CONVOLVE

$x[n] = \{1 \quad 2 \quad 3 \; 4]$ $\qquad x[0] = 1$

$h[n] = [5 \; 6]$ $\qquad h[0] = 5$

1

Expect $L_y = L_x + L_h - 1 = 4 + 2 - 1 = 5$

so

$$y[n] = \sum_{k=0} h[k] \, x[n-k]$$

So we have

$x[0] \quad x[1] \quad x[2] \quad x[3]$

$h[0] \quad h[1]$

$$y[0] = \overset{5}{h[0]}\,\overset{1}{x[0]} = 5$$

$$y[1] = \overset{5}{h[0]}\,\overset{2}{x[1]} + \overset{6}{h[1]}\,\overset{1}{x[0]} = 16$$

$$y[2] = \overset{5}{h[0]}\,\overset{3}{x[2]} + \overset{6}{h[1]}\,\overset{2}{x[1]} + \cancel{h[2]}\cdots = 27$$

$$y[3] = \overset{5}{h[0]}\,\overset{4}{x[3]} + \overset{6}{h[1]}\,\overset{3}{x[2]} + \cancel{h[2]}\cdots = 38$$

$$y[4] = h[0]\,\overset{0}{x[4]} + \overset{6}{h[1]}\,\overset{4}{x[3]} + \overset{0}{h[2]}\,\overset{3}{x[2]} + \cancel{h[3]}\cdots = 24$$

OR

1	2	3	4	
5	6			
5	10	15	20	
	6	12	18	24
5	16	27	38	24

NOTE

$h[n] * x[n] =$

$x[n] * h[n]$

$*$ = CONVOLVE

OR MATLAB

NEXT

convolve

3

```
% discreteconvHW5_3315.m   finite convolution
diary discreteconv.txt
x=[1 2 3 4 ]
h=[5 6]
y=conv(h,x)     % y =      5     16     27     38     24
Ly=length(y)
diary off
```

Pg. 152

$$\sum_{\ell=n-2}^{n} \tfrac{1}{3} x(\ell) =$$

$$\tfrac{1}{3} [x[n-2] + x[n-1] + x[n]]$$

write as

$$\tfrac{1}{3} (x[n] + x[n-1] + x[n-2])$$

$$= \sum_{k=0}^{2} \tfrac{1}{3} x(n-k)$$

Take a case

n	-2	-1	0	1	2	3	4	5	6	7
x[n]	0	0	2	4	6	4	2	0	0	0

So $y[0] = \sum_{k=0}^{2} \tfrac{1}{3} x(-k) = \tfrac{1}{3} \{x(0) + x(-1) + x(-2)\}$

$= \tfrac{1}{3} x[0] = \tfrac{2}{3}$

$y[1] = \sum_{k=0}^{2} \tfrac{1}{3} x[1-k] = \tfrac{1}{3} \{x[1] + x[0] + x[-1]\}$

$= \tfrac{1}{3} (4+2) = 2$

⋮

So $y[n] = \tfrac{1}{3} [x[n] + x[n-1] + x(n-2)]$

$L_y = L_x + L_h - 1 = 10 + 3 - 1 = \underline{\underline{12}}$

TRY MATLAB CONV(h,x) $\quad h[n] = [\tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}]$

UNIT IMPULSE Pg 160

P153

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] =$$

$$b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]$$

NOTE SUM FROM $K=0$ <u>CAUSAL</u>

TERM BY TERM

$$y[0] = b_0 x[0-0] + b_1 \overset{=0}{x[-1]} + \cdots = 0 \quad \text{(ALL ZEROS)}$$

$$y[1] = b_0 x[1-0] + b_1 x[1-1] + \cancel{0}$$

$x[m<0]$

$$= b_0 x[1] + b_1 x[0]$$

$\vdots$

SPECIAL CASE – IMPULSE RESPONSE

$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$$

$$h[n] = \sum_{k=0}^{M} b_k \delta[n-k] \qquad \underset{n<0}{h[n]=0}$$

So $h[0] = b_0 \delta(0) = b_0$

$$h[1] = b_0 \delta[1] + b_1 \delta[0] = b_1$$

$\vdots$

$$h[M] = \underbrace{b_0 \delta[M] + \cdots}_{\text{zeros}} + b_M \delta[M-M] = b_M$$

FILTER BY CONVOLUTION

NOTE: $x(n) = 0 \begin{cases} n < 0 \\ n > 5 \end{cases}$; $b[k] = 0 \quad k \geq 2$

n	0	1	2	3	4	5		
x	1	2	3	4	5	6		
b	$1/3$	$1/3$	$1/3$					
	$1/3$	$2/3$	$3/3$	$4/3$	$5/3$	$6/3$		
		$1/3$	$2/3$	$3/3$	$4/3$	$5/3$	$6/3$	
			$1/3$	$2/3$	$3/3$	$4/3$	$5/3$	$6/3$
	$1/3$	1	2	3	4	5	$4/3$	2

$$L = L(x) + L(b) - 1 = 6 + 3 - 1 = 8$$

$\frac{1}{3}$ $\frac{2+1}{3}$ $\frac{3+2+1}{3}$ $\frac{4+3+2}{3}$ ...

