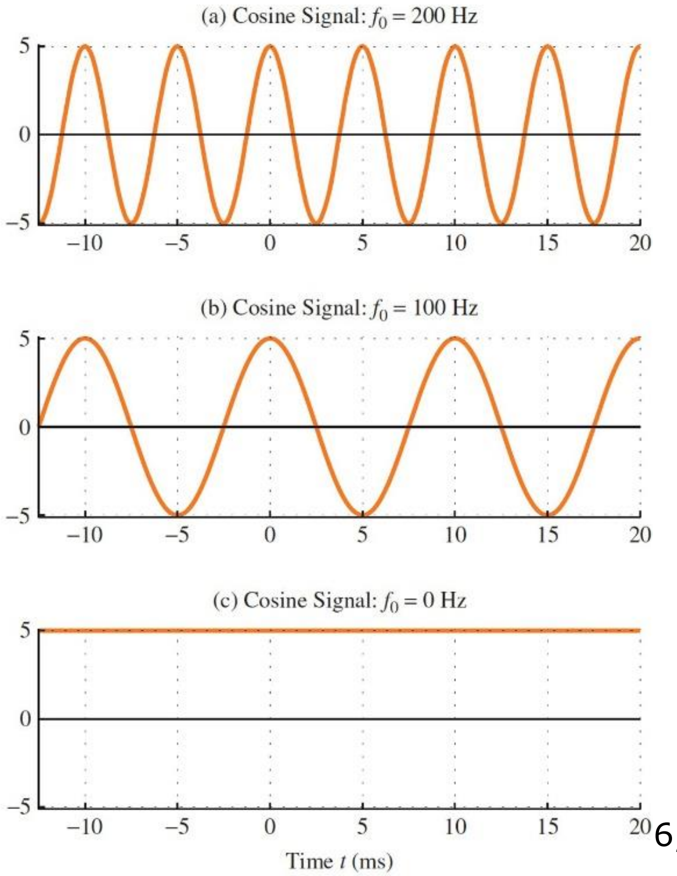
1/40 = 0.025 sec

Relation of Frequency to Period (2 of 2)

Figure 2-7: Cosine Signals $X(t) = 5Cos(2\pi f_0 t)$ for Several Values of $F_0 : (A) F_0 = 200\text{ Hz}; (B) F_0 = 100\text{ Hz}; (C) F_0 = 0$

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Seconds



$$\omega = 2\pi f \quad Radians/sec$$

f in Hz or cycles/second

PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(\underline{0.3\pi}t + 1.2\pi)$$

- Determine **period**:

$$\boxed{T = 2\pi/\omega = 2\pi/0.3\pi = 20/3}$$ SECONDS

- Determine a **peak** location by solving

$$\boxed{\begin{array}{l}(\omega t + \varphi) = 0 \\ 0.3\pi t + 1.2\pi = 0\end{array}}$$

