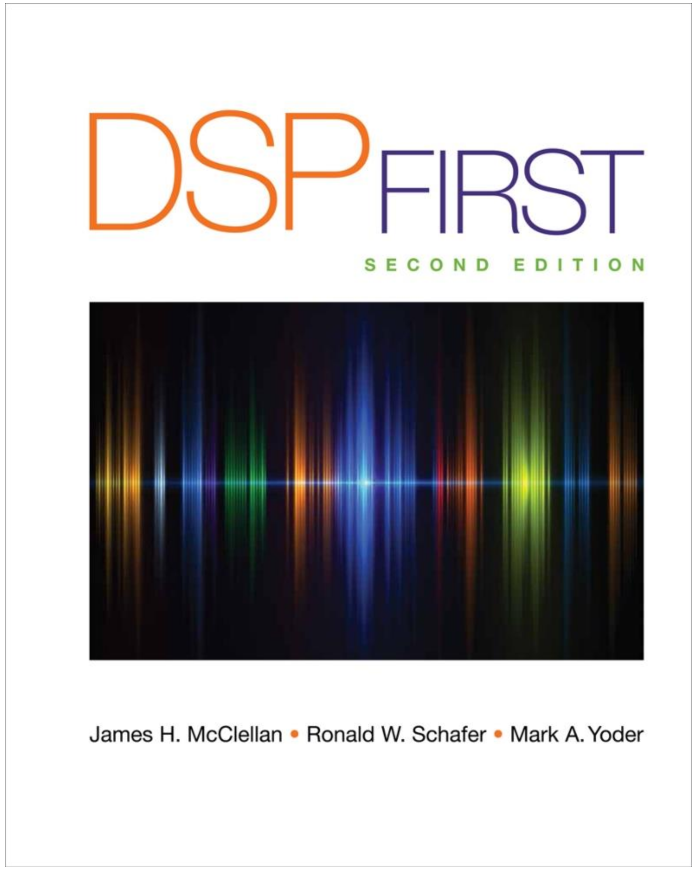
DSP First

Second Edition

- Let’s Master Sinusoids

TLH Modified CENG 3315
CHAPTER 2 2-1 TO 2-3



Chapter 2

Sinusoids

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, Sections 2-1 and 2-2 and 2-3
  - See References for Sinusoids on our website

LET'S VIEW A FEW VIDEOS - SINUSOIDAL REVIEW

1. Dr. Van Veen and Sinusoids 11 Minutes

Introduction to Signal Processing
137,979 views
https://www.youtube.com/watch?v=YmSvQe2FDKs&feature=youtu.be

2. Why Study Sinusoids?
https://www.youtube.com/watch?v=yXjXJ5OlNyQ&feature=youtu.be

3. Example Finding Parameters of a Sinusoid from a Graph **6:19**
https://www.youtube.com/watch?v=h72Eax1jQkw&feature=youtu.be

TUNING FORK EXAMPLE

- IPhone demo
- “A” is at 440 Hertz (Hz)
- Waveform is a SINUSOIDAL SIGNAL
- Computer plot looks like a sine wave
- This should be the mathematical formula:

$$A\cos(2\pi(440)t + \varphi)$$

Tuning-Fork Experiment (1 of 2)

Figure 2-2: Picture of a Tuning Fork for 440 Hz

TRY IT ON THE PHONE!

