REVIEW OF CH5 LECTURE 1

Finite Impulse Response

Finite Impulse Response

- Each output value y[n] is the some of a FINITE number of weighted values of the input sequence x[n]

- The FIR filter can be represented in various ways:
- By a difference Equation Page 150
- By the Impulse Response Page 158
- By the Convolution Sum Page 162

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GENERAL CAUSAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

NOTE: Index k = 0, 1,2,...

- For example,

$$b_k = \{3, -1, 2, 1\}$$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

DIFFERENCE EQUATION

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

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GENERAL CAUSAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- FILTER **ORDER** is M
- FILTER **"LENGTH"** is L = M+1
  - NUMBER of FILTER COEFFS is L

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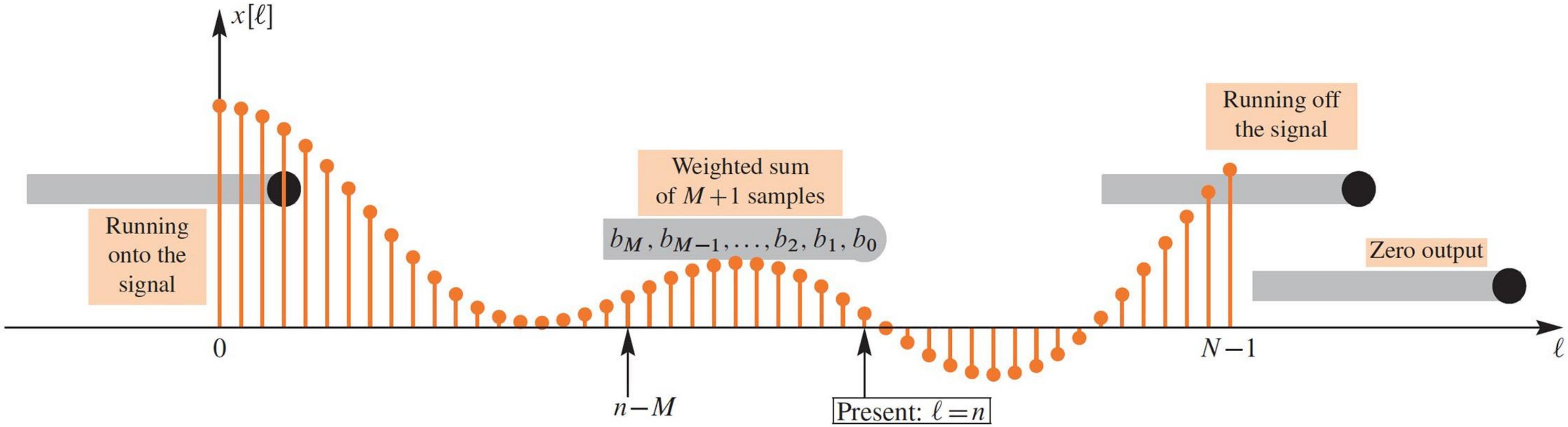
GENERAL CAUSAL FIR FILTER

- SLIDE a WINDOW across x[n]

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]$$



$x[n\text{-}M]$ **$x[n]$**

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FILTERED STOCK SIGNAL

SPECIAL INPUT SIGNALS

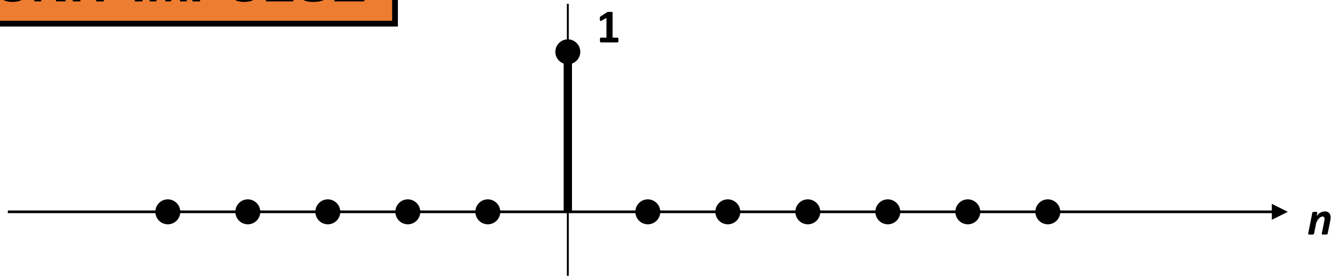
- x[n] = SINUSOID
- x[n] has only one NON-ZERO VALUE

FREQUENCY RESPONSE (LATER)

