CONVOLUTION

DISCRETE

&

RC CIRCUIT

$$y[n] = h[n] * x[n]$$

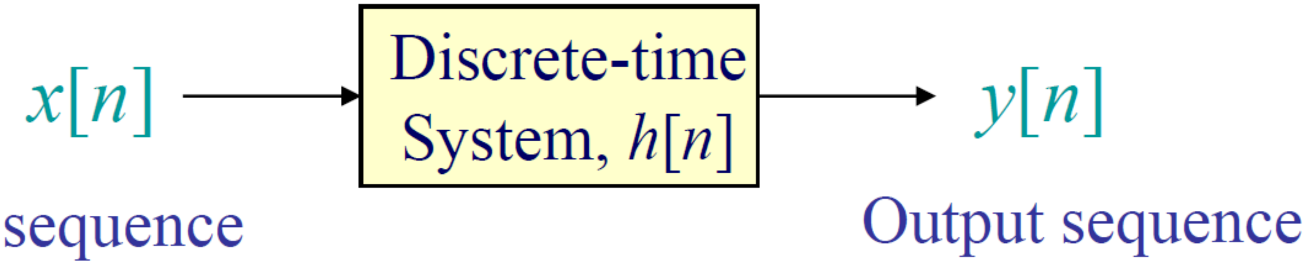
LTI: Convolution Sum

- Delay the signal x[n] & then multiply by filter coefficients that come from h[n]

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

$$= h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + \ldots$$

➲ At the heart of any DSP system:



$$
\boxed{\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{m=-\infty}^{\infty} x[m] \cdot h[n-m] \\ &= \sum_{m=-\infty}^{\infty} h[m] \cdot x[n-m] \end{aligned}}
$$

$h[n]$: Impulse response of the system

Assuming that $x(\tau) = 0$ for $\tau < 0$, adding the response to all the past inputs leads to the integral

$$y(t) = \int_0^t x(\tau)h(t-\tau)\,d\tau. \qquad (1)$$

This integral is called the *convolution* or *superposition* integral and the operation is said to be the convolution of x, the input, and h, the impulse response of the system. The convolution is often written $x * h$.

System Relationships Digital

Input	*System*	*Output*
$x[n]$	$h[n]$	$\sum_{i=0}^{n} h[i]x[n-i]$

CONTINUOUS AND DISCRETE CONVOLUTION - SAME PRINCIPLE!

INTEGRATE SHIFTED FUNCTION OR MULTIPLY AND SHIFT

https://www.analog.com/media/en/technical-documentation/dsp-book/dsp_book_Ch6.pdf

