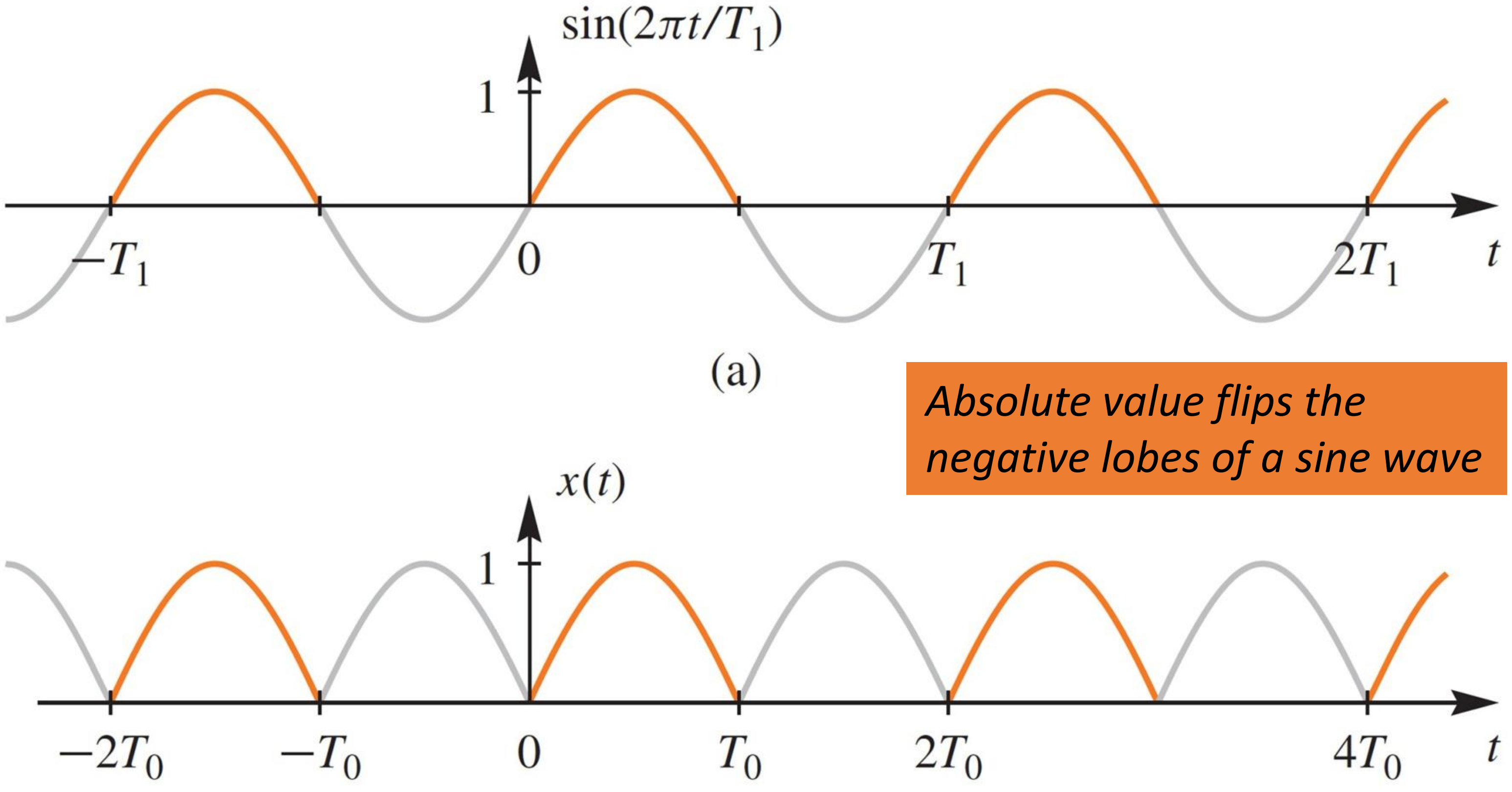
Full Wave Rectified Sinewave

Theory and MATLAB for FWRS

Recall FWRS

$$x(t) = \left|\sin(2\pi t / T_1)\right| \qquad \text{Period is } T_0 = \tfrac{1}{2}T_1$$



(a)

(b)