Test Signal

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



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UNIT IMPULSE SIGNAL **δ[n]**

n	$\dots$	-2	-1	0	1	2	3	4	5	6	$\dots$
$\delta[n]$	0	0	0	1	0	0	0	0	0	0	0
$\delta[n-3]$	0	0	0	0	0	0	1	0	0	0	0

$n = 3$

δ[n] is NON-ZERO When its argument is equal to ZERO

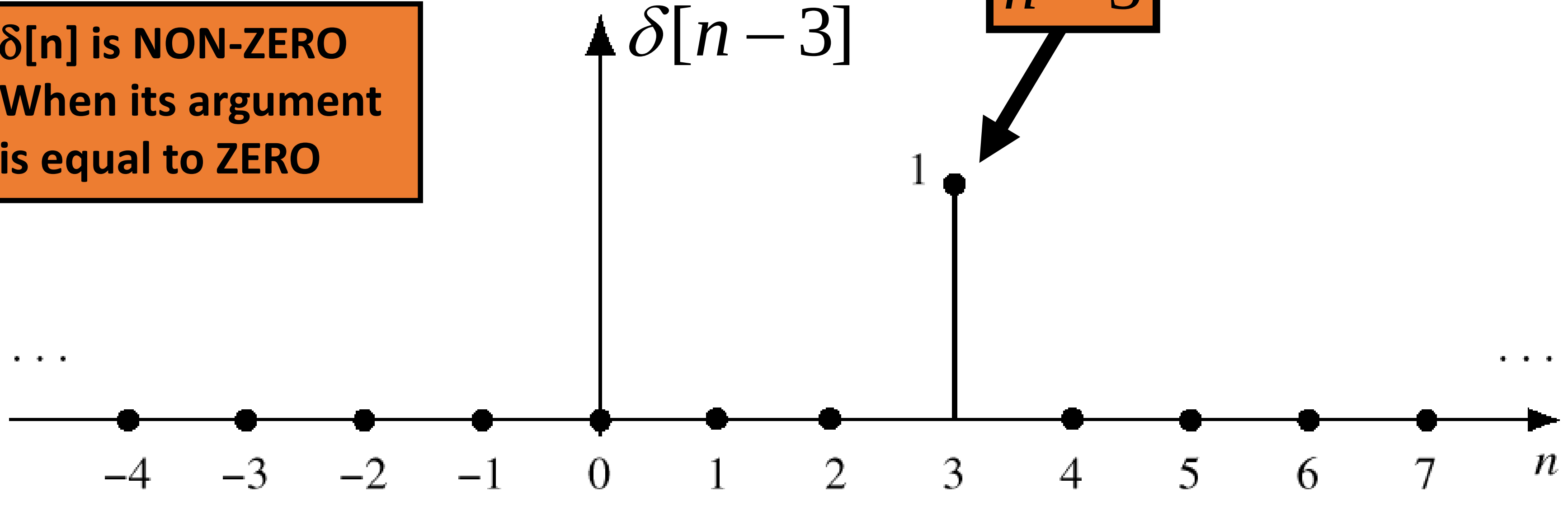
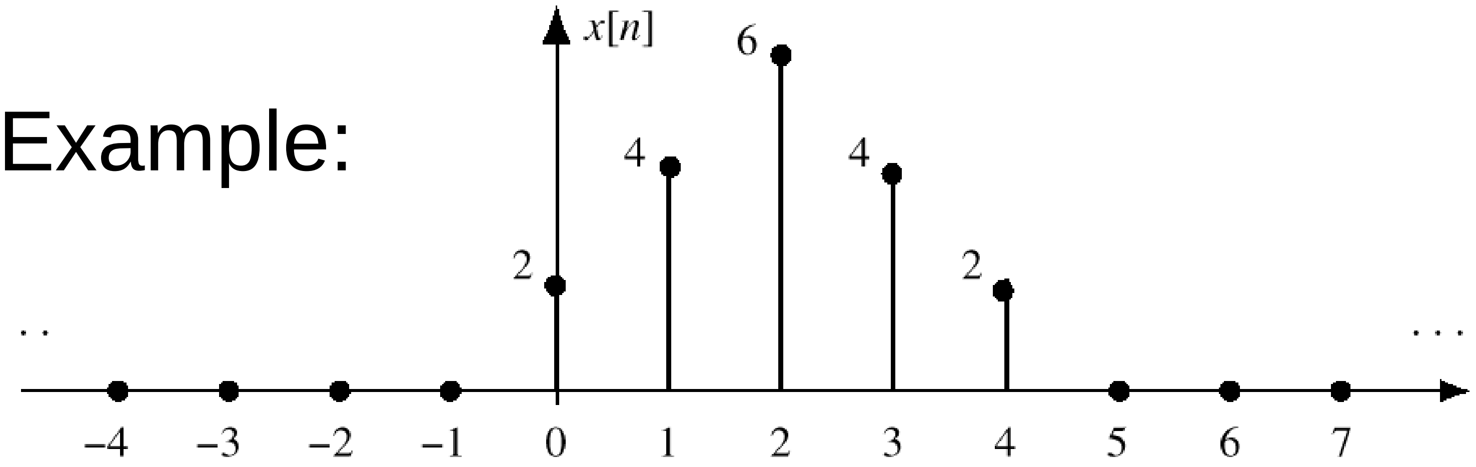


