DSP First 2/e

CENG 3315
Review3_FrequencySpecturm.pptx

Lecture 5A:
Operations on the Spectrum

Section 3-3 Page 61

LECTURE OBJECTIVES

- **Operations** on a time-domain signal x(t) have a **SIMPLE form** in the frequency-domain

- SPECTRUM Representation has lines at:

$$(A_k, \varphi_k, f_k)$$

- Represents Sinusoid with DIFFERENT Frequencies

$$x(t) = \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \varphi_k)$$

POSITIVE SPECTRUM

Euler's formula says:

$$e^{it} = \cos(t) + i\sin(t)$$

and

$$e^{-it} = \cos(t) - i\sin(t).$$

By adding and subtracting we get:

$$\cos(t) = \frac{e^{it} + e^{-it}}{2}$$

and

$$\sin(t) = \frac{e^{it} - e^{-it}}{2i}.$$

Please take note of these formulas we will use them frequently!

General Spectrum USING Aexp[j(wt+Φ)]

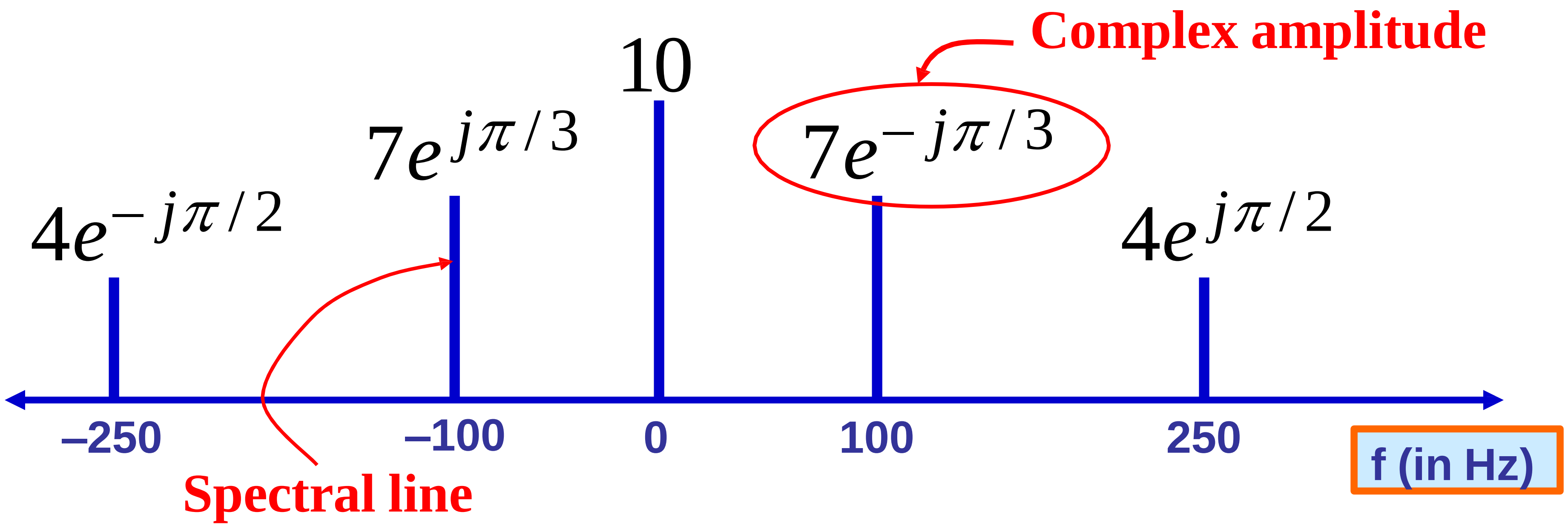
- 2M + 1 spectrum components:

$$x(t) = \sum_{k=-M}^{M} a_k e^{j2\pi f_k t}$$

- At $f = f_k$ the complex amplitude is a_k
  - usually, for real x(t) $f_0 = 0$

FREQUENCY DIAGRAM

- Used to visualize relationship between frequencies, amplitudes and phases
- Plot Complex Amplitude vs. Freq



OPERATIONS on SPECTRUM

- Adding DC, or amplitude scaling
- Adding two (or more) signals
- Time-Shifting
  - Multiply in frequency by complex exponential
- Differentiation of x(t)
  - Multiply in frequency-domain by ($j\omega$)
- Frequency Shifting
  - Multiply in time-domain by sinusoid

Scaling or Adding a constant

- Adding DC

$$x(t) + c = \sum_{k \neq 0} a_k e^{j2\pi f_k t} + \underbrace{a_0 e^{j2\pi(0)t} + c e^{j2\pi(0)t}}_{\text{new DC is } a_0 + c}$$

- Scaling

$$\gamma x(t) = \gamma \sum_{k=-M}^{M} a_k e^{j2\pi f_k t} = \sum_{k=-M}^{M} (\gamma a_k) e^{j2\pi f_k t}$$

Time Shifting x(t)