Fig 6.3

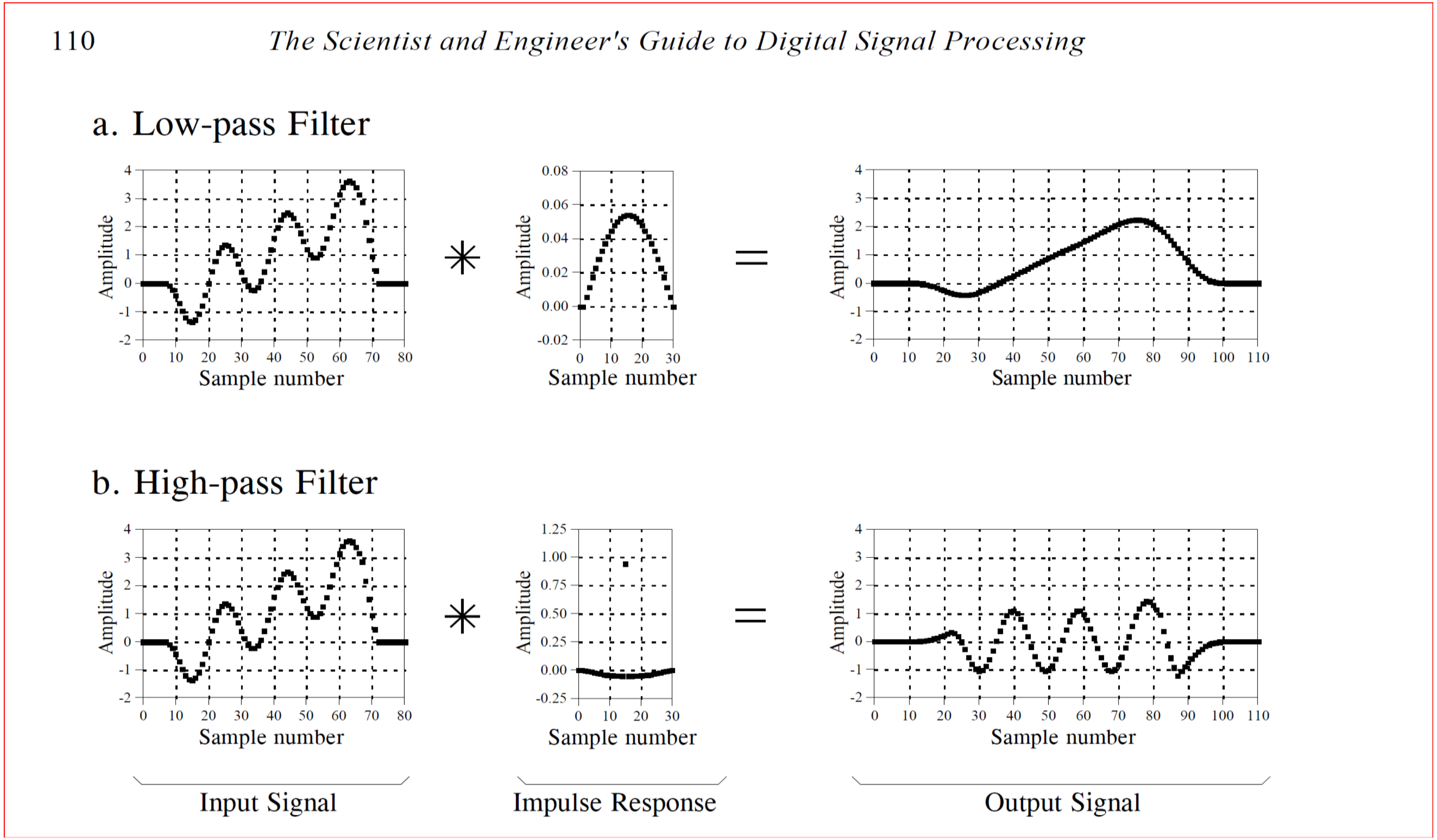


FIGURE 6-3
Examples of low-pass and high-pass filtering using convolution. In this example, the input signal is a few cycles of a sine wave plus a slowly rising ramp. These two components are separated by using properly selected impulse responses.

Figure 6-3 shows convolution being used for low-pass and high-pass filtering. The example input signal is the sum of two components: three cycles of a sine wave (representing a high frequency), plus a slowly rising ramp (composed of low frequencies). In (a), the impulse response for the low-pass filter is a smooth arch, resulting in only the slowly changing ramp waveform being passed to the output. Similarly, the high-pass filter, (b), allows only the more rapidly changing sinusoid to pass.

