3315_Ch4_6_Ch6_ Presentation3

FREQUENCY RESPONSE OF FIR FILTERS

RECORD ON

LECTURE OBJECTIVES

- SINUSOIDAL INPUT SIGNAL
  - DETERMINE the FIR FILTER OUTPUT

- FREQUENCY RESPONSE of FIR
  - PLOTTING vs. Frequency
  - MAGNITUDE vs. Freq
  - PHASE vs. Freq

MAG

PHASE

$$H(e^{j\hat{\omega}}) = \left|H(e^{j\hat{\omega}})\right| e^{j\angle H(e^{j\hat{\omega}})}$$

Amplitude and magnitude , both these terms are similar with a minute difference.
The amplitude of a variable is the measure of how far, and in what direction, that variable differs from zero. Thus, signal amplitudes can be either positive or negative.

The **magnitude** of a variable, on the other hand, is the measure of how far, regardless of direction, its quantity differs from zero. So magnitudes are always positive values.
Magnitude of x is represented by | | symbol. In 1D this is Absolute value. In 2D, it is length of a vector
SOURCE :
UNDERSTANDING DSP by RICHARD G. LYONS

1. ***INPUT*** $A\cos(2\pi(100)t)$ TO A LINEAR SYSTEM

OUTPUT $B\cos(2\pi(100)t+\varphi)$

CHANGE IN AMPLITUDE from A to B

AND PHASE from 0 to φ (TYPICALLY A TIME DELAY)

2. $Ae^{j\omega}$ = $A\cos(2\pi f) + jA\sin(2\pi f)$ = $|A| \angle \omega$

THIS IS A VECTOR WITH **MAGNITUDE** (LENGTH) A AND **ANGLE IN ω 2D**

TIME & FREQUENCY

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k] x[n-k]$$

FIR DIFFERENCE EQUATION is the TIME-DOMAIN

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega} k}$$

$$\begin{aligned} H(e^{j\hat{\omega}}) &= h[0] + h[1]e^{-j\hat{\omega}} + h[2]e^{-j2\hat{\omega}} + h[3]e^{-j3\hat{\omega}} + \cdots \\ &= \left|H(e^{j\hat{\omega}})\right| e^{j\angle H(e^{j\hat{\omega}})} \end{aligned}$$

COMPLEX EXPONENTIAL

$$x[n] = Ae^{j\varphi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

x[n] is the input signal—a complex exponential

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

FIR DIFFERENCE EQUATION

COMPLEX EXP OUTPUT

- Use the FIR "Difference Equation"

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} b_k A e^{j\varphi} e^{j\hat{\omega}(n-k)}$$

$$= \left(\sum_{k=0}^{M} b_k e^{j\hat{\omega}(-k)} \right) A e^{j\varphi} e^{j\hat{\omega} n}$$

$$= H(\hat{\omega}) A e^{j\varphi} e^{j\hat{\omega} n}$$

H IS THE TRANSFER FUNCTION

FREQUENCY RESPONSE

Example 6.1 $b_k = \{\underset{\uparrow}{1}, 2, 1\}$ so

$$H(e^{j\hat{\omega}}) = 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2}$$

write as

$$= e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})$$

To plot

$$-\pi < \hat{\omega} < \pi$$

This is $|H(e^{j\hat{\omega}})| \angle{-\hat{\omega}}$

EXAMPLE 6.1

REMEMBER - PI < $\hat{\omega}$ < PI

$$\{b_k\} = \{1,\ 2,\ 1\}$$

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \\ &= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}) \\ &= e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega}) \end{aligned}$$