Figure 5.7 Shifted impulse sequence, $\delta[n-3]$.

Sequence Representation

Example:



$$x[n=0]=x[0]=2 \qquad x[n=1]=x[1]=4$$

$$x[n=2]=x[2]=6 \qquad x[n=3]=x[3]=4$$

$$x[n]=\cdots+\ \mathbf{0}\ \delta[n+1]+\ \mathbf{2}\ \delta[n]+\ \mathbf{4}\ \delta[n-1]$$
$$+\ \mathbf{6}\ \delta[n-2]+\mathbf{4}\ \delta[n-3]+\cdots$$

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UNIT IMPULSE RESPONSE

- FIR filter description usually given in terms of coefficients b_k

$$\boxed{y[n] = \sum_{k=0}^{M} b_k x[n-k]}$$

- Can we describe the filter using a **SIGNAL** instead?
- What happens if input is a unit impulse?

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Example: 4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES

$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

- INPUT = UNIT IMPULSE SIGNAL = $\delta[n]$

$$x[n] = \delta[n]$$
$$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$

- OUTPUT is called **"IMPULSE RESPONSE"**
  - Denoted $h[n]=y[n]$ when $x[n]=\delta[n]$

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FIR IMPULSE RESPONSE

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$h[n] = \sum_{k=0}^{M} b_k \delta[n-k]$$

n	$n < 0$	0	1	2	3	$\ldots$	M	$M+1$	$n > M+1$
$x[n] = \delta[n]$	0	1	0	0	0	0	0	0	0
$y[n] = h[n]$	0	b_0	b_1	b_2	b_3	$\ldots$	b_M	0	0

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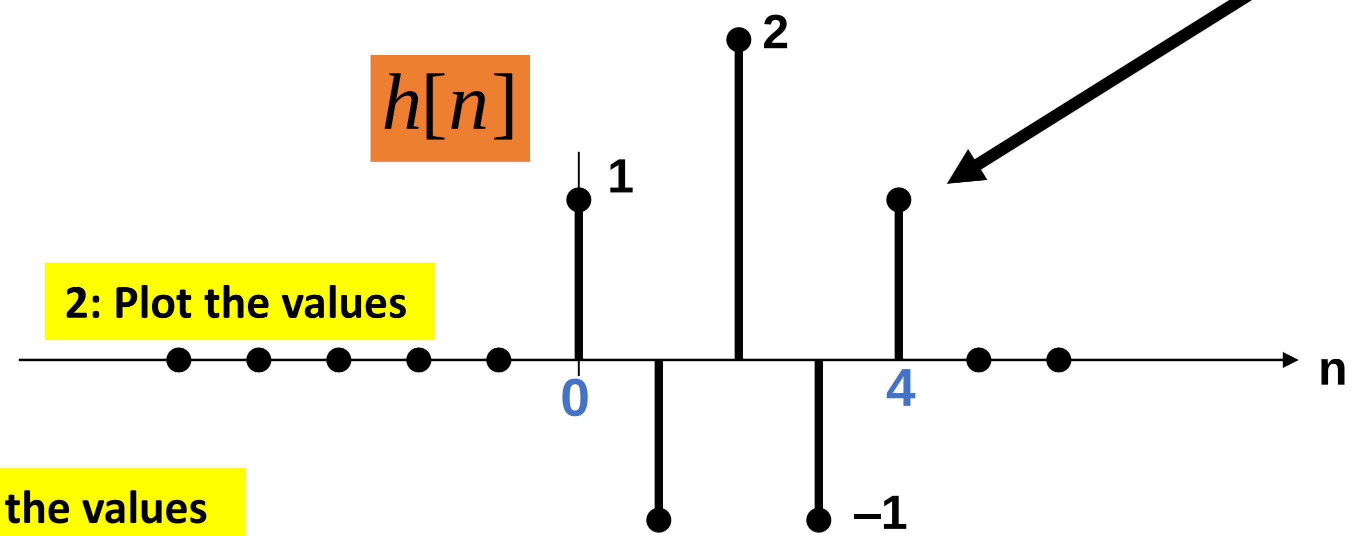
3 Ways to Represent the FIR filter