Absolute value flips the negative lobes of a sine wave

FWRS Fourier Integral → $\{a_k\}$

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-j(2\pi/T_0)kt}\,dt$$

$$a_k = \frac{1}{T_0}\int_0^{T_0} \sin\left(\tfrac{\pi}{T_0}t\right) e^{-j(2\pi/T_0)kt}\,dt$$

$$= \frac{1}{T_0}\int_0^{T_0} \frac{e^{j(\pi/T_0)t} - e^{-j(\pi/T_0)t}}{2j} e^{-j(2\pi/T_0)kt}\,dt$$

$$= \left.\frac{e^{-j(\pi/T_0)(2k-1)t}}{j2T_0(-j(\pi/T_0)(2k-1))}\right|_0^{T_0} - \left.\frac{e^{-j(\pi/T_0)(2k+1)t}}{j2T_0(-j(\pi/T_0)(2k+1))}\right|_0^{T_0}$$

$$a_k = \frac{-2}{\pi(4k^2-1)}$$

Full-Wave Rectified Sine

$$x(t) = \left|\sin(2\pi t/T_1)\right|$$

$$Period: T_0 = \tfrac{1}{2}T_1$$

$$\Rightarrow x(t) = \left|\sin(\pi t/T_0)\right|$$

FWRS Fourier Coeffs: a_k

- a_k is a function of k
  - Complex Amplitude for k-th Harmonic

$$a_k = \frac{-2}{\pi(4k^2 - 1)}$$

  - Does not depend on the period, T_0
  - DC value is

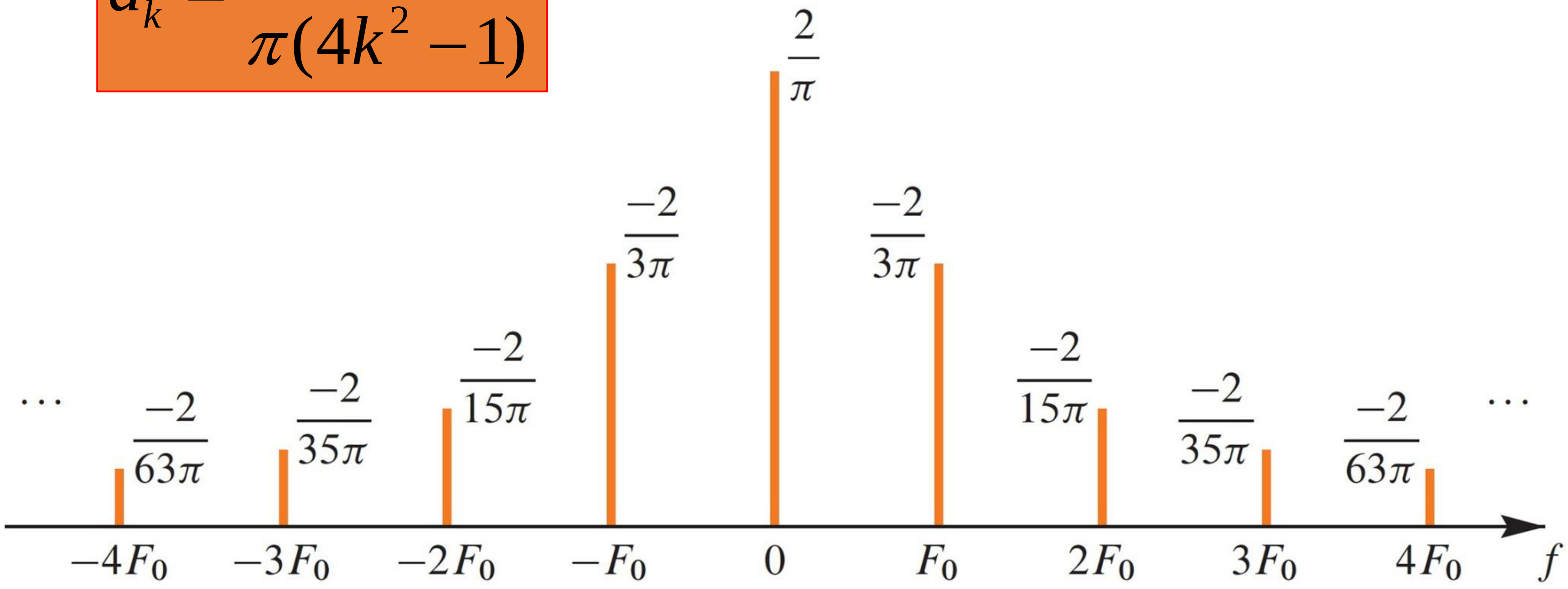
$$a_0 = 2/\pi = 0.6336$$

Spectrum from Fourier Series

Plot a_k for Full-Wave Rectified Sinusoid

$$a_k = \frac{-2}{\pi(4k^2-1)}$$

$F_0 = 1/T_0$ and $\omega_0 = 2\pi F_0$



Fourier Series Synthesis

- HOW do you **APPROXIMATE** x(t) ?

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j(2\pi/T_0)kt}dt$$