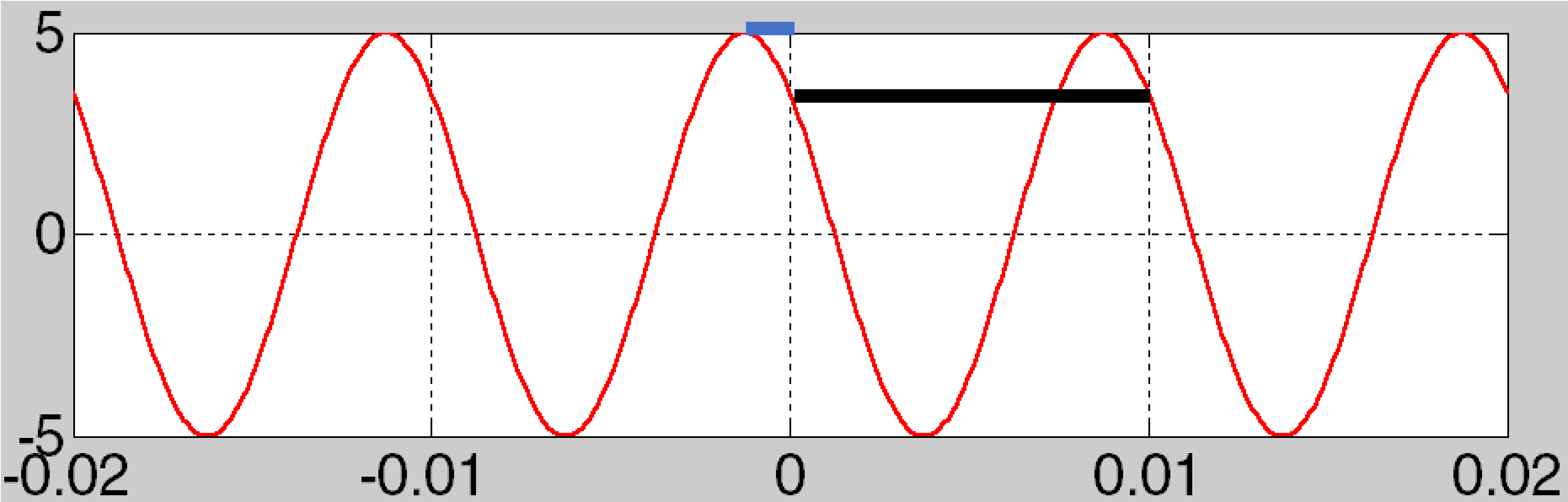
- **Peak at t= -4 SEC.**

```
% Lecture Ch2_2
%
% 5*cos(0.3*pi*t +1.2*pi)
% Find the radian frequency, the frequency, and period
omega = 0.3*pi              %  0.9425 rad/sec
omega_deg = 0.3*180         %  54 degrees per second
f = omega/(2*pi)            %  0.1500 Hertz (cycles/sec)
T = 1/f                     %  6.6667 seconds in a period
```

```
%
% Find phase shift and time shift 0.3*pi*t+1.2*pi =0
%
phi_shift = 1.2*pi       %  3.7699 rad
tpeak= -1.2*pi/(0.3*pi) % -4 seconds (shift to LEFT)
% CHECK 1.2*pi/2*pi  and -4/T
rad_shift_ratio = -1.2*pi/(2*pi)  %  0.6000  (60%)
t_shift_ratio  = -4/T                 %  0.6000  Same ratio
```

Same ratio

(A, ω, ϕ) from a PLOT



$$T = \frac{0.01\text{sec}}{1\,\text{period}} = \frac{1}{100}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.01} = 200\pi$$