CONVOLUTION SUM $y(n) = \sum_{k=0}^{n} b_k x[n-k]$

$$y[0] = \tfrac{1}{3}x(0) + \tfrac{1}{3}x(-1) + \tfrac{1}{3}x(-2) = \tfrac{1}{3}\cdot 1 + 0 + 0 = \tfrac{1}{3}$$

$$y[1] = \tfrac{1}{3}x[1] + \tfrac{1}{3}x[0] + \tfrac{1}{3}x[-1] = \tfrac{1}{3}\cdot 2 + \tfrac{1}{3}\cdot 1 + 0 = 1$$

$$y[2] = \tfrac{1}{3}x[2] + \tfrac{1}{3}x[1] + \tfrac{1}{3}x[0] = \tfrac{1}{3}3 + \tfrac{1}{3}2 + \tfrac{1}{3}1 = 2$$

NOTE THAT $b[k] = 0 \quad k \geq 3$ SO

$$y[4] = \sum_{k=0}^{4} b[k]\,x[4-k] = b_0x_4 + b_1x_3 + b_2x_2 + \overset{=0}{b_3}x_1 + \overset{=0}{b_4}x_0$$
$$= \tfrac{1}{3}\cdot 5 + \tfrac{1}{3}4 + \tfrac{1}{3}3 = \tfrac{12}{3} = 4$$

$$y[7] = \sum_{k=0}^{2} b_k x[7-k] = b_0 \underset{\to 0}{x[7]} + b_1 \underset{\to 0}{x[6]} + b_2 x(5) = \tfrac{1}{3}6 = 2$$

MATCAB ON WEB

SMOOTHING FILTER

$a(1)\,y(n) = b(1)\,x(n)$ ← INPUT

OUTPUT $\quad - a(2)\,y(n-1)$

So $a(1) = 1$

$a(2) = a$

$b(1) = (1-a)$

$a = [1 \; a]$

$b = [1-a]$

SO FILTER

yfilt = filter(b, a, x)