EXPLOIT SYMMETRY

Since $(2 + 2\cos\hat{\omega}) \geq 0$

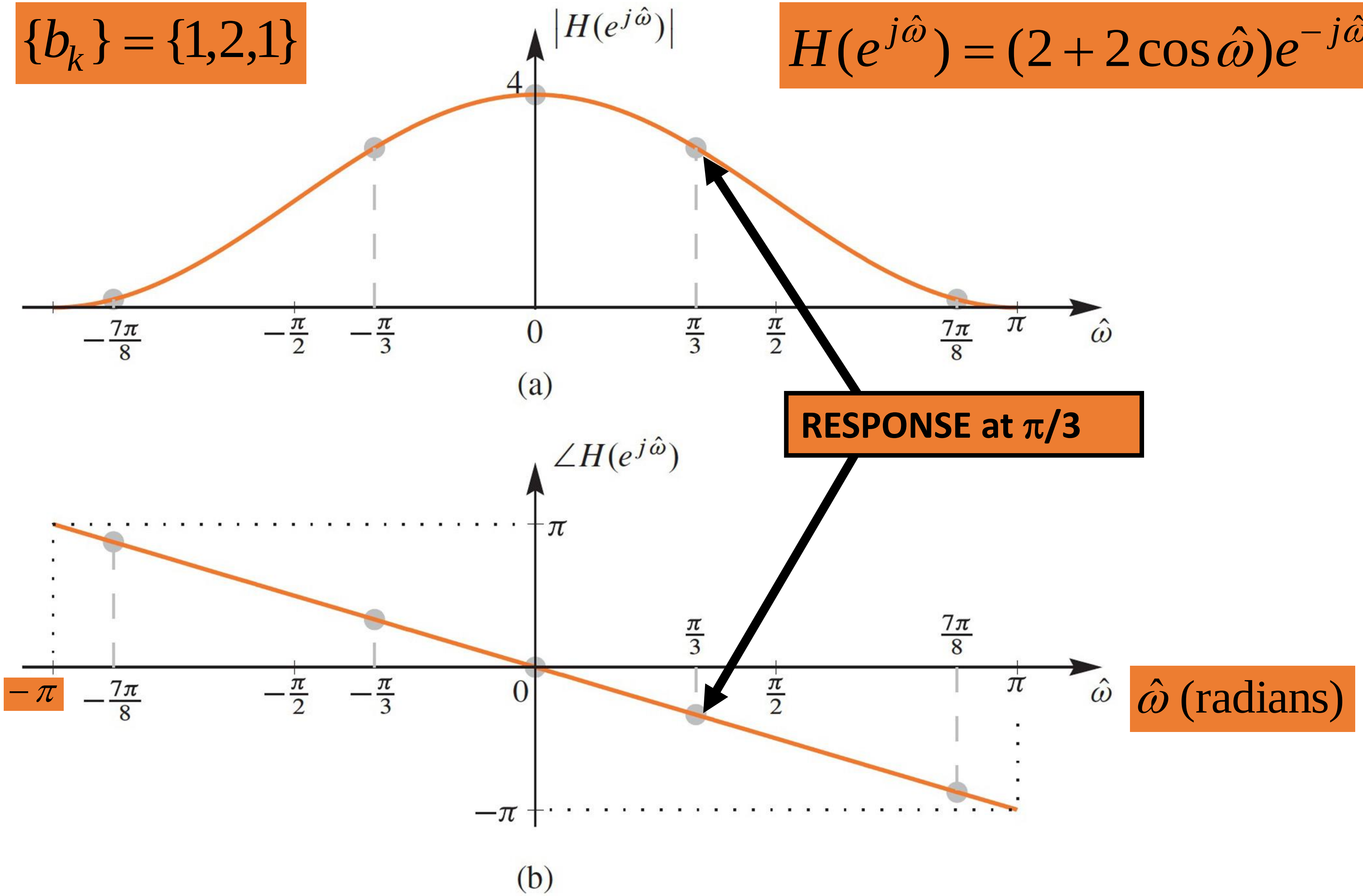
Magnitude is $\left|H(e^{j\hat{\omega}})\right| = (2 + 2\cos\hat{\omega})$

