DFT Ch8_ProblemSession 1

Example 8-1: In order to compute the 4-point DFT of the sequence $x[n] = \{1, 1, 0, 0\}$, we carry out the sum (**??**) four times, once for each value of $k = 0, 1, 2, 3$. When $N = 4$, all the exponents in (**??**) are integer multiples of $\pi/2$ because $2\pi/N = \pi/2$.

$$\begin{aligned}
X[0] &= \overset{1}{x[0]}e^{-j0} + \overset{1}{x[1]}e^{-j0} + \overset{0}{x[2]}e^{-j0} + \overset{0}{x[3]}e^{-j0} \\
&= 1 + 1 + 0 + 0 = 2 \\
X[1] &= 1e^{-j0} + 1e^{-j\pi/2} + 0e^{-j\pi} + 0e^{-j3\pi/2} \\
&= 1 + (-j) + 0 + 0 = 1 - j = \sqrt{2}e^{-j\pi/4} \\
X[2] &= 1e^{-j0} + 1e^{-j\pi} + 0e^{-j2\pi} + 0e^{-j3\pi} \\
&= 1 + (-1) + 0 + 0 = 0 \\
X[3] &= 1e^{-j0} + 1e^{-j3\pi/2} + 0e^{-j3\pi} + 0e^{-j9\pi/2} \\
&= 1 + (j) + 0 + 0 = 1 + j = \sqrt{2}e^{j\pi/4}
\end{aligned}$$

Thus, we obtain the four DFT coefficients $X[k] = \{2,\ \sqrt{2}e^{-j\pi/4},\ 0,\ \sqrt{2}e^{j\pi/4}\}$.

Note: The term "coefficient" is commonly applied to DFT values. This is appropriate because $X[k]$ is the (complex amplitude) coefficient of $e^{j(2\pi/N)kn}$ in the IDFT (**??**).

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Example 8-2: The 4-point DFT in Example 8-1 is the sequence $X[k] = \{2,\ \sqrt{2}e^{-j\pi/4},\ 0,\ \sqrt{2}e^{j\pi/4}\}$. If we compute the 4-point IDFT of this $X[k]$, we should recover $x[n]$ when we apply the IDFT summation (**??**) for each value of $n = 0, 1, 2, 3$. As before, the exponents in (**??**) are all integer multiples of $\pi/2$ when $N = 4$.

$$\begin{aligned}
x[0] &= \tfrac{1}{4}\left(X[0]e^{j0} + X[1]e^{j0} + X[2]e^{j0} + X[3]e^{j0}\right) \\
&= \tfrac{1}{4}\left(2 + \sqrt{2}e^{-j\pi/4} + 0 + \sqrt{2}e^{j\pi/4}\right) = \tfrac{1}{4}(2 + (1-j) + 0 + (1+j)) = 1 \\
x[1] &= \tfrac{1}{4}\left(X[0]e^{j0} + X[1]e^{j\pi/2} + X[2]e^{j\pi} + X[3]e^{j3\pi/2}\right) \\
&= \tfrac{1}{4}\left(2 + \sqrt{2}e^{j(-\pi/4+\pi/2)} + 0 + \sqrt{2}e^{j(\pi/4+3\pi/2)}\right) = \tfrac{1}{4}(2 + (1+j) + (1-j)) = 1 \\
x[2] &= \tfrac{1}{4}\left(X[0]e^{j0} + X[1]e^{j\pi} + X[2]e^{j2\pi} + X[3]e^{j3\pi}\right) \\
&= \tfrac{1}{4}\left(2 + \sqrt{2}e^{j(-\pi/4+\pi)} + 0 + \sqrt{2}e^{j(\pi/4+3\pi)}\right) = \tfrac{1}{4}(2 + (-1+j) + (-1-j)) = 0 \\
x[3] &= \tfrac{1}{4}\left(X[0]e^{j0} + X[1]e^{j3\pi/2} + X[2]e^{j3\pi} + X[3]e^{j9\pi/2}\right) \\
&= \tfrac{1}{4}\left(2 + \sqrt{2}e^{j(-\pi/4+3\pi/2)} + 0 + \sqrt{2}e^{j(\pi/4+9\pi/2)}\right) = \tfrac{1}{4}(2 + (-1-j) + (-1+j)) = 0
\end{aligned}$$

Thus, we have verified that the length-4 signal $x[n] = \{1, 1, 0, 0\}$ can be recovered from its 4-point DFT coefficients, $X[k] = \left\{2,\ \sqrt{2}e^{-j\pi/4},\ 0,\ \sqrt{2}e^{j\pi/4}\right\}$.

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