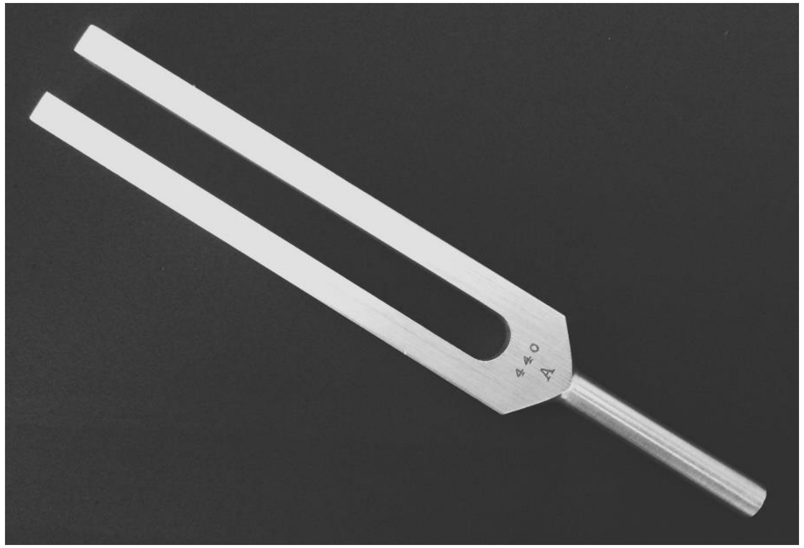
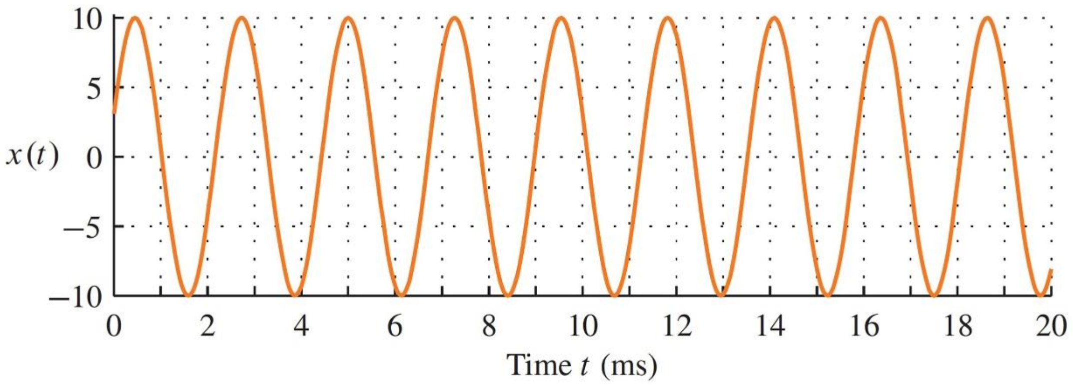


Figure 2-1: Sinusoidal Signal Generated From the Formula: $X(t) = 10\cos\left(2\pi(440)t - 0\cdot 4\pi\right)$