- Time Shifting

$$x(t-\tau_d) = \sum_{k=-M}^{M} a_k e^{j2\pi f_k (t-\tau_d)} = \sum_{k=-M}^{M} \underbrace{(a_k \boxed{e^{-j2\pi f_k \tau_d}})}_{b_k} e^{j2\pi f_k t}$$

$$y(t) = \sum_{k=-M}^{M} b_k e^{j2\pi f_k t}$$

- Multiply Spectrum complex amplitudes by a complex exponential

Differentiating x(t)

- Take **derivative** of the Signal x(t)

$$\tfrac{d}{dt}x(t) = \sum_{k=-M}^{M} a_k (j2\pi f_k) e^{j2\pi f_k t} = \sum_{k=-M}^{M} \underbrace{\boxed{(j2\pi f_k)}\, a_k}_{b_k} e^{j2\pi f_k t}$$

$$y(t) = \sum_{k=-M}^{M} b_k e^{j2\pi f_k t}$$

Example 3-6

- Multiply complex amplitudes by "$j\omega$"="$j2\pi f$"

Frequency Shifting x(t)

Multiply x(t) by Complex Exponential

➔Frequency Shifting

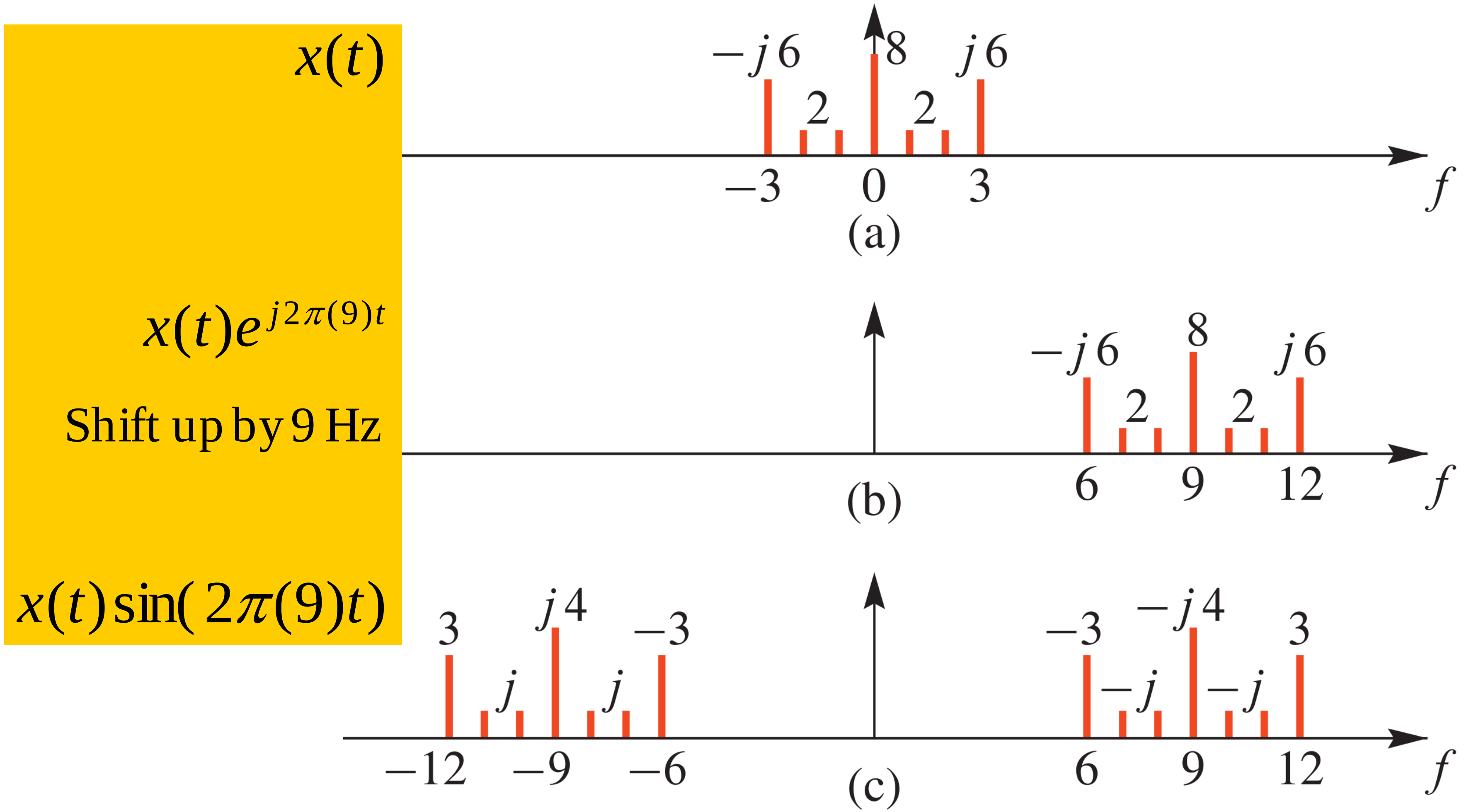
$$y(t) = Ae^{j\varphi}e^{j2\pi f_c t}x(t)$$

$$y(t) = \sum_{k=-M}^{M} Ae^{j\varphi}e^{j2\pi f_c t}a_k e^{j2\pi f_k t}$$

$$= \sum_{k=-M}^{M} (a_k Ae^{j\varphi})e^{j2\pi(f_k + f_c)t}$$

- Spectrum components shifted: $f_k \rightarrow f_k + f_c$

Frequency Shifting x(t)