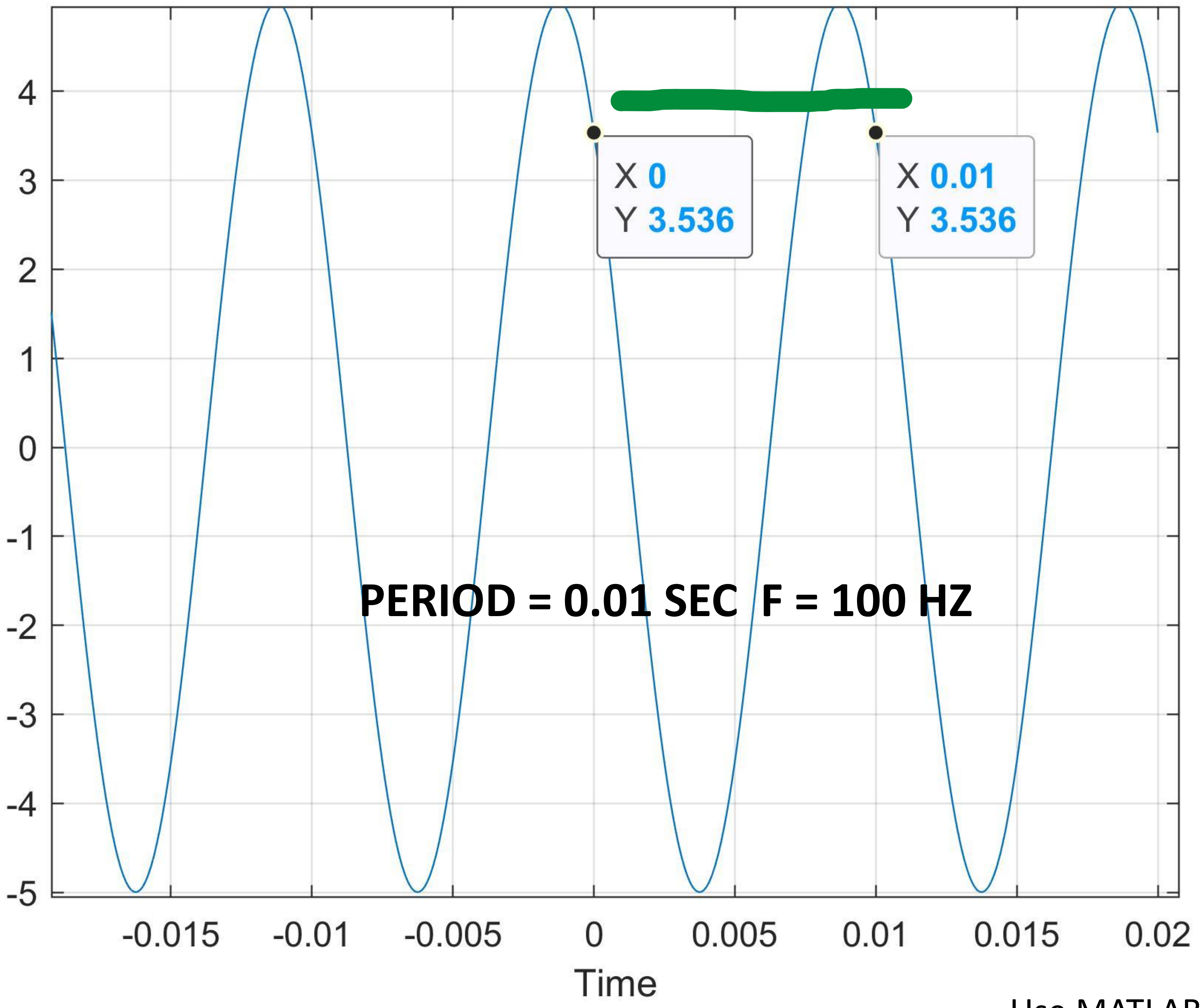
$$t_m = -0.00125\text{sec}$$

$$\varphi = -\omega t_m = -(200\pi)(t_m) = 0.25\pi$$

```
%
format long    % Get full precision
figure(1)
t=-0.02:.0001:.02;
y=5*cos(200*pi*t + 0.25*pi);

plot(t,y),grid,xlabel('Time')


t_shift = -.25*pi/(2*pi)*(1/100)  % -0.00125000000000000 s

sprintf('%0.5f', t_shift)     %  ans = '-0.00125'
```

IF YOU HAVE THE DATA VALUES – MOST SAMPLING O-SCOPES CAN PROVIDE IT!

Use MATLAB tools on the plot



Use MATLAB tools on the plot

Figure 2-9: Plotting the 40-hz Sampled Cosine 2.8(b) for (A) $T_s = 0 \cdot 005\ S$; (B) $\mathrm{T_s} = 0 \cdot 0025\ \mathrm{S}$;(C) $\mathrm{T_s} = 0 \cdot 0005\ \mathrm{S}$