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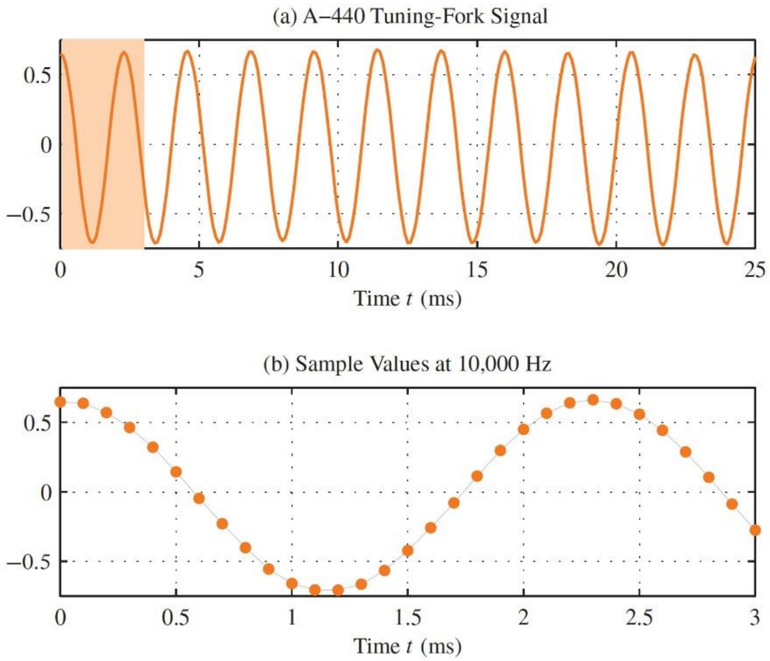
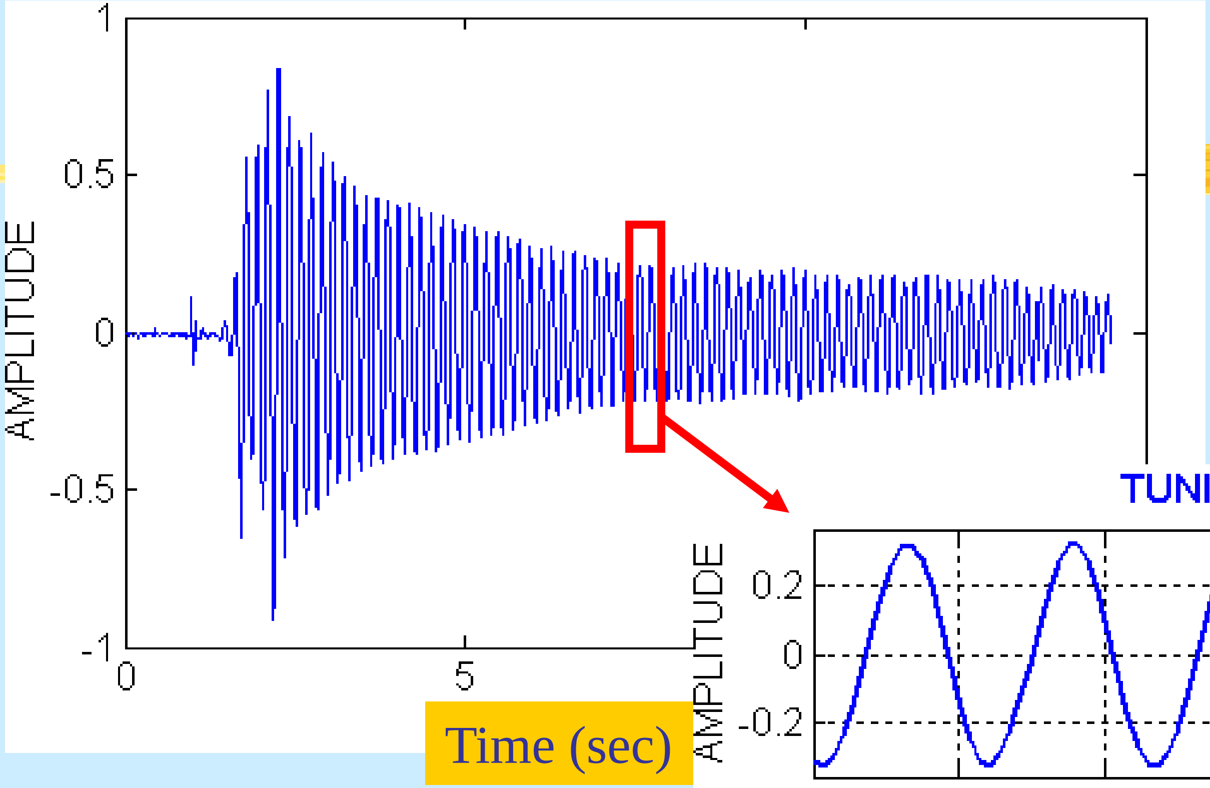
Tuning-Fork Experiment (2 of 2)