- BEAT Notes, Sum and Difference frequencies

$$\cos(2\pi(660)t)\sin(2\pi(12)t)$$

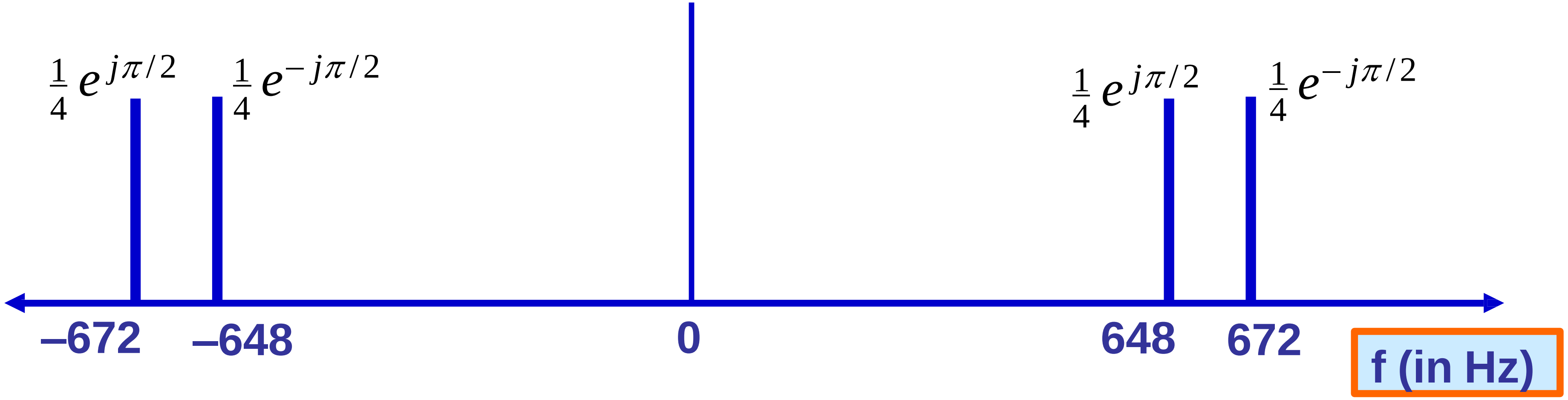
$$\frac{1}{2}\left(e^{j2\pi(660)t}+e^{-j2\pi(660)t}\right)\frac{1}{2j}\left(e^{j2\pi(12)t}-e^{-j2\pi(12)t}\right)$$

$$\frac{1}{4j}\left(e^{j2\pi(672)t}-e^{-j2\pi(672)t}-e^{j2\pi(648)t}+e^{-j2\pi(648)t}\right)$$

$$\frac{1}{2}\cos(2\pi(672)t-\frac{\pi}{2})+\frac{1}{2}\cos(2\pi(648)t+\frac{\pi}{2})$$

SPECTRUM of AM
(Amplitude Modulation)

- **<u>SUM</u>** of 4 complex exponentials:



What is the fundamental frequency?

