DSP-First, 2/e

MODIFIED TLH

LECTURE 4 # Ch2
Phasor Addition Theorem

ADDING PHASORS WITH THE SAME FREQUENCY

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, Section 2-6

- Other Reading:
  - Appendix A: Complex Numbers

Dr. Van Veen Strikes Again

3. Complex Numbers Review (Wouldn't hurt to review) 10:22

Review of how to work with complex numbers in rectangular and polar coordinates.

https://www.youtube.com/watch?v=UAn9uah7puU&list=PLGI7M8vwfrFNO-gQ1xoJmN3bJy2-wp2J3

4. Complex Sinusoid Representations for Real Sinusoids 13:21

https://www.youtube.com/watch?v=Tm3gI6PQOYo&feature=youtu.be

PHASOR ADDITION RULE

Page 29

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \varphi_k)$$

$$= A\cos(\omega_0 t + \varphi)$$

Get the new complex amplitude by complex addition

Find Amplitude and Phase – ω is Known!

$$\sum_{k=1}^{N} A_k e^{j\varphi_k} = Ae^{j\varphi}$$

LECTURE OBJECTIVES

- Phasors = Complex Amplitude
  - Complex Numbers **represent** Sinusoids

$$A\cos(\omega t + \varphi) = \Re\{(Ae^{j\varphi})e^{j\omega t}\}$$

- Develop the ABSTRACTION:
  - Adding Sinusoids = Complex Addition
  - **PHASOR ADDITION THEOREM**

Adding Complex Numbers

- Polar Form
  - Could convert to Cartesian and back out
  - Use MATLAB
  - Visualize the vectors

Cos = REAL PART

What about sinusoidal signals over time?

Real part of Euler's

$$\cos(\omega t) = \Re\{e^{j\omega t}\}$$

General Sinusoid

$$A\cos(\omega t + \varphi) = \Re\{Ae^{j(\omega t + \varphi)}\}$$

$$= \Re\{Ae^{j\varphi}e^{j\omega t}\}$$

Complex Amplitude: Constant — Varies with time

POP QUIZ: Complex Amp

- Find the COMPLEX AMPLITUDE for:

$$x(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

- Use EULER's FORMULA:

$$x(t) = \Re\{\sqrt{3}e^{j(77\pi t + 0.5\pi)}\}$$

$$= \Re\{\sqrt{3}e^{j0.5\pi}e^{j77\pi t}\}$$

$$X = \sqrt{3}e^{j0.5\pi}$$

POP QUIZ-2: Complex Amp

- Determine the 60-Hz sinusoid whose COMPLEX AMPLITUDE is:

$$X = \sqrt{3} + j3$$

- Convert ***X*** to **POLAR**:

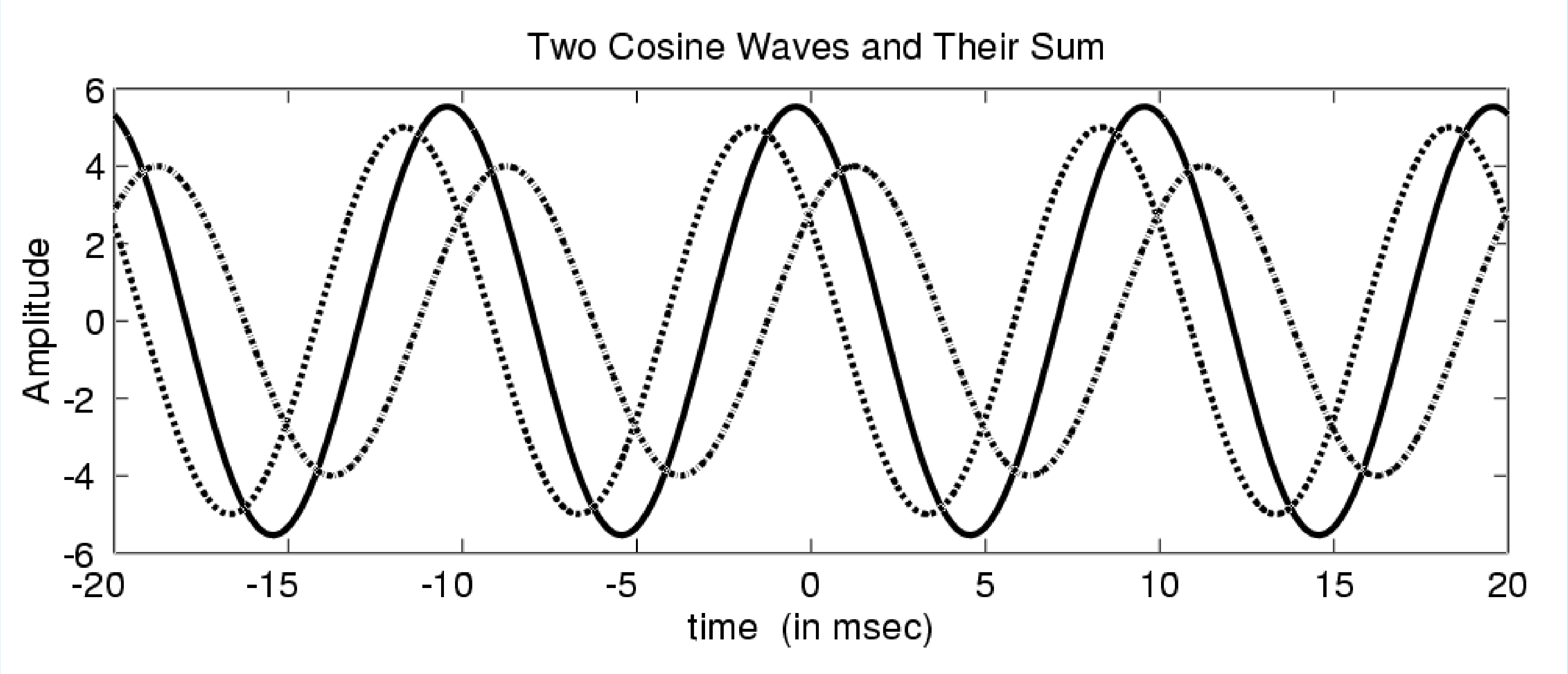
$$x(t) = \Re\{(\sqrt{3} + j3)e^{j(120\pi t)}\}$$

$$= \Re\{\sqrt{12}e^{j\pi/3}e^{j120\pi t}\}$$

$$\Rightarrow x(t) = \sqrt{12}\cos(120\pi t + \pi/3)$$

WANT to ADD SINUSOIDS

- **Main point to remember**: Adding sinusoids of common frequency results in sinusoid with **SAME** frequency



PHASOR ADDITION RULE

Page 29

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \varphi_k)$$

$$= A\cos(\omega_0 t + \varphi)$$

Get the new complex amplitude by complex addition

$$\sum_{k=1}^{N} A_k e^{j\varphi_k} = Ae^{j\varphi}$$

Phasor Addition Proof

$$\sum_{k=1}^{N} A_k \cos(\omega_0 t + \varphi_k) = \sum_{k=1}^{N} \Re\{A_k e^{j(\omega_0 t + \varphi_k)}\}$$

$$= \Re\left\{ \sum_{k=1}^{N} A_k e^{j\varphi_k} e^{j\omega_0 t} \right\}$$

$$= \Re\left\{ \left(\sum_{k=1}^{N} A_k e^{j\varphi_k} \right) e^{j\omega_0 t} \right\}$$

$$= \Re\left\{ \left(A e^{j\varphi} \right) e^{j\omega_0 t} \right\} = A\cos(\omega_0 t + \varphi)$$