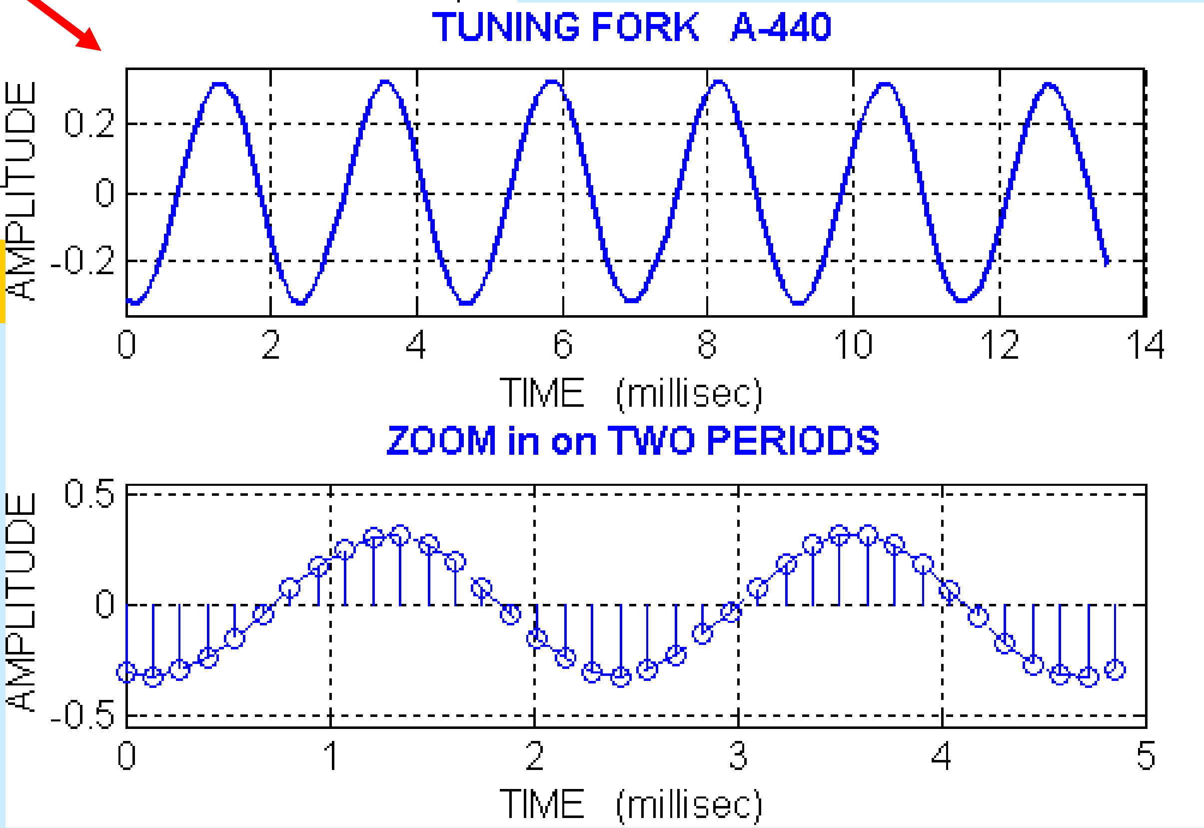
Figure 2-3: (a) Recording of an A–440 tuning-fork signal sampled at a sampling rate of 10,000 Samples/s.b) Zoom in to the first 3ms taken from the top plot (shaded region), showing the individual sample values (connected by a thin gray line).



