MODIFIED TLH

# REVIEW 5 SAMPLING

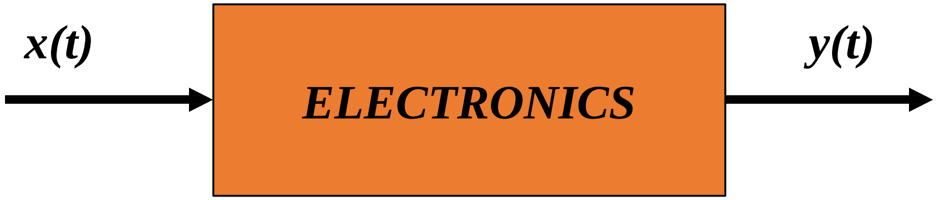
# DSP First, 2/e

## Sampling & Aliasing

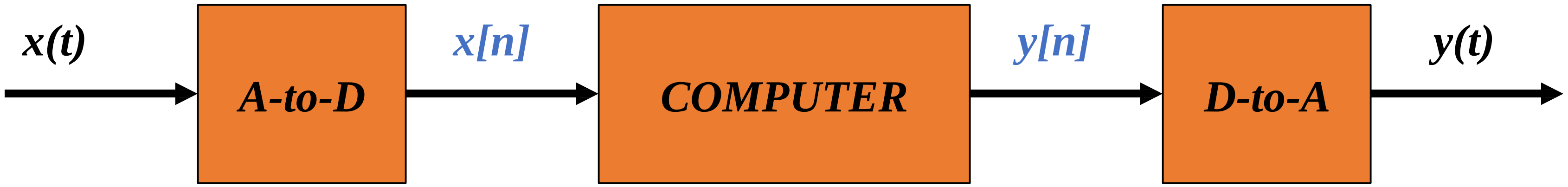
### CHAPTER 4 PRESENTATION 2

# System IMPLEMENTATION

- ANALOG/ELECTRONIC:
  - Circuits: resistors, capacitors, op-amps




- DIGITAL/MICROPROCESSOR
  - Convert x(t) to numbers stored in memory



# SAMPLING THEOREM THE BIG DEAL!!

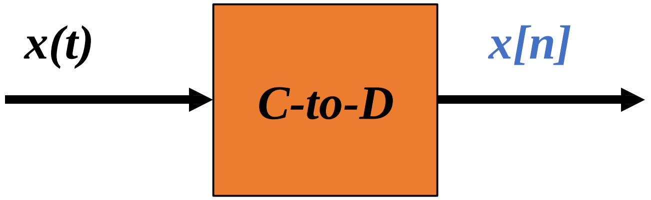
- HOW OFTEN DO WE NEED TO SAMPLE?
  - DEPENDS on FREQUENCY of SINUSOID
  - ANSWERED by SHANNON/NYQUIST Theorem
  - ALSO DEPENDS on "**RECONSTRUCTION**"

*Shannon Sampling Theorem*

A continuous-time signal $x(t)$ with frequencies no higher than $f_{\max}$ can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{\max}$.

# SAMPLING x(t)

- SAMPLING PROCESS
  - Convert x(t) to numbers x[n]
  - "n" is an <u>integer index</u>; x[n] is a sequence of values
  - Think of "n" as the storage address in memory
- UNIFORM SAMPLING at $t = nT_s$
  - IDEAL: $x[n] = x(nT_s)$

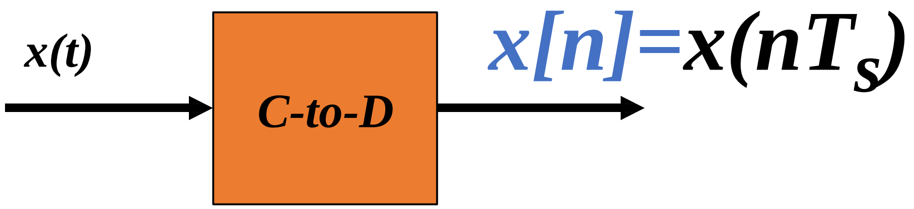


# SAMPLING RATE, $f_s$

- SAMPLING RATE ($f_s$)
  - $f_s = 1/T_s$
    - NUMBER of SAMPLES PER SECOND
  - $T_s$ = 125 microsec ❼ $f_s$ = 8000 samples/sec
    - UNITS of $f_s$ ARE HERTZ: 8000 Hz
- UNIFORM SAMPLING at $t = nT_s = n/f_s$
  - IDEAL: $x[n] = x(nT_s) = x(n/f_s)$