- Use FINITE number of coefficients

$$\boxed{x(t) = \sum_{k=-N}^{N} a_k e^{j2\pi kF_0 t}}$$

$$\boxed{a_{-k} = a_k^* \text{ when } x(t) \text{ is real}}$$

Reconstruct From Finite Number of Harmonic Components

Full-Wave Rectified Sinusoid

$$x(t) = \left|\sin(\pi t / T_0)\right|$$

$$T_0 = 10\,\text{ms}$$
$$\Rightarrow F_0 = 100\,\text{Hz}$$

$$a_k = \frac{-2}{\pi(4k^2 - 1)}$$

$$a_0 = 2/\pi = 0.6336$$

$$x_N(t) = a_0 + \sum_{k=1}^{N} \left\{ a_k\, e^{j2\pi k F_0 t} + a_k^*\, e^{-j2\pi k F_0 t} \right\}$$

How close is $x_N(t)$ to $x(t) = \left|\sin(\pi t / T_0)\right|$?

Full-Wave Rectified Sine $\{a_k\}$

EVEN

$$a_k = \frac{-2}{\pi(4k^2-1)} = \frac{-2}{\pi(4k^2-1)} \quad \text{is real} - \text{valued}$$

$$x_N(t) = \sum_{k=-N}^{N} \frac{-2}{\pi(4k^2-1)} e^{jk\omega_0 t}$$

$$= \frac{-2}{-\pi} + \frac{-2}{\pi(4-1)} e^{j\omega_0 t} + \frac{-2}{\pi(4-1)} e^{-j\omega_0 t} + \frac{-2}{\pi(16-1)} e^{j2\omega_0 t} + \frac{-2}{\pi(16-1)} e^{-j2\omega_0 t} \ldots$$

$$= \frac{2}{\pi} - \frac{2}{3\pi} e^{j\omega_0 t} - \frac{2}{3\pi} e^{-j\omega_0 t} - \frac{2}{15\pi} e^{j2\omega_0 t} - \frac{2}{15\pi} e^{-j2\omega_0 t} + \ldots$$

$$= \frac{2}{\pi} - \frac{4}{3\pi}\cos(\omega_0 t) - \frac{4}{15\pi}\cos(2\omega_0 t) - \ldots - \frac{4}{(4N^2-1)\pi}\cos(N\omega_0 t)$$