648 Hz ?

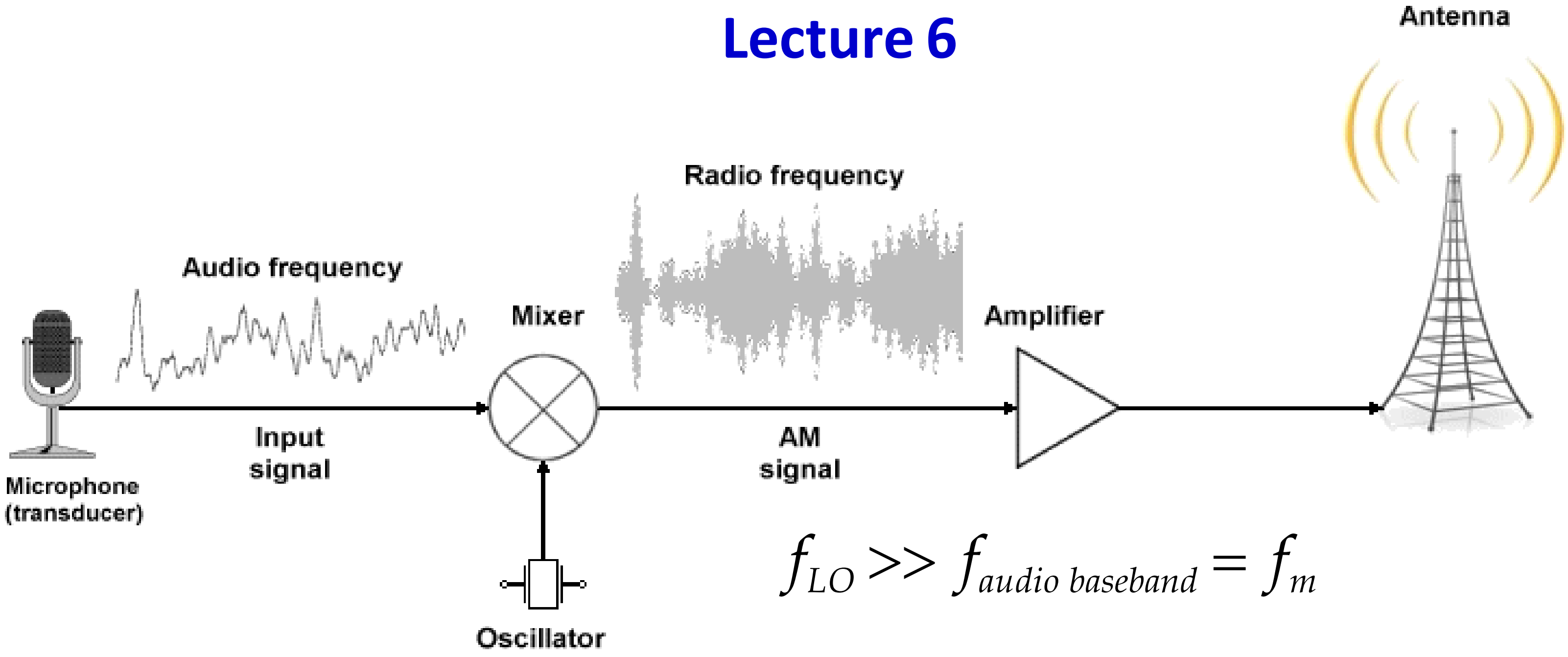
24 Hz ?

Amplitude Modulation and demodulation

Amplitude Modulation – Early Radio

EE 442 – Spring Semester

Lecture 6



$$f_{LO} >> f_{audio\ baseband} = f_m$$

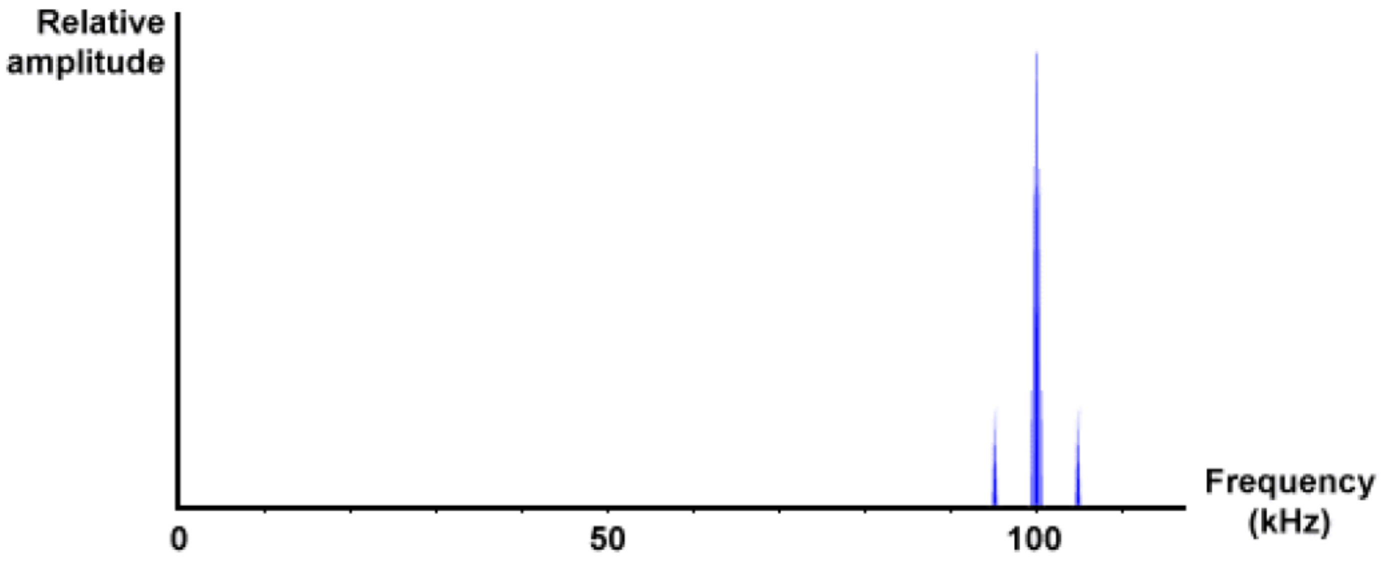
http://www.technologyuk.net/telecommunications/telecom_principles/amplitude_modulation.shtml

Modern Mechanix (December 1952)

Today it is reality!