$y(1), y(2), \ldots\ldots y(n)$_max

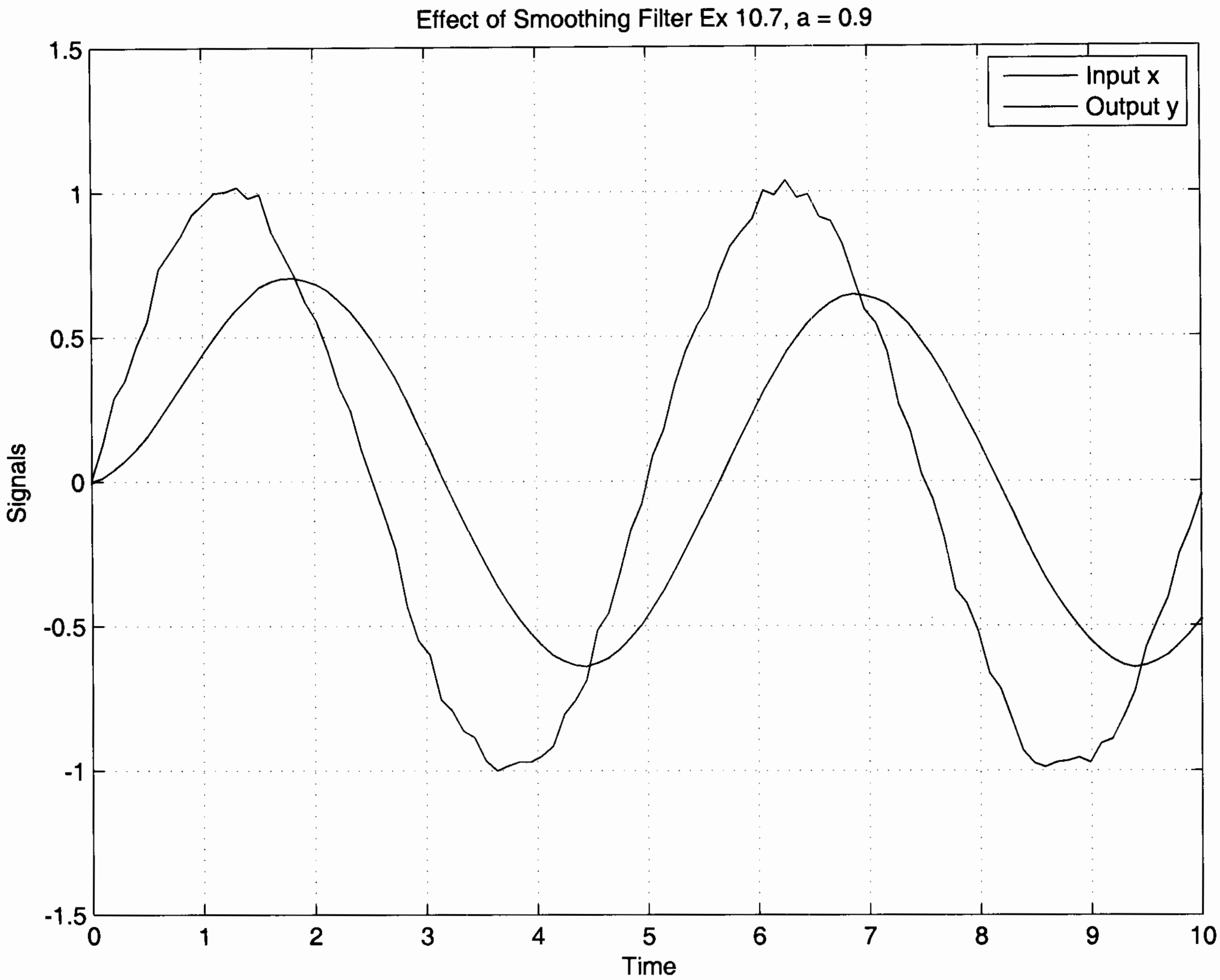
WATCH INDEX

MATH $\quad \hat{y}(0), \hat{y}(1), \hat{y}(2) - \ldots y(N-1)$

MATLAB $\quad y(1), y(2), y(3) \quad\quad y(N)$

```
% smootherEx10_7filt.m A smoothing filter defined as
%   y(n) = a*y(n-1) +(1-a)*x(n) , y(-1)=0
%   x(n) is input signal, y(n) is smoothed output
%
% Test signal is sin(w*t) with random noise
%  INPUT:  Weighing factor a
%  OUTPUT: Plot of x and y
%
clear, clf
w=2*pi/5;
t = linspace(0,10,100);      % Time steps
s = sin(w*t);                % Noiseless signal
% Add random noise
len=size(t);
na = 0.1;                    % Noise amplitude
noise = na*(rand(len)-.5);    % (-.05 to +.05)
x = s + noise;
%
% Weighing factor
a = input('Weighing factor a= ')
%
y(1)=(1-a)*x(1);
for I=2:100
 y(I) = a*y(I-1) + (1-a)*x(I);  % Digital Filter
end
figure(1),plot(t,x,t,y),grid
xlabel('Time'), ylabel('Signals')
title(['Effect of Smoothing Filter Ex 10.7, a = ', num2str(a)])
legend('Input x','Output y')
% Filter
avec=[1 -a]
bvec=[1-a]
yfilt=filter(bvec,avec,x)
figure(2),plot(t,x,t,yfilt),grid,xlabel('Time'),axis([0 10 -1.5
1.5])
title(['Effect of MATLAB Filter Ex 10.7, a = ', num2str(a)])
legend('Input x','Output y')
```