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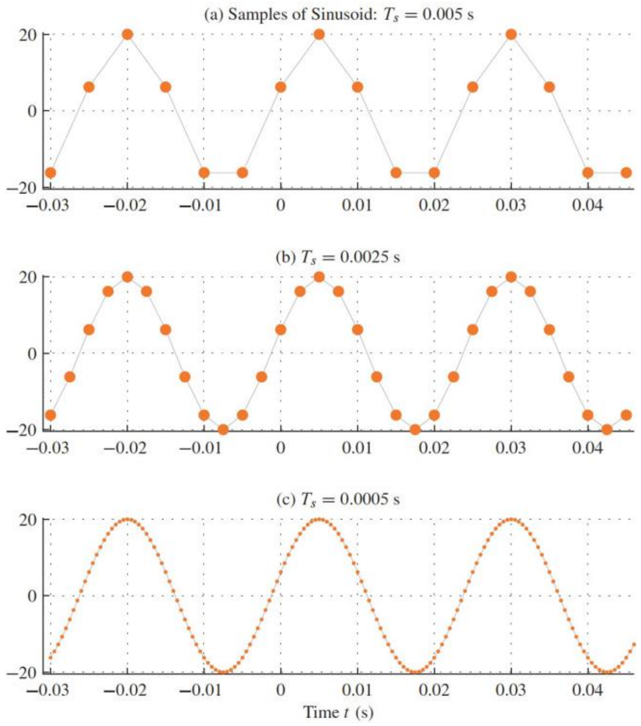

STRAIGHT LINE INTERPOLATION

This is NOT Shannon's Sampling Theorem

PHASOR ADDITION RULE

Sinusoids of the same frequency!

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \varphi_k)$$

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$$= A\cos(\omega_0 t + \varphi)$$

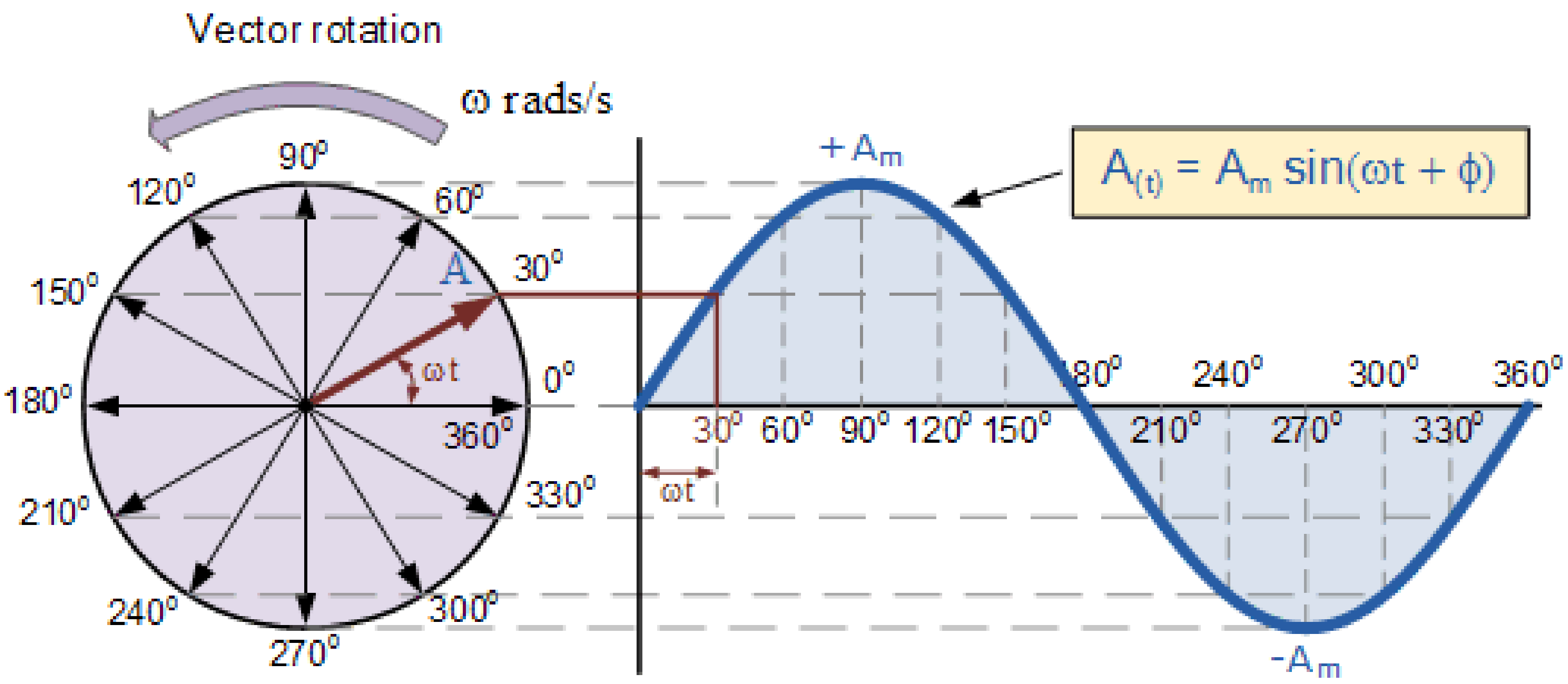
<u>Get the new complex amplitude by complex addition</u>