**SOMETIMES GIVEN IN Hz**

**Thus - Fmax = 4000 Hz**



# STORING DIGITAL SOUND

- $\boldsymbol{x[n]}$ is a SAMPLED SISIGNAL
  - A list of numbers stored in memory
- EXAMPLE: audio CD
- CD rate is 44,100 samples per second
  - 16-bit samples
  - Stereo uses 2 channels

**THUS – Frequency range of 22,050 Hz is beyond (most) humans hearing range.**

- Number of bytes for 1 minute is
  - 2 X (16/8) X 60 X 44100 = 10.584 Mbytes

# DIGITAL FREQUENCY $\hat{\omega}$

$$\boxed{\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s}}$$

- $\hat{\omega}$ VARIES from **0** to **2π**, as f varies from 0 to the sampling frequency
- UNITS are radians, **not** rad/sec
  - DIGITAL FREQUENCY is NORMALIZED

# DISCRETE-TIME SINUSOID

- Change x(t) into x[n]      DERIVATION

$$x(t) = A\cos(\omega t + \varphi)$$

$$x[n] = x(nT_s) = A\cos(\omega n T_s + \varphi)$$

$$x[n] = A\cos(\boxed{(\omega T_s)}\, n + \varphi)$$

$$x[n] = A\cos(\boxed{\hat{\omega}}\, n + \varphi)$$

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

***DEFINE DIGITAL FREQUENCY***

- SAMPLING can cause ALIASING
  - **Sampling Theorem**
  - Sampling Rate > 2(Highest Frequency)
- Spectrum for digital signals, x[n]
  - Normalized Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

***ALIASING***

Example

$\cos(200\pi t + \theta)$ $\quad f = 100\ Hz$

a) Need $f_s = S > 200$ samples/sec

b) frequencies

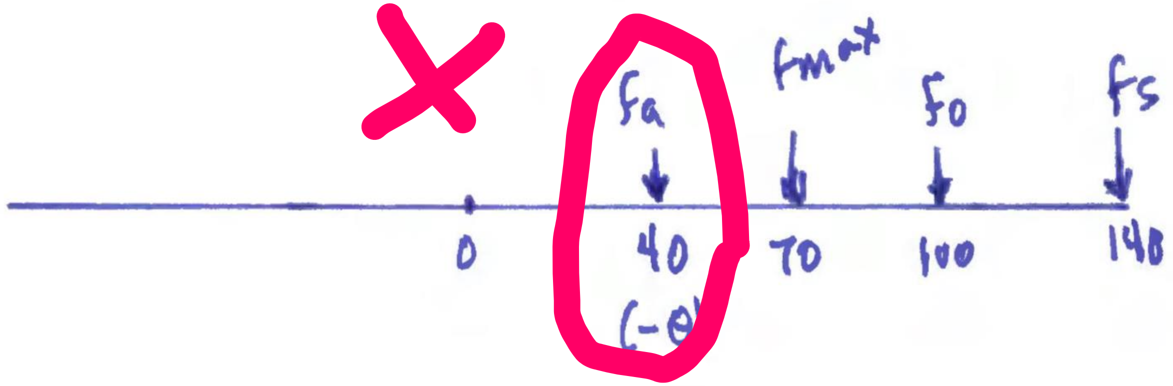
$f_s = 240\ Hz$ $\quad f = 100\ Hz$ $\quad$ OK

$f_s = 140\ Hz$ - Aliased

$f_a = 100 - 140 = -40\ Hz$

$x_a(t) = \cos(-2\pi(40)t + \theta) = \cos(80\pi t - \theta)$

Phase Reversal



# ALIASING CONCLUSIONS

- Adding an **INTEGER multiple** of $f_s$ or $-f_s$ to the frequency of a continuous sinusoid $x_c(t)$ gives **exactly the same values** for the sampled signal $x[n] = x_c(n/f_s)$

- **GIVEN $x[n]$, we CAN'T KNOW whether it came from a sinusoid at $f_o$ or $(f_o + f_s)$ or $(f_o + 2f_s)$ …**

- **This is called ALIASING**

# Video Aliasing

**Why car wheels rotate backwards in movies 4:25**

https://www.youtube.com/watch?v=SFbINinFsxk&feature=youtu.be