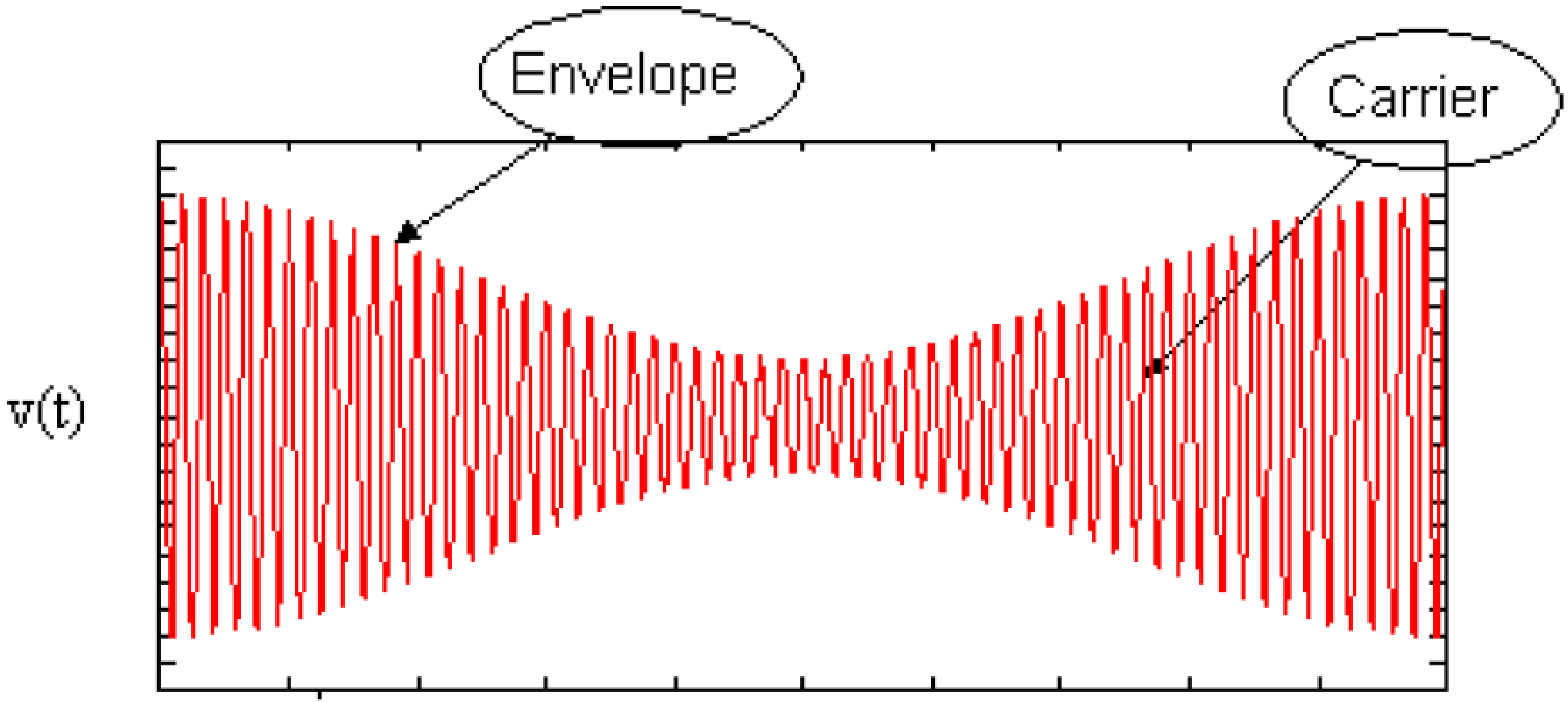
http://blog.modernmechanix.com/dick-tracy-wrist-radio/

Consider, for example, a 100 kHz carrier that is modulated by a steady audio signal (or *tone*) of 5 kHz. When these signals are added, two sidebands are produced. One sideband has a frequency equal to the *sum* of the carrier and the modulating signal (100 kHz + 5 kHz = 105 kHz), while the other sideband has a frequency equal to the *difference* between the carrier and the modulating signal (100 kHz - 5 kHz = 95 kHz). The two sidebands are 5 kHz equidistant from the carrier (one above it and one below it), giving a total bandwidth for the modulated signal of 10 kHz (105 kHz - 95 kHz). The resulting frequency spectrum is illustrated below.