TUNING FORK A-440 Waveform



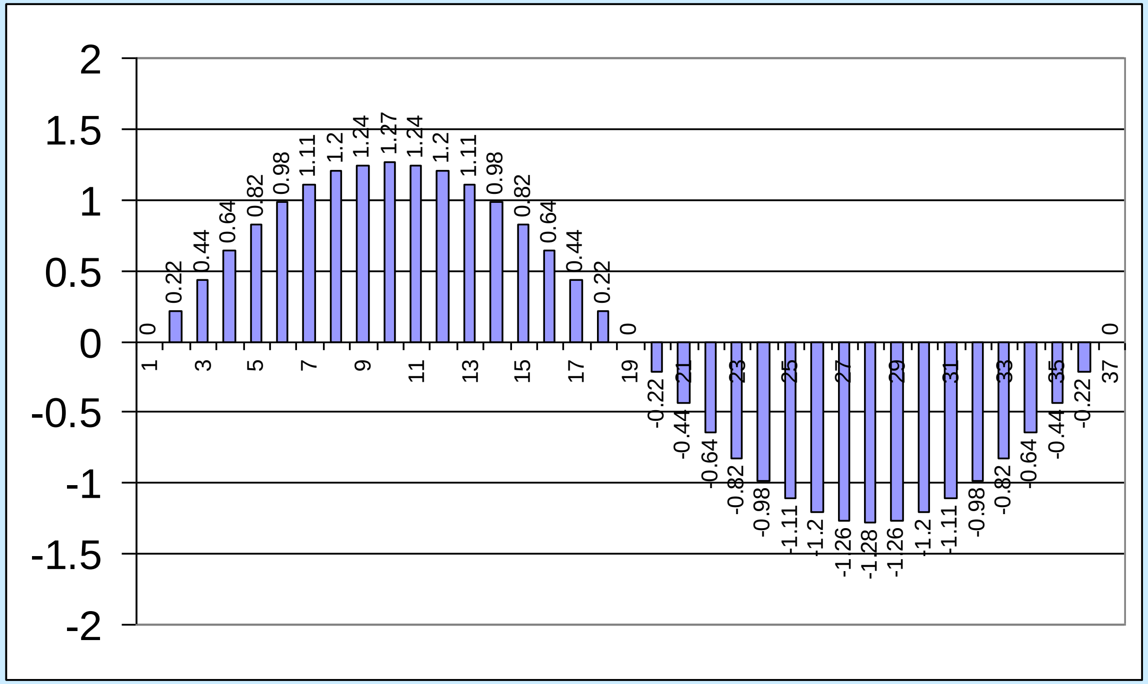
$$T \approx 8.15 - 5.85$$
$$= 2.3\,\mathrm{ms}$$



$$f = 1/T$$
$$= 1000/2.3$$
$$\approx 435\,\mathrm{Hz}$$

Sampled Sinusoid signal might look like this In Compuer.