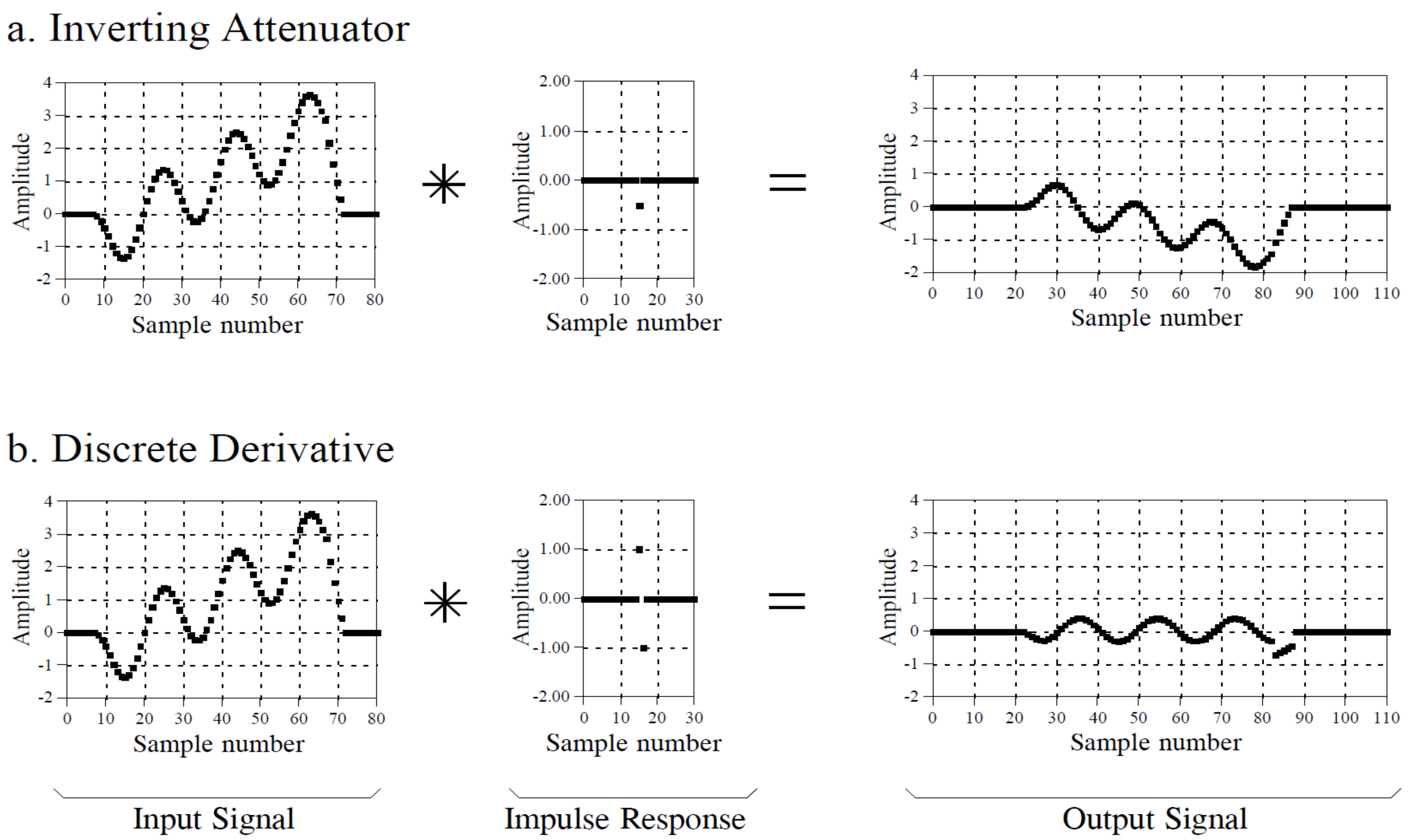
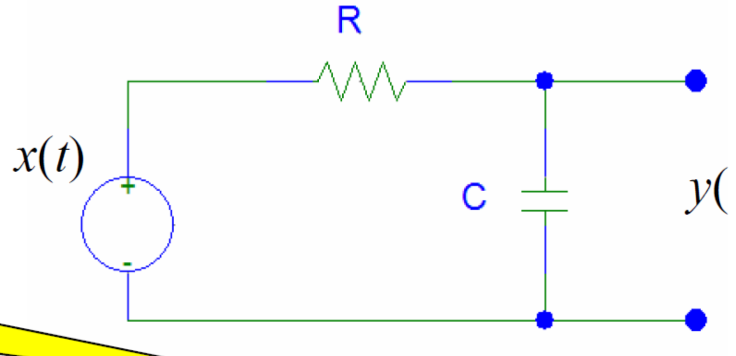


FIGURE 6-4
Examples of signals being processed using convolution. Many signal processing tasks use very simple impulse responses. As shown in these examples, dramatic changes can be achieved with only a few nonzero points.

http://www.ws.binghamton.edu/fowler/fowler%20personal%20page/EE301_files/EECE%20301%20Note%20Set%2010%20CT%20Convolution.pdf

Example 2.14 Output of RC Circuit with Unit Step Input

u(t)



LTI SYSTEM

The book considers a different case… $x(t)$ is a pulse

Problem: Find the zero-state response of this circuit to a unit step input… i.e., let $x(t) = u(t)$ and find $y(t)$ for the case of the ICs set to zero (for this case that means $y(0^-) = 0$).

Zero State – No Initial Conditions

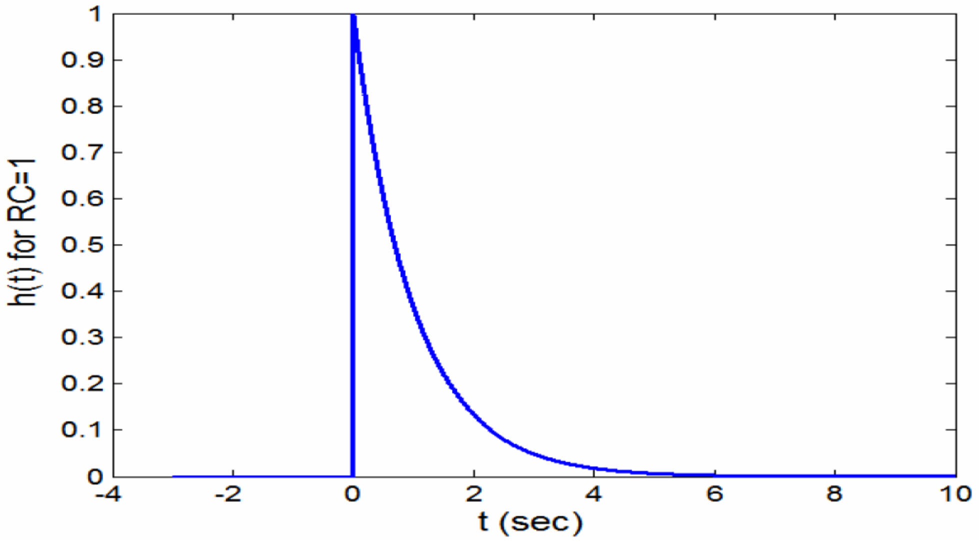
We have seen that this circuit is modeled by the following Differential Equation:

$$\frac{dy(t)}{dt} + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)$$

But… to do that we need to know the impulse response $h(t)$ for this system (i.e., for this differential equation)!!!

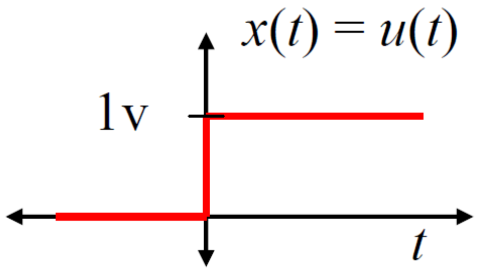
In Chapter 6 we will learn how to find the impulse response by applying the Laplace Transform to the differential equation. The result is:

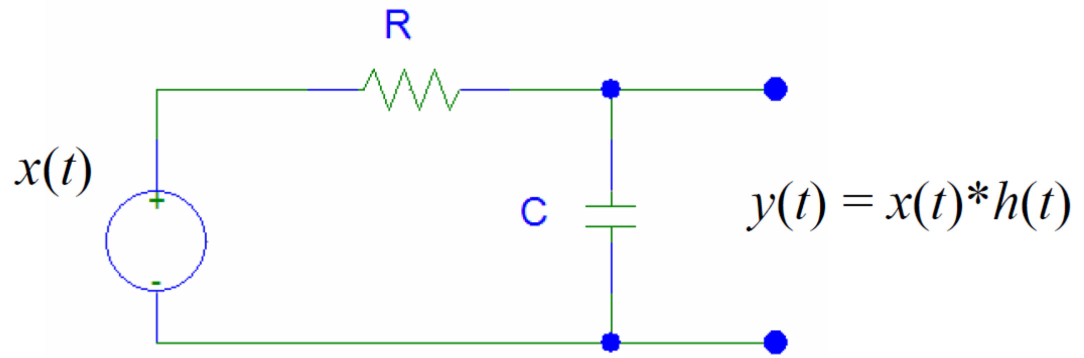
$$h(t) = \begin{cases} \dfrac{1}{RC} e^{-(1/RC)t}, t \geq 0 \\ 0, \quad t < 0 \end{cases} \qquad or \qquad h(t) = \frac{1}{RC} e^{-(1/RC)t} u(t)$$



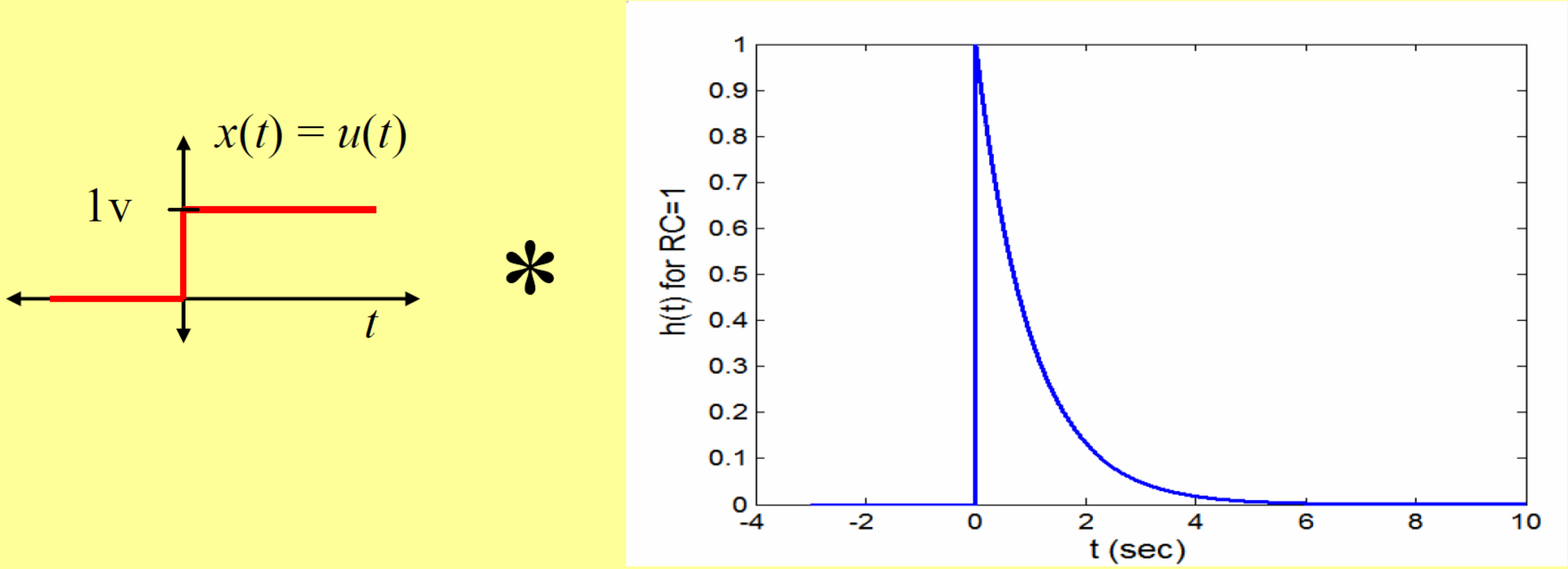
Impulse Applied,
Cap charges,
Circuit closes,
R bleeds off charge