- Plots for N=4 and N=9 are shown next
- Excellent Approximation for N=9

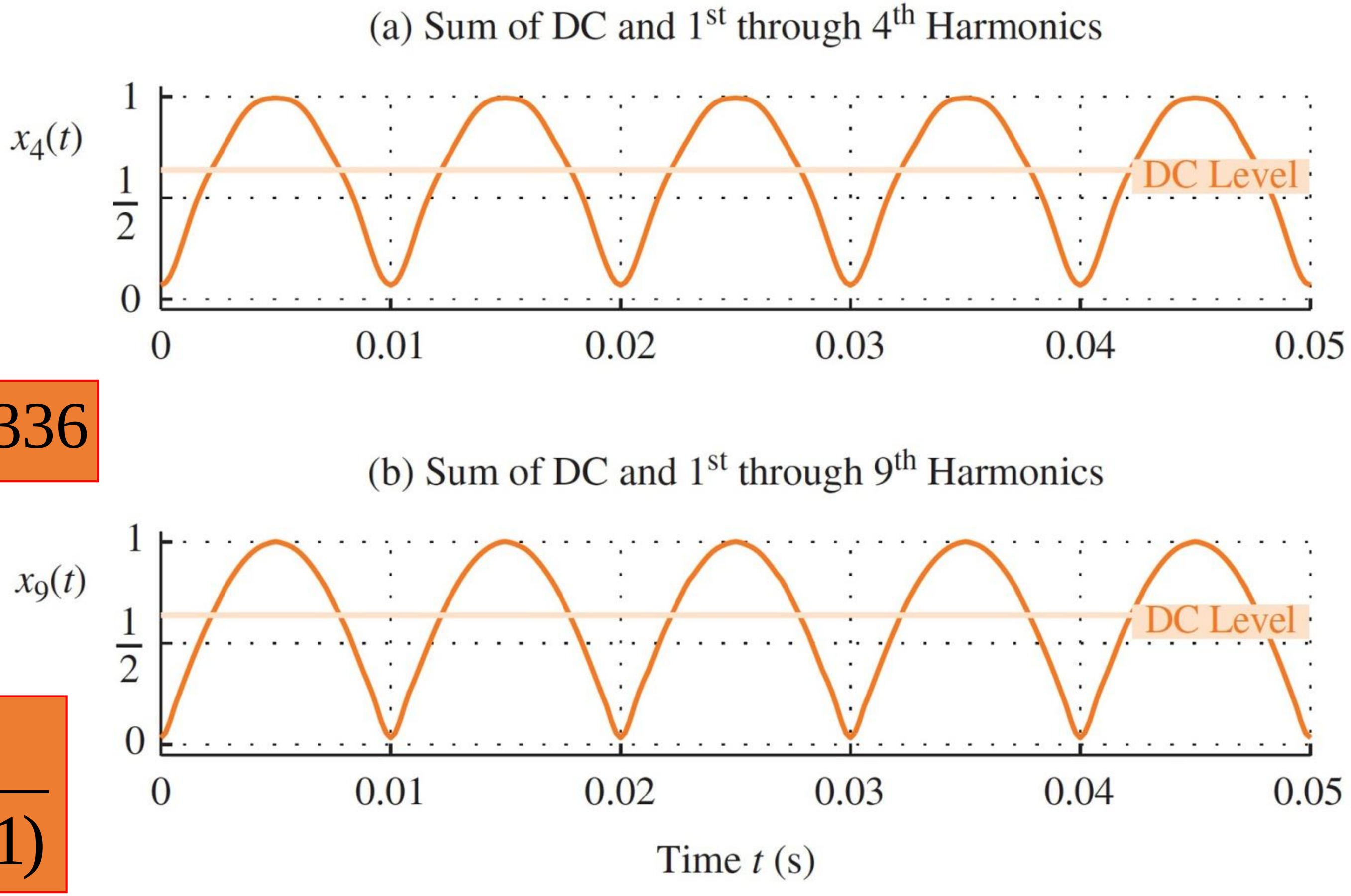
Reconstruct From Finite Number of Spectrum Components