Find Amplitude and
Phase – ω is Known!

$$\sum_{k=1}^{N} A_k e^{j\varphi_k} = Ae^{j\varphi}$$

https://www.electronics-tutorials.ws/accircuits/phasors.html

Basically a rotating vector, simply called a "**Phasor**" is a scaled line whose length represents an AC quantity that has both magnitude ("peak amplitude") and direction ("phase") which is "frozen" at some point in time.

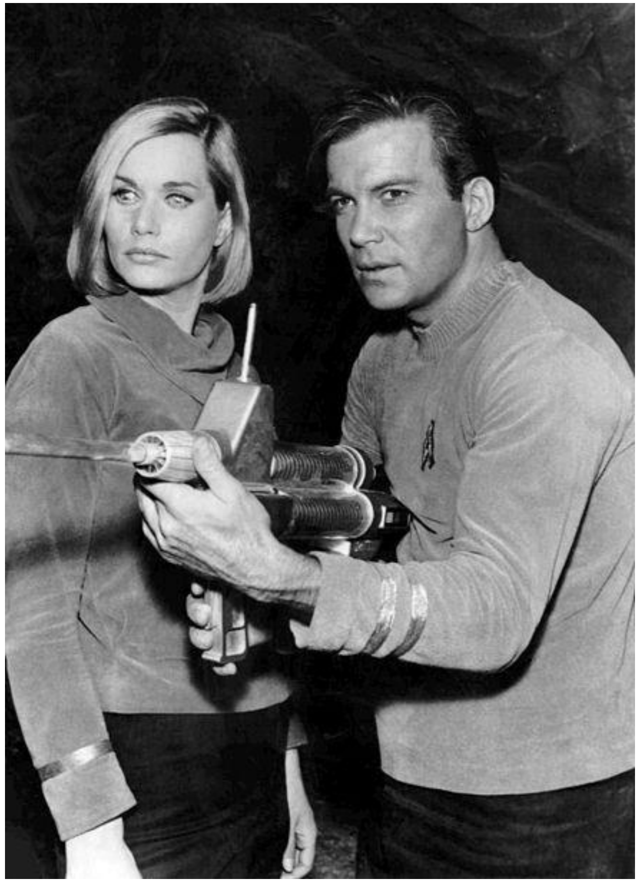


Rotating Phasor

Sinusoidal Waveform in the Time Domain

Phasors Not Phasers

Captain Kirk &
Sally Kellerman



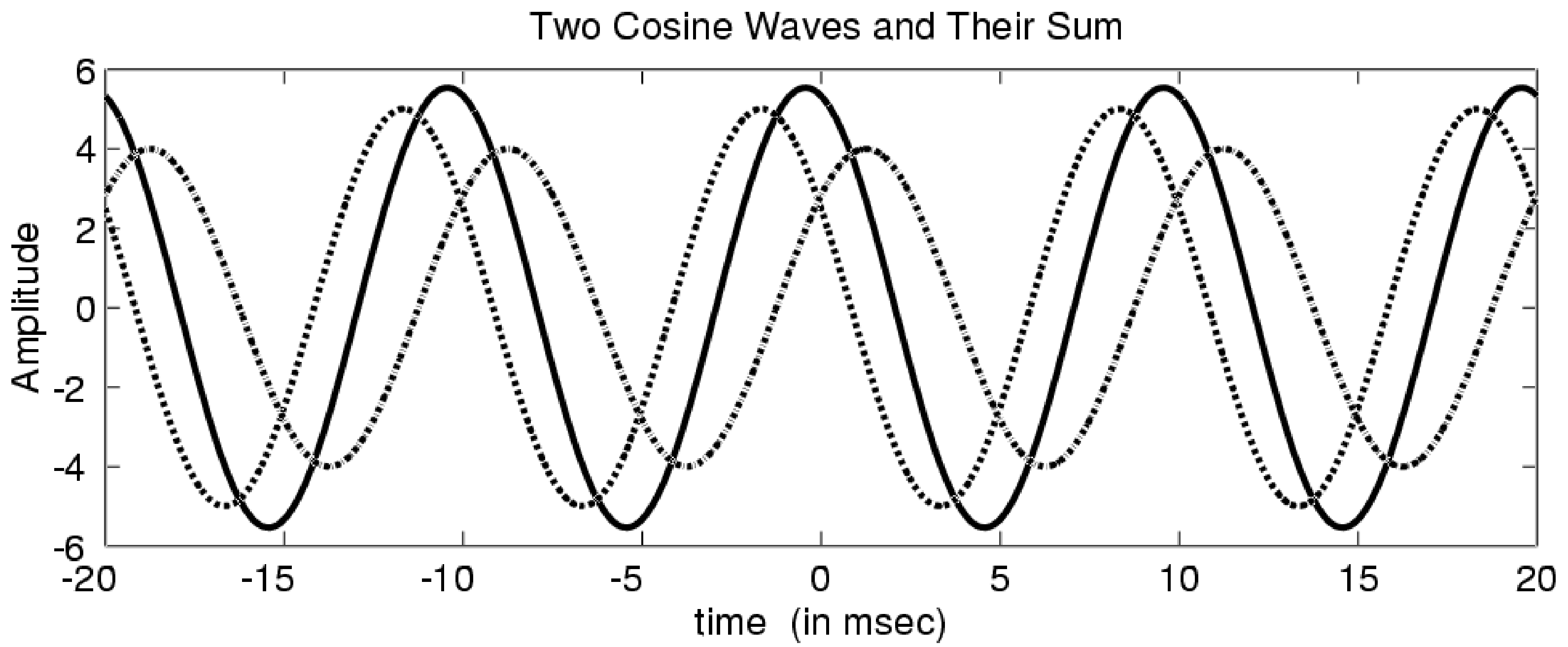
STAR TREK

Linear Systems –
Linear Time Invariant(LTI)
Hold the frequency term – Add it in after determining the Complex Amplitude!

In a sinusoidally excited LTI circuit,
all branch voltages and currents
are **sinusoids at the same frequency**
as the excitation signal.

WANT to ADD SINUSOIDS

- **Main point to remember**: Adding sinusoids of common frequency results in sinusoid with **SAME** frequency



It is important to go between the sinusoidal form and the phasor form. Assume the frequencies of the sinusoids are the same. We know the sum of such sinusoids will be a sinusoid of the same frequency.

Take the 10 Hz sinusoids (DSP First Page 31)

$$\begin{aligned} x_1(t) &= 1.7\cos(20\pi t + 70\pi/180) \\ x_2(t) &= 1.9\cos(20\pi t + 200\pi/180) \end{aligned} \tag{2}$$

The phasors involved are

$$\begin{aligned} X_1 &= A_1 e^{j\phi_1} = 1.7\ e^{j70\pi/180} \\ X_2 &= A_2 e^{j\phi_2} = 1.9\ e^{j200\pi/180} \end{aligned} \tag{3}$$

Then the steps to form $x_3(t) = x_1(t) + x_2(t)$ is as follows:

1. Convert both phasors to Rectangular form
2. Add the real and imaginary parts
3. Convert back to polar for the phasor X_3
4. Convert to the cosine form.