A 100 kHz carrier modulated by a 5kHz audio tone

AM Radio Signal



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Lecture 6

Periodic Signals, Harmonics & Time-Varying Sinusoids

Section 3-4

LECTURE OBJECTIVES

- Signals with **HARMONIC** Frequencies
  - Add Sinusoids with $f_k = kf_0$

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

Second Topic: FREQUENCY can change vs. TIME

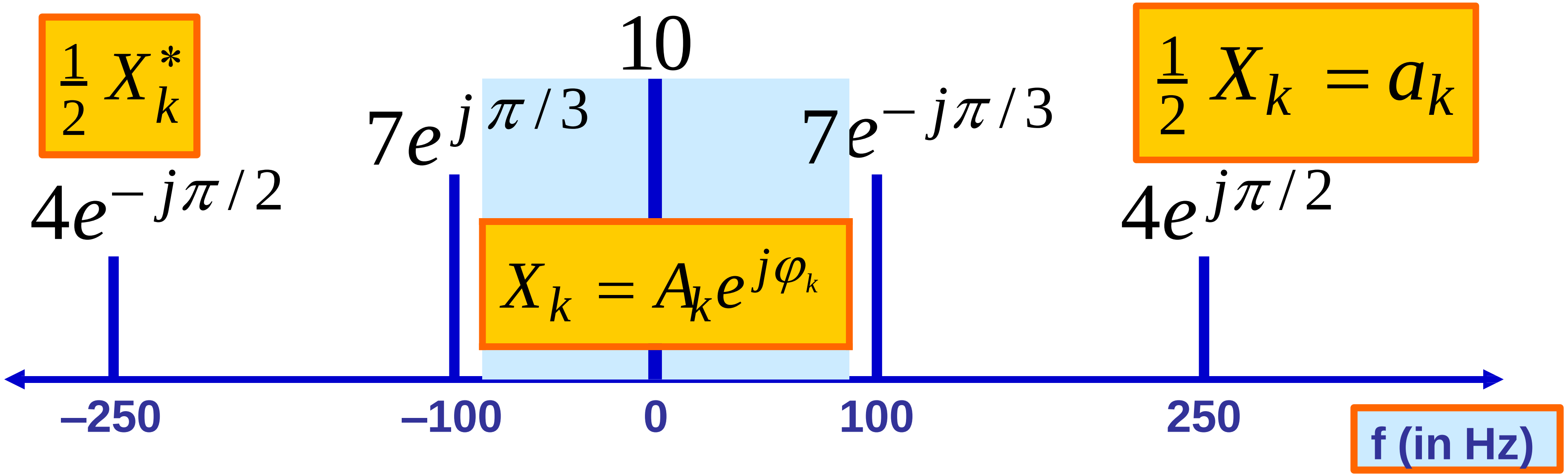
Introduce Spectrogram Visualization

`(spectrogram.m)` `(plotspec.m)`

Chirps: $x(t) = \cos(\alpha t^2)$

SPECTRUM DIAGRAM

- Recall Complex Amplitude vs. Freq



$$x(t) = 10 + 14\cos(2\pi(100)t - \pi/3) + 8\cos(2\pi(250)t + \pi/2)$$

Harmonic Signal

$$\text{Periodic signal}: x(t) = x(t+T)$$

$$\text{Can only have } \textbf{\textit{harmonic}} \text{ freqs}: f_k = k f_0$$

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$\boxed{f_0 T = 1}$$

$x(t)$ is periodic if

$$\cos(2\pi k f_0 (t+T) + \varphi_k) = \cos(2\pi k f_0 t + 2\pi k f_0 T + \varphi_k)$$

Define FUNDAMENTAL FREQ

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$f_0 = \frac{1}{T_0}$$

Largest f_0 such that

$f_k = k f_0 \qquad (\omega_0 = 2\pi f_0)$

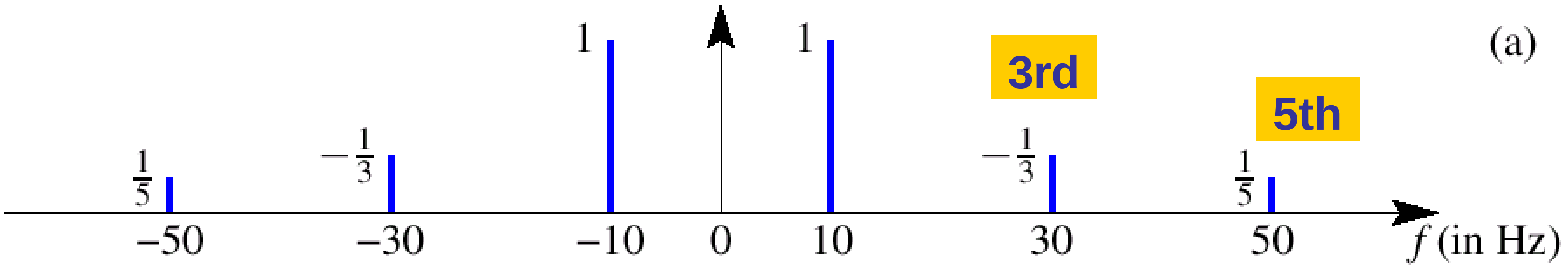
$f_0 =$ fundamental Frequency

$f_k / f_0 =$ integer, for all k

$T_0 =$ fundamental Period

Main point:
for periodic signals, all spectral lines have frequencies that are *integer* multiples of the fundamental frequency

Harmonic Spectrum (3 Freqs)