Full-Wave Rectified Sinusoid

$$x(t) = |\sin(\pi t / T_0)|$$

$$T_0 = 10\,\text{ms}$$

$$\Rightarrow F_0 = 100\,\text{Hz}$$



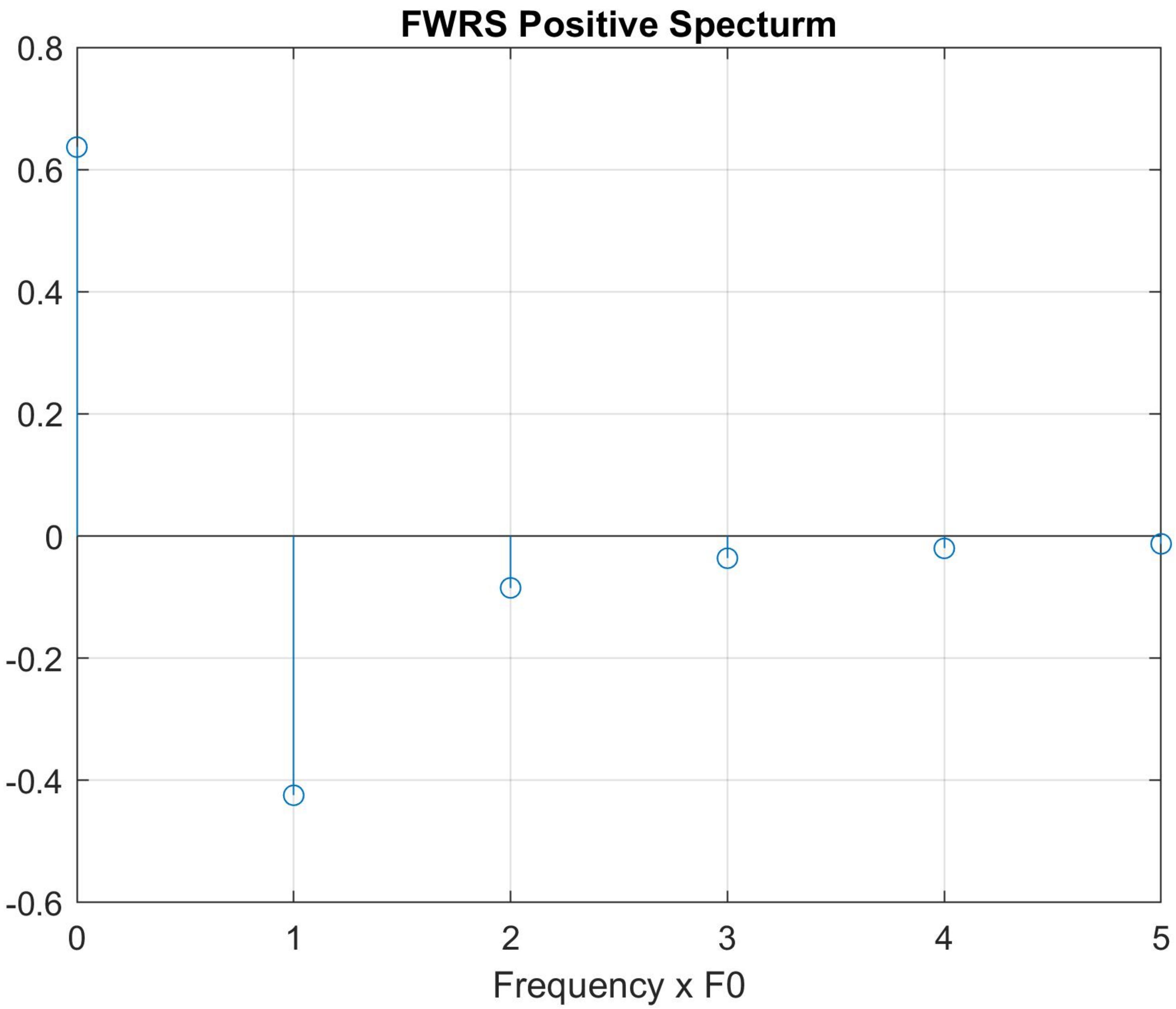
$$a_0 = 2/\pi = 0.6336$$

$$a_k = \frac{-2}{\pi(4k^2 - 1)}$$

MATLAB Vectors Practice

```
>> n= 1:5

n =      1     2     3     4     5
>> Fpos = n.*exp(n)   Multiply element by element

Fpos =     2.7183   14.7781   60.2566  218.3926  742.0658

K>> exp(n)
ans =       2.7183    7.3891    20.0855   54.5982    148.4132
```

```
%
clear all, clc, close all
k=1:5      % k =  1     2     3     4     5
a0 = 2/pi  % a0 = 0.6366
ak=2./(pi*(1-4*k.^2))  %  Element by Element
%  ak =    -0.2122   -0.0424   -0.0182   -0.0101   -0.0064

fplus  =2* [-2/(3*pi) -2/(15*pi)  -2/(35*pi) -2/(63*pi) -2/(99*pi)]
% fplus = -0.4244   -0.0849   -0.0364   -0.0202   -0.0129
%
fpos = [2/pi fplus]
% fpos =  0.6366   -0.4244   -0.0849   -0.0364   -0.0202   -0.0129

k1= [0 k] % k1 = 0     1     2     3     4     5
figure(1)
stem(k1,fpos); grid, xlabel('Frequency x F0')
title('FWRS Positive Specturm')
```


FWRS Positive Specturm
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
1
2
3
4
5
Frequency x F0