POP QUIZ: Add Sinusoids

- ADD THESE 2 SINUSOIDS:

$$x_1(t) = \cos(77\pi t - \pi)$$

$$x_2(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

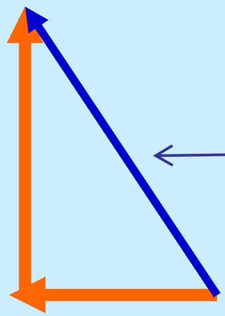
- COMPLEX (PHASOR) ADDITION:

$$1e^{-j\pi} + \sqrt{3}e^{j0.5\pi}$$

POP QUIZ (answer)

- COMPLEX ADDITION: $1e^{-j\pi} + \sqrt{3}e^{j0.5\pi}$

$\sqrt{3}e^{j\pi/2} = j\sqrt{3}$



$-1 + j\sqrt{3} = 2e^{j2\pi/3}$

$e^{-j\pi} = -1$

- CONVERT back to cosine form:

$$x_3(t) = 2\cos(77\pi t + \tfrac{2\pi}{3})$$

ADD SINUSOIDS EXAMPLE

- ALL SINUSOIDS have SAME FREQUENCY
- HOW to GET {Amp,Phase} of RESULT ?

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t) = A\cos(\omega t + \varphi)$$

$$= \Re\{Ae^{j\varphi}e^{j20\pi t}\}$$

It is important to go between the sinusoidal form and the phasor form. Assume the frequencies of the sinusoids are the same. We know the sum of such sinusoids will be a sinusoid of the same frequency.

Take the 10 Hz sinusoids (DSP First Page 31)

$$\begin{aligned} x_1(t) &= 1.7\cos(20\pi t + 70\pi/180) \\ x_2(t) &= 1.9\cos(20\pi t + 200\pi/180) \end{aligned} \tag{2}$$

The phasors involved are

$$\begin{aligned} X_1 &= A_1 e^{j\phi_1} = 1.7\, e^{j70\pi/180} \\ X_2 &= A_2 e^{j\phi_2} = 1.9\, e^{j200\pi/180} \end{aligned} \tag{3}$$

Then the steps to form $x_3(t) = x_1(t) + x_2(t)$ is as follows:

1. Convert both phasors to Rectangular form
2. Add the real and imaginary parts
3. Convert back to polar for the phasor X_3
4. Convert to the cosine form.

Can we do these steps?
Sure but MATLAB saves us!

```
% Convert Phasor to rectangular
format short
x1=1.7*exp(j*70*pi/180)
  % x1= 0.5814+ 1.5975i
x2=1.9*exp(j*200*pi/180)
 % x2 = -1.7854 - 0.6498i
x3=x1+x2  % x3 = -1.2040 + 0.9476i
 % Convert x3 to polar
magx3=abs(x3)  % magx3   = 1.5322
x3theta=angle(x3) % x3theta = 2.4748 rad
thetadeg=x3theta*180/pi
% thetadeg =   141.7942 degrees
```

Piece of Cake!

Convert Sinusoids to Phasors

- Each sinusoid → Complex Amp

$$1.7\cos(20\pi t + 70\pi/180) \rightarrow 1.7e^{j70\pi/180}$$

$$1.9\cos(20\pi t + 200\pi/180) \rightarrow 1.9e^{j200\pi/180}$$

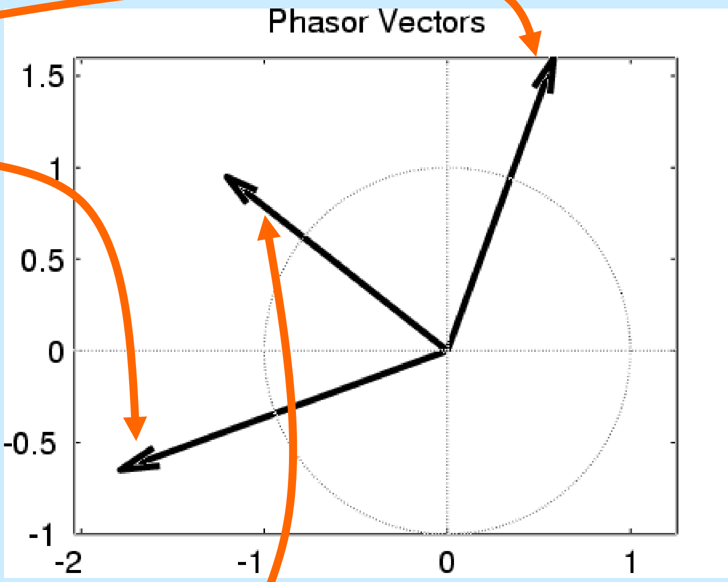
$$1.7e^{j70\pi/180} + 1.9e^{j200\pi/180} = ?$$