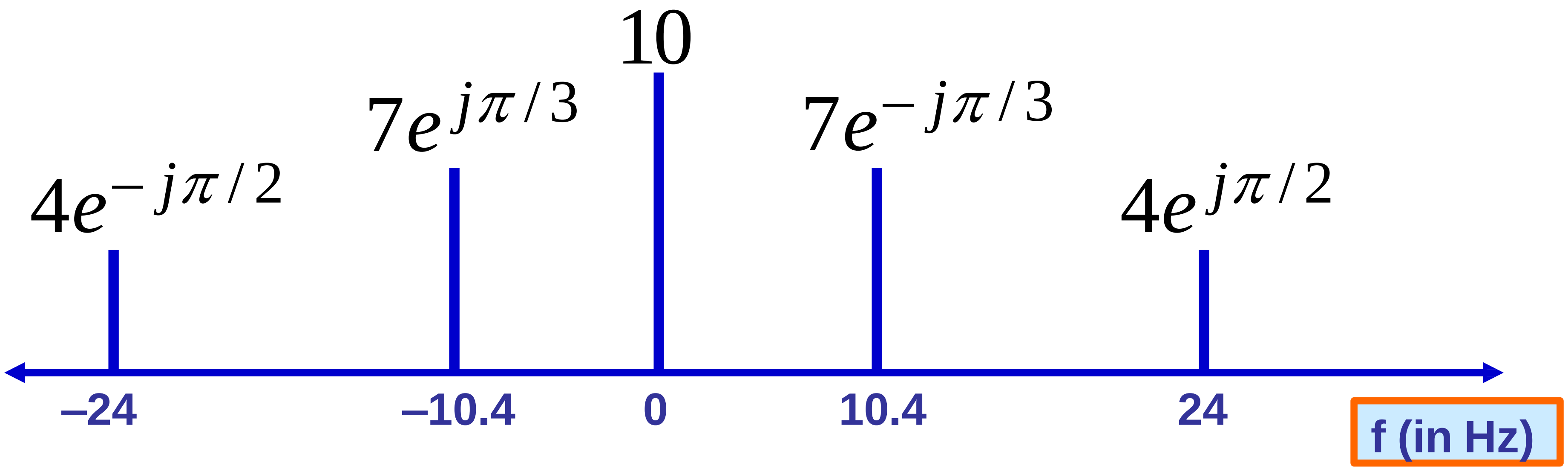


What is the fundamental frequency?

10 Hz

POP QUIZ: FUNDAMENTAL

- Here's another spectrum:



What is the fundamental frequency?