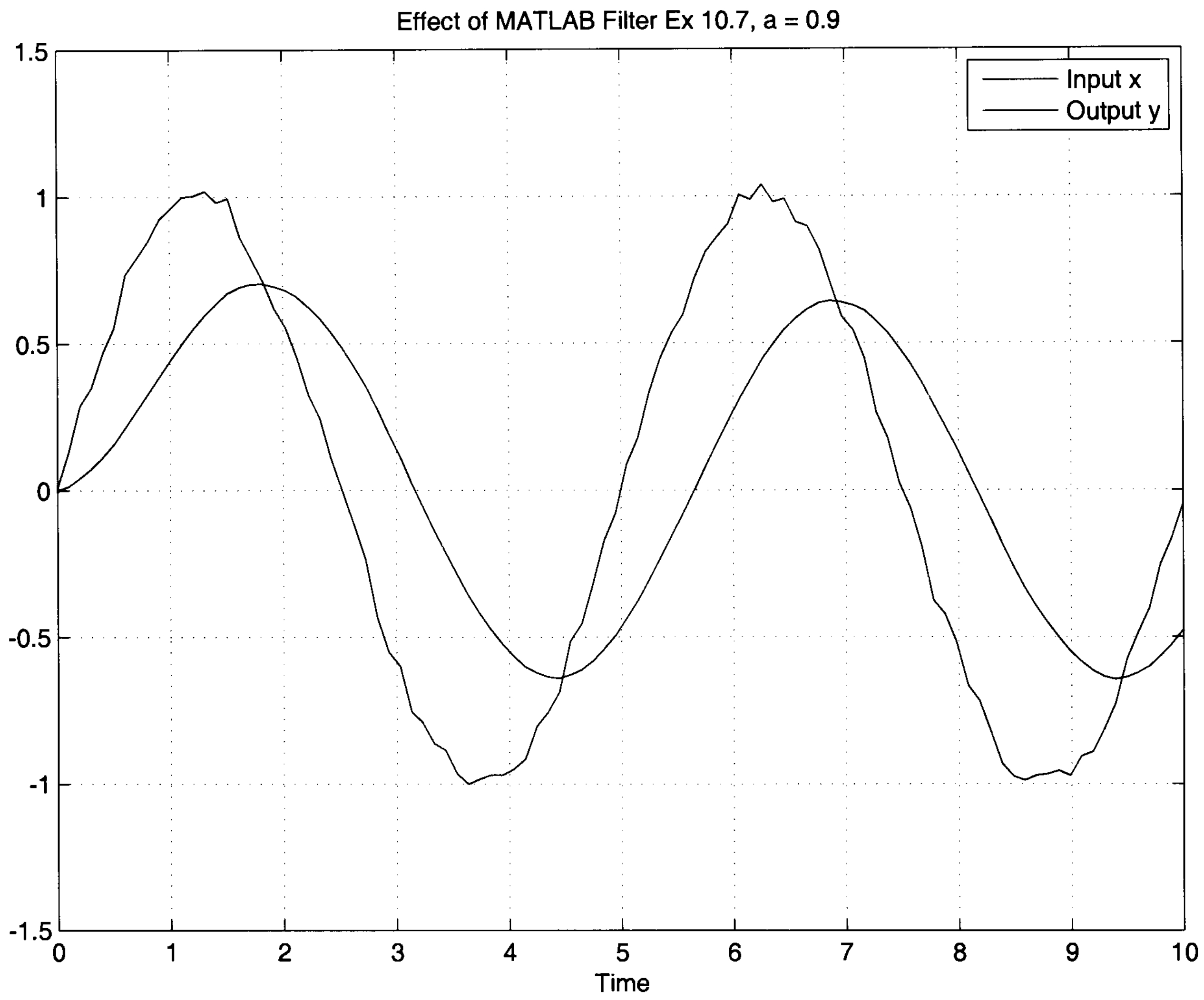
FIG 1



Effect of Smoothing Filter Ex 10.7, a = 0.9
1.5
1
0.5
0
-0.5
-1
-1.5
Signals
0
1
2
3
4
5
6
7
8
9
10
Time
Input x
Output y

FIG 2

FILTER


Effect of MATLAB Filter Ex 10.7, a = 0.9
Input x
Output y
1.5
1
0.5
0
-0.5
-1
-1.5
0
1
2
3
4
5
6
7
8
9
10
Time

Signal processing toolbox ~~Matlab~~ Problem Session
versions

MA FILTER

DSP FIRST
Pg 164-165

convolve P1-14

If power is missing — add a 0!

$(x^2+x+1)(x^2+x+1)$

$$\begin{array}{ccccc} & & x^2 & +x & +1 \\ & & x^2 & +x & +1 \\ \hline & & 1 & 1 & 1 \\ & 1 & 1 & 1 & \\ 1 & 1 & 1 & & \\ \hline 1 & 2 & 3 & 2 & 1 \\ x^4 & x^3 & x^2 & x & \end{array}$$

MATLAB
conv([1 1 1], [1 1 1])

$$y(n) = h(n) * x(n) = \sum_{m=-\infty}^{\infty} h(n-m)\,x(m)$$

so

$$y(0) = h(0-m)\,x(m) = h(0)\,x(0) = 1$$

$$y(1) = \sum_{0}^{1} h(1-m)\,x(m) = h(1)x(0) + h(0)x(1) = 1+1 = 2$$

$$y(2) = \sum_{0}^{2} h(2-m)\,x(m) = h(2)x(0) + h(1)x(1) + h(0)x(2) = 1+1+1 = 3$$

$$y(3) = \sum_{0}^{3} h(3-m)\,x(m) = h(3)x(0) + h(2)x(1) + h(1)x(2) + h(0)x(3) = 0+1+1+0 = 2$$

$$y(4) = \sum_{0}^{4} h(4-m)\,x(m) = h(4)^{=0}x(0) + h(3)^{=0}x(1) + \underline{h(2)x(2)} + h(1)x(3)^{=0} + h(0)x(4)^{=0} = 1$$