Can we do these steps?
Sure but MATLAB saves us!

```
% Convert Phasor to rectangular
format short
x1=1.7*exp(j*70*pi/180)
  % x1= 0.5814+ 1.5975i
x2=1.9*exp(j*200*pi/180)
 % x2 = -1.7854 - 0.6498i
x3=x1+x2  % x3 = -1.2040 + 0.9476i
 % Convert x3 to polar
magx3=abs(x3)  % magx3   = 1.5322
x3theta=angle(x3) % x3theta = 2.4748 rad
thetadeg=x3theta*180/pi
% thetadeg =   141.7942 degrees
```

Piece of Cake!

Convert Sinusoids to Phasors

- Each sinusoid → Complex Amp

$$1.7\cos(20\pi t + 70\pi/180) \rightarrow 1.7e^{j70\pi/180}$$

$$1.9\cos(20\pi t + 200\pi/180) \rightarrow 1.9e^{j200\pi/180}$$

$$1.7e^{j70\pi/180} + 1.9e^{j200\pi/180} = ?$$

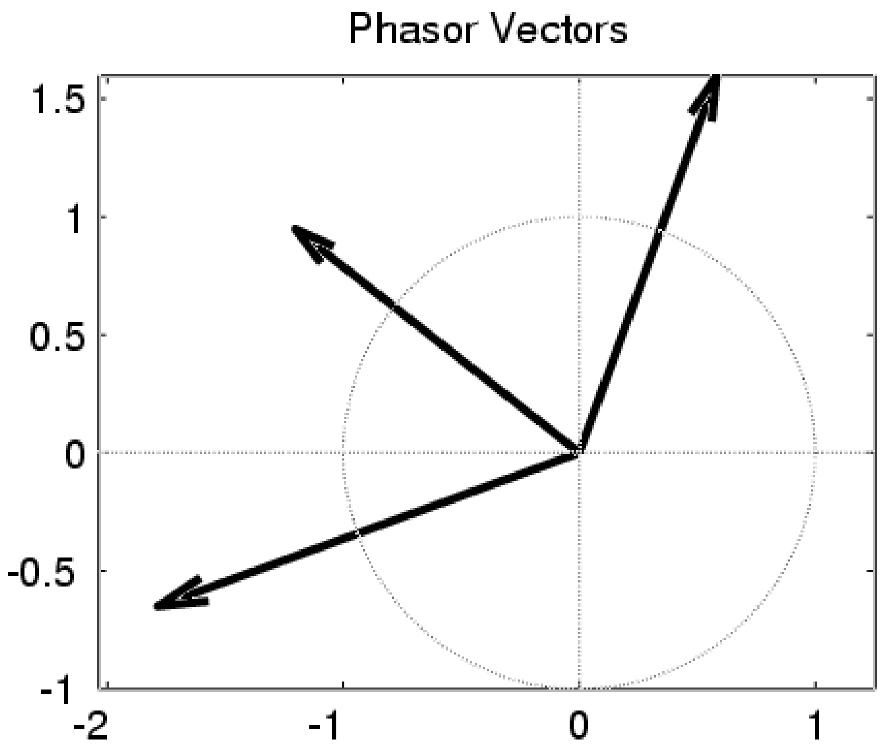
$$1.532e^{j141.79\pi/180}$$

$$\rightarrow 1.532\cos(20\pi t + 141.79\pi/180)$$

Phasor Add: Numerical

- Convert Polar to Cartesian
  - $X_1 = 0.5814 + j1.597$
  - $X_2 = -1.785 - j0.6498$
  - sum =
  - $X_3 = -1.204 + j0.9476$

- Convert back to Polar
  - $X_3 = 1.532$ at angle $141.79\pi/180$
  - This is the sum



ADDING SINUSOIDS IS COMPLEX ADDITION

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$
$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

VECTOR (PHASOR) ADD