and Phase is $\angle H(e^{j\hat{\omega}}) = -\hat{\omega}$

PLOT of FREQ RESPONSE

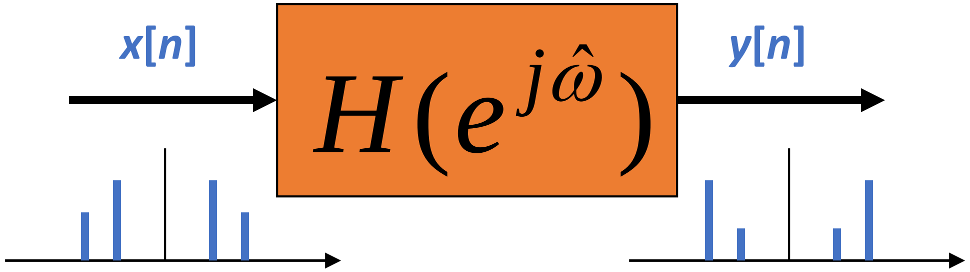
$\{b_k\} = \{1,2,1\}$

$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$



EXAMPLE 6.2

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EXAMPLE 6.2 (answer)

$$\text{Find } y[n] \text{ when } x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$$

$$\text{Evaluate } H(e^{j\hat{\omega}}) \text{ at } \hat{\omega} = \pi/3$$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @\,\hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6e^{-j\pi/12}e^{j(\pi/3)n}$$

component separately.

EXAMPLE 6-3 Cosine Input

For the FIR filter with coefficients $\{b_k\} = \{1, 2, 1\}$, find the output when the input is

$$x[n] = 3\cos\left(\frac{\pi}{3}n - \frac{\pi}{2}\right)$$

The frequency response of the system was determined in Example 6-1 to be

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

Note that $H(e^{-j\hat{\omega}}) = H^*(e^{j\hat{\omega}})$, so $H(e^{j\hat{\omega}})$ has conjugate symmetry. Solution of this problem requires just one evaluation of $H(e^{j\hat{\omega}})$ at the frequency $\hat{\omega} = \pi/3$:

$$\begin{aligned} H(e^{j\pi/3}) &= e^{-j\pi/3}(2 + 2\cos(\pi/3)) \\ &= e^{-j\pi/3}(2 + 2(\tfrac{1}{2})) = 3e^{-j\pi/3} \end{aligned}$$

Therefore, the magnitude is $|H(e^{j\pi/3})| = 3$ and the phase is $\angle H(e^{j\pi/3}) = -\pi/3$, so the output is

$$\begin{aligned} y[n] &= (3)(3)\cos\left(\frac{\pi}{3}n - \frac{\pi}{3} - \frac{\pi}{2}\right) \\ &= 9\cos\left(\frac{\pi}{3}(n-1) - \frac{\pi}{2}\right) \end{aligned}$$

Notice that the magnitude of the frequency response multiplies the amplitude of the cosine signal, and the phase angle of the frequency response adds to the phase of the ...
signal. This problem could also be studied ...

Example: Lowpass Averager

- Consider a 5-point moving average filter wrapped up between a C-to-D and D-to-C system
- We assume a sampling rate of 1000 Hz and an input composed of two sinusoids

$$x(t) = \cos[2\pi(100)t] + 3\cos[2\pi(300)t]$$

- Find the system frequency response in terms of the analog frequency variable f, and find the steady-state output $y(t)$
- We will use `freqz()` to obtain the frequency response

```
>> w = -pi:pi/100:pi;
>> H = freqz(ones(1,5)/5,1,w);
>> subplot(211)
```

SOLVE THE PROBLEM BEFORE MATLAB – IF POSSIBLE!
AT LEAST KNOW THE RANGES INVOLVED

$f_s = 1000\text{ Hz}$ so $f_{max} = 500\text{ Hz}$

$x(t) = \cos\{2\pi(100t)\} + 3\cos\{2\pi\, 300t\}$

$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$ $\quad \hat{\omega}_{100} = 2\pi \frac{100}{1000} = 0.2\pi \quad \hat{\omega}_{300} = 0.6\pi$