ARRL



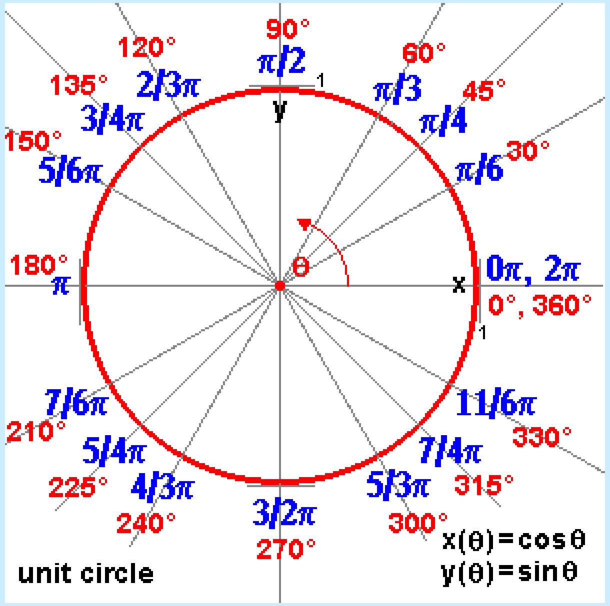
Let’s Master Sinusoids

1. **Circular Motion and Waves (Video :19)** **Shubha Raj Kharel**

   Rotating Vector to Sinusoid :19 Seconds

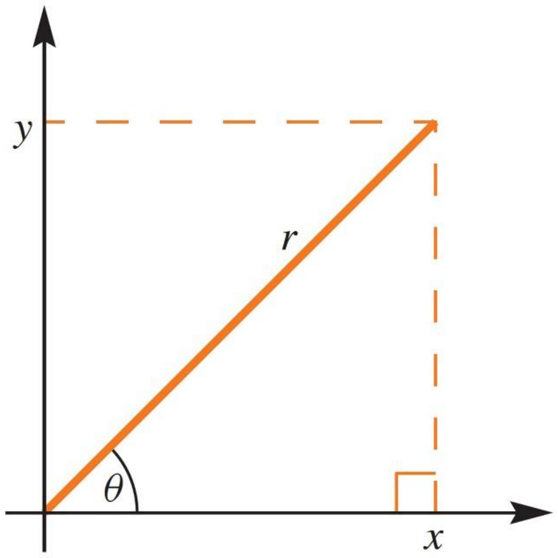
https://youtu.be/EZFlxXPLgr4

Chapter 2 TLH Sinusoids
KNOW THIS!



Angle θ				
Degrees	Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	undefined
180	π	0	-1	0
270	$\frac{3\pi}{2}$	-1	0	undefined
360	2π	0	1	0

Figure 2-4: Definition of Sine and Cosine of an Angle θ within a Right Triangle



$$\sin\theta = \frac{y}{r}$$

$$\Longrightarrow \quad y = r\sin\theta$$

$$\cos\theta = \frac{x}{r}$$

$$\Longrightarrow \quad x = r\cos\theta$$

Figure 2-5: (A) Sine Function and (B) Cosine Function Plotted Versus Angle Θ. Both Functions Have a Period of 2π

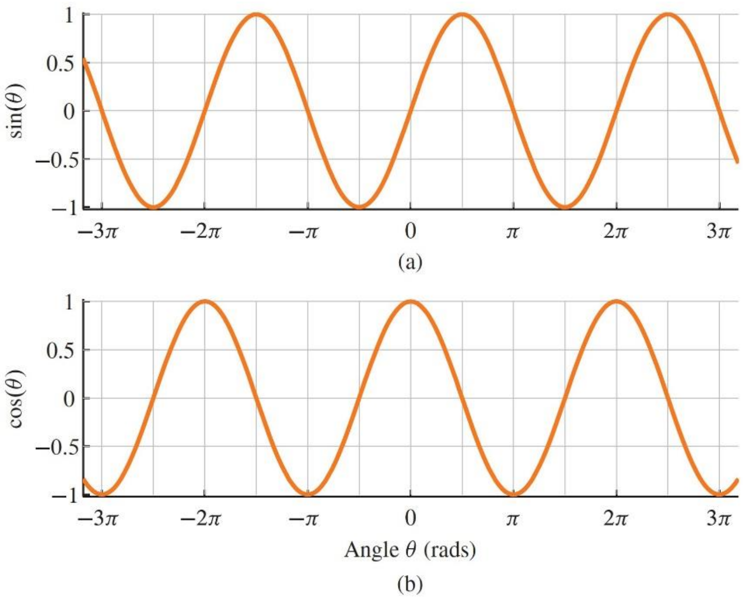


Table 2-1: Basic Properties of the Sine and Cosine Functions

Property	Equation
Equivalence	$\sin\theta = \cos(\theta - \pi/2)$ or $\cos(\theta) = \sin(\theta + \pi/2)$
Periodicity	$\cos(\theta + 2\pi k) = \cos\theta$, when k is an integer
Evenness of cosine	$\cos(-\theta) = \cos\theta$
Oddness of sine	$\sin(-\theta) = -\sin\theta$
Zeros of sine	$\sin(\pi k) = 0$, when k is an integer
Ones of cosine	$\cos(2\pi k) = 1$, when k is an integer
Minus ones of cosine	$\cos[2\pi(k + \frac{1}{2})] = -1$, when k is an integer

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Table 2-2: Some Basic Trigonometric Identities