(0.1)GCD(104,240) = (0.1)(8)=0.8 Hz

Time-Varying FREQUENCIES Diagram

Frequency is the vertical axis

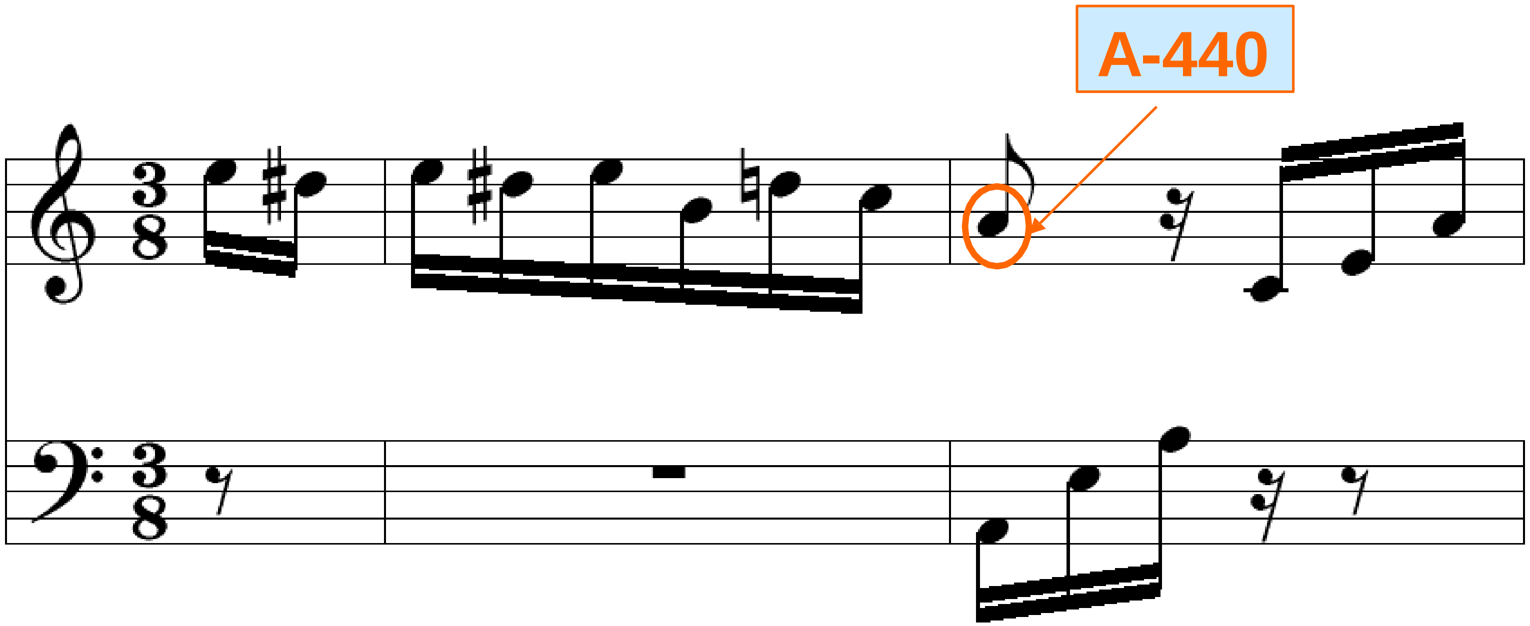
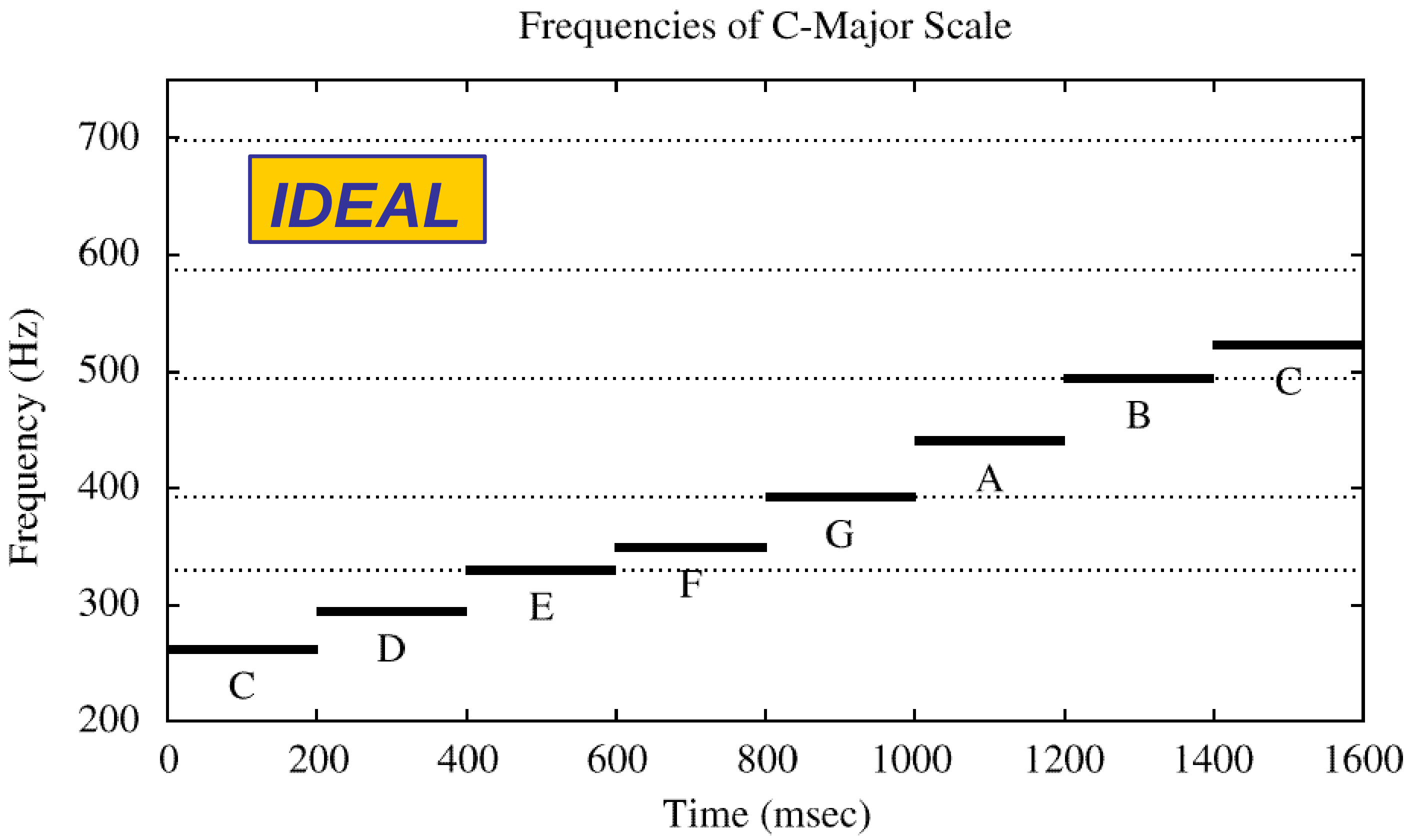


Figure 3.18 Sheet-music notation is a time–frequency diagram.

Time is the horizontal axis

SIMPLE TEST SIGNAL

- C-major SCALE: stepped frequencies
  - Frequency is constant for each note



SPECTROGRAM

- SPECTROGRAM Tool
  - MATLAB function is **`spectrogram.m`**
  - SP-First has **`plotspec.m`** & **`spectgr.m`**
- **ANALYSIS** program
  - Takes x(t) as input
  - Produces spectrum values X_k
  - Breaks x(t) into SHORT TIME SEGMENTS
    - Then uses the FFT (Fast Fourier Transform)

https://www.mathworks.com/help/signal/ref/spectrogram.html

Generate a quadratic chirp, x, sampled at 1 kHz for 2 seconds. The frequency of the chirp is 100 Hz initially and crosses 200Hz at t = 1 s.`t = 0:0.001:2; x = chirp(t,100,1,200,'quadratic');`
Compute and display the spectrogram of x.