$$1.532e^{j141.79\pi/180}$$

$$\rightarrow 1.532\cos(20\pi t + 141.79\pi/180)$$

Phasor Add: Numerical

- Convert Polar to Cartesian
  - $X_1 = 0.5814 + j1.597$
  - $X_2 = -1.785 - j0.6498$
  - sum =
  - $X_3 = -1.204 + j0.9476$



- Convert back to Polar
  - $X_3 = 1.532$ at angle $141.79\pi/180$
  - This is the sum

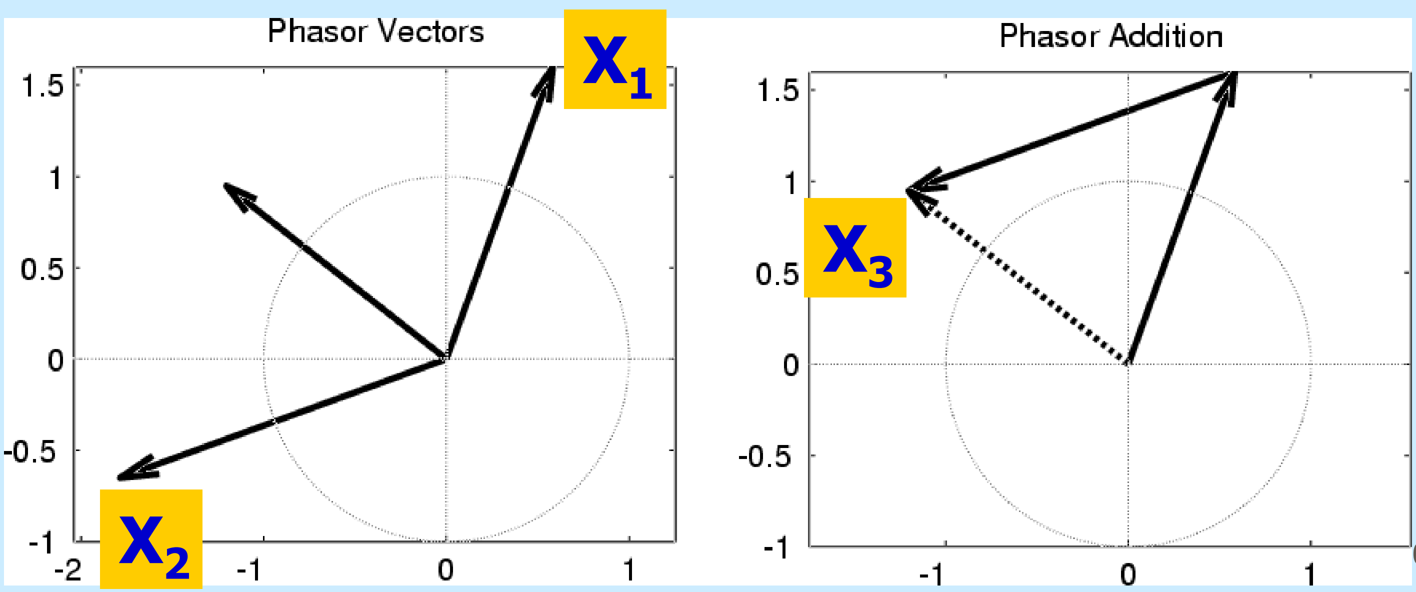
ADDING SINUSOIDS IS COMPLEX ADDITION

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$
$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

VECTOR (PHASOR) ADD



Phasor Addition Rule: Example

Figure 2-15: (a) Adding sinusoids by doing a phasor addition, which is actually a graphical vector sum. (b) The time of the signal maximum is marked on each $X_j(t)$ plot.