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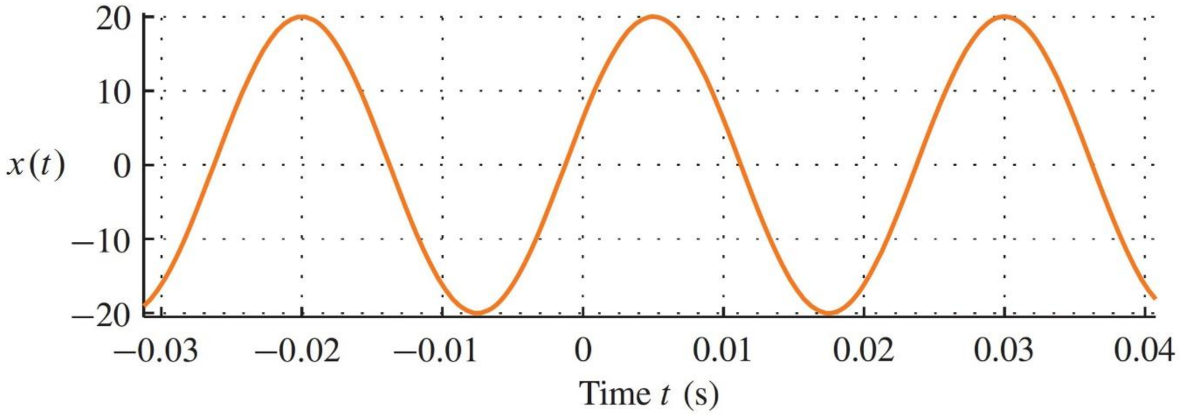
Number	Equation
1	$\sin^2\theta + \cos^2\theta = 1$
2	$\cos 2\theta = \cos^2\theta - \sin^2\theta$
3	$\sin 2\theta = 2\sin\theta\cos\theta$
4	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
5	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Relation of Frequency to Period (1 of 2)

Time-Domain versus Frequency-Domain

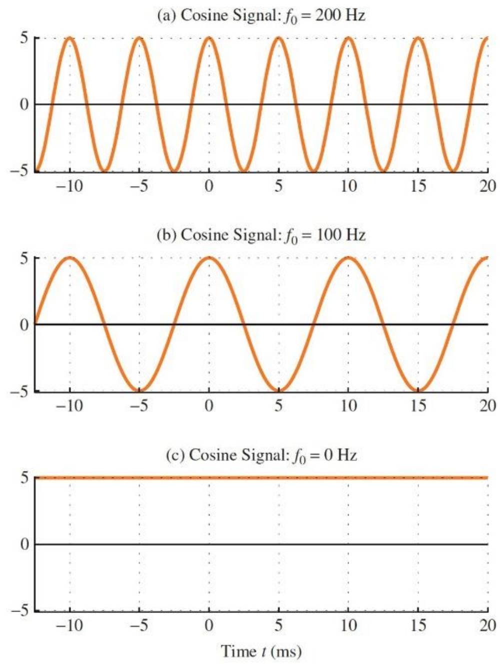
Figure 2-6: Sinusoidal signal with parameters $A = 20, \Omega_0 = 2\pi(40), F_0 = 40\ \text{Hz},\ \text{and}$ $\phi = -0.4\pi rad.$



Relation of Frequency to Period (2 of 2)

Figure 2-7: Cosine Signals $X(t) = 5Cos(2\pi f_0 t)$ for Several Values of $F_0 : (A) F_0 = 200 \text{ Hz};$ $(B) F_0 = 100 \text{ Hz}; (C) F_0 = 0$

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$



$$\omega = 2\pi f$$

SINUSOIDAL SIGNAL

$$A\cos(\omega t+\varphi)$$

- **FREQUENCY** ω
  - Radians/sec
  - Hertz (cycles/sec)

$$\omega=(2\pi)f$$

- **PERIOD** (in sec)

$$T=\frac{1}{f}=\frac{2\pi}{\omega}$$

- **AMPLITUDE** A
  - Magnitude
- **PHASE** φ

Phase and Time Shift

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Figure 2-8: Illustration Of Time-shifting: (A) The Triangular Signal $S(t)$; (B) Shifted to The Right By 2 S, $X_1(t) = S(t-2)$; (C) Shifted To The Left By 1 S, $X_2(t) = S(t+1)$