For our step input:





This is the convolution we need to do…



$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$$

This is the general form for convolution

$$= \int_{-\infty}^{\infty} \underbrace{\left[\tfrac{1}{RC} e^{-(1/RC)\lambda} u(\lambda)\right]} \underbrace{u(t-\lambda)} d\lambda$$

Plug in given forms for $h(t)$ and $x(t)$

This makes the integrand = 0 whenever λ < 0. And… it is 1 otherwise.

This makes the integrand = 0 whenever t- λ < 0 or in other words whenever λ > t. And… it is 1 otherwise.

(Note that if t < 0 then the integrand is 0 for all λ)

So exploiting these facts we see that the only thing the unit steps do here is to limit the range of integration…

$$y(t) = \begin{cases} \frac{1}{RC} \int_0^t e^{-(1/RC)\lambda} d\lambda, \ t > 0 \\ 0, \ t \le 0 \end{cases}$$

So… to find the output for this problem all we have to do is evaluate this integral to get a function of t

This integral is the easiest one you learned in Calc I!!!

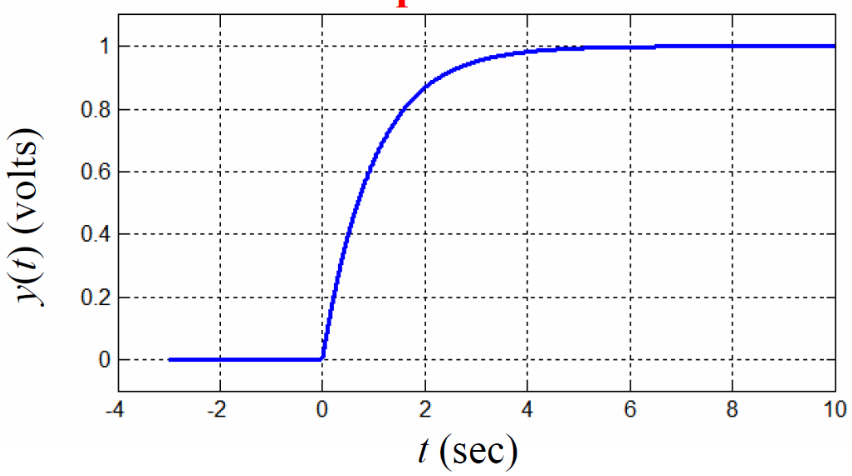
$$\tfrac{1}{RC}\int_0^t e^{-(1/RC)\lambda}d\lambda = \tfrac{1}{RC}\left[-RCe^{-(1/RC)\lambda}\right]_0^t = \left[-e^{-(1/RC)\lambda}\right]_0^t = \left[-e^{-(1/RC)t}\right]-\left[-e^{0}\right]$$

$$= 1 - e^{-(1/RC)t}$$

$$\boxed{y(t) = \begin{cases} 1-e^{-(1/RC)t}, t>0 \\ 0, t \le 0 \end{cases}}$$

Once we can compute this kind of integral in general we can find out what the output looks like for any given input!

Output for *RC* = 1



Recall Time-Constant Rules:
- ▶ 63% after 1 TC
- ▶ ≈100% after 5 TCs

10/1

NOTE: STEP RESPONSE Y_s(t) IS INTEGRAL OF IMPULSE RESPONSE