1 Use SHIFTED IMPULSES to write $\boldsymbol{h[n]}$

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$



2: Plot the values

3: List the values

$$b_k = \{\, 1, -1, 2, -1, 1 \,\}$$

True for any signal, x[n]

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FILTERING EXAMPLE

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER
  - Changes A slightly

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

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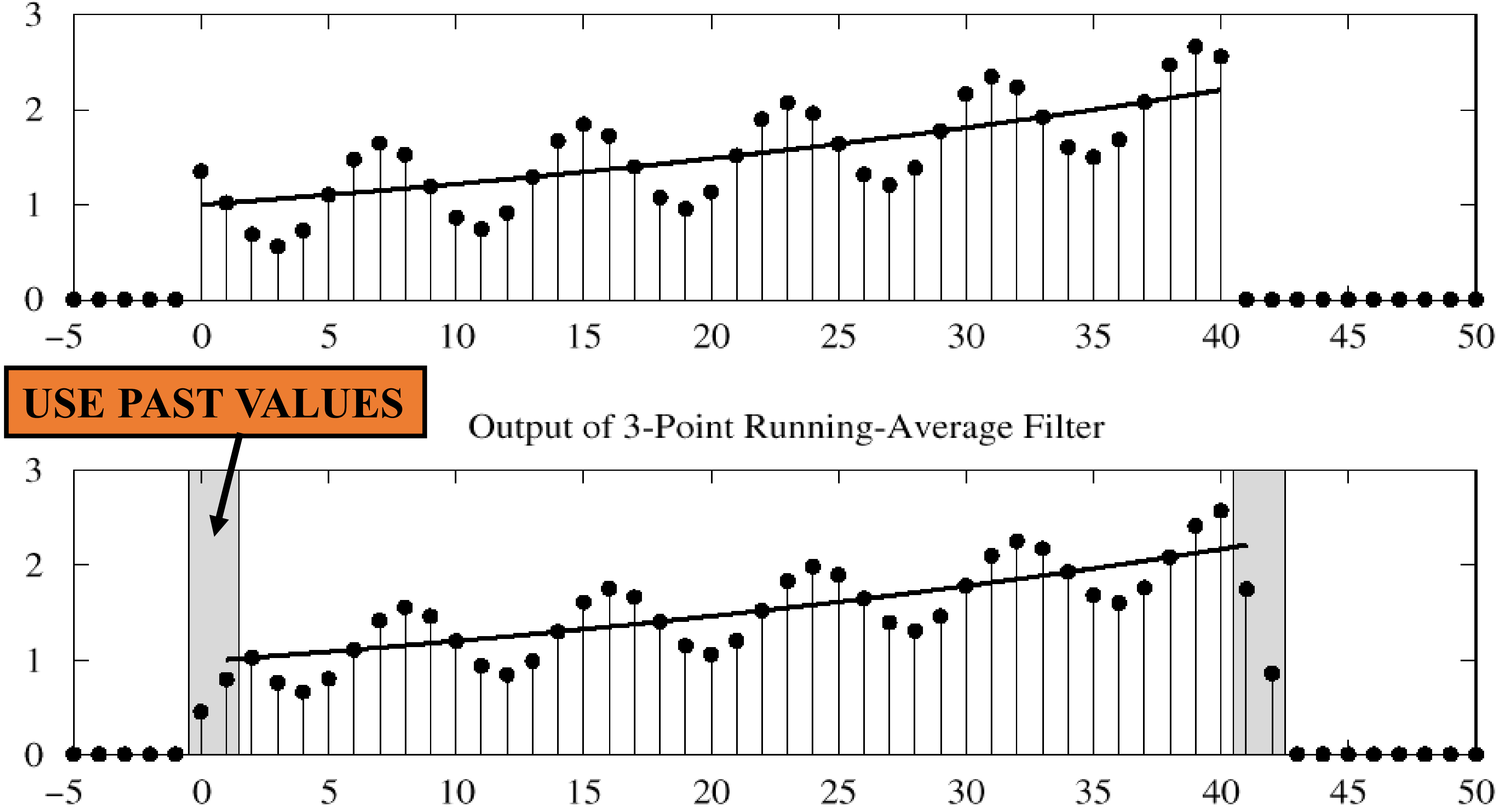
3-pt AVG EXAMPLE

$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$



7-pt FIR EXAMPLE (AVG)

$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$