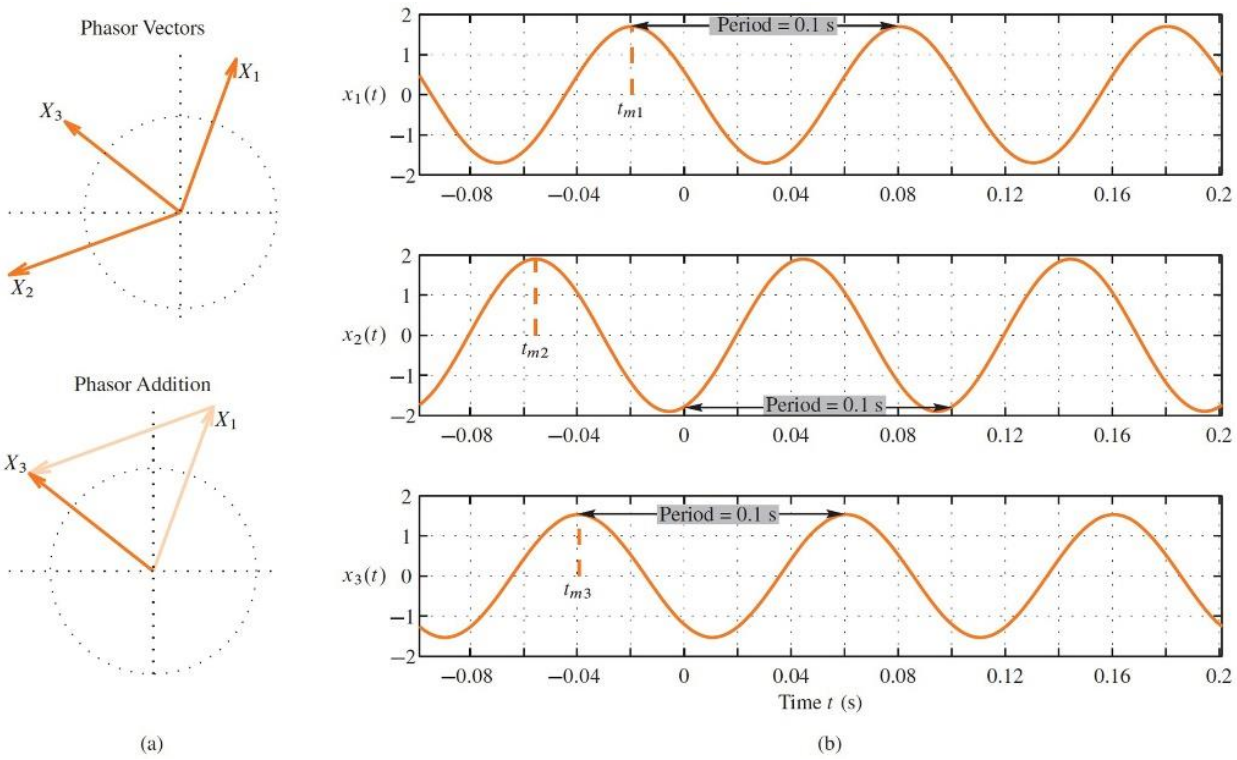


Table 2-3: Phasor Addition Example

Z	= X	+ jY	Magnitude	Phase	Ph/pi	Ph(deg)
Z1	0.5814	1.5975	1.7	1.222	0.389	70.00
Z2	-1.7854	-0.6498	1.9	-2.793	-0.889	-160.00
Z3	-1.204	0.9477	1.5322	2.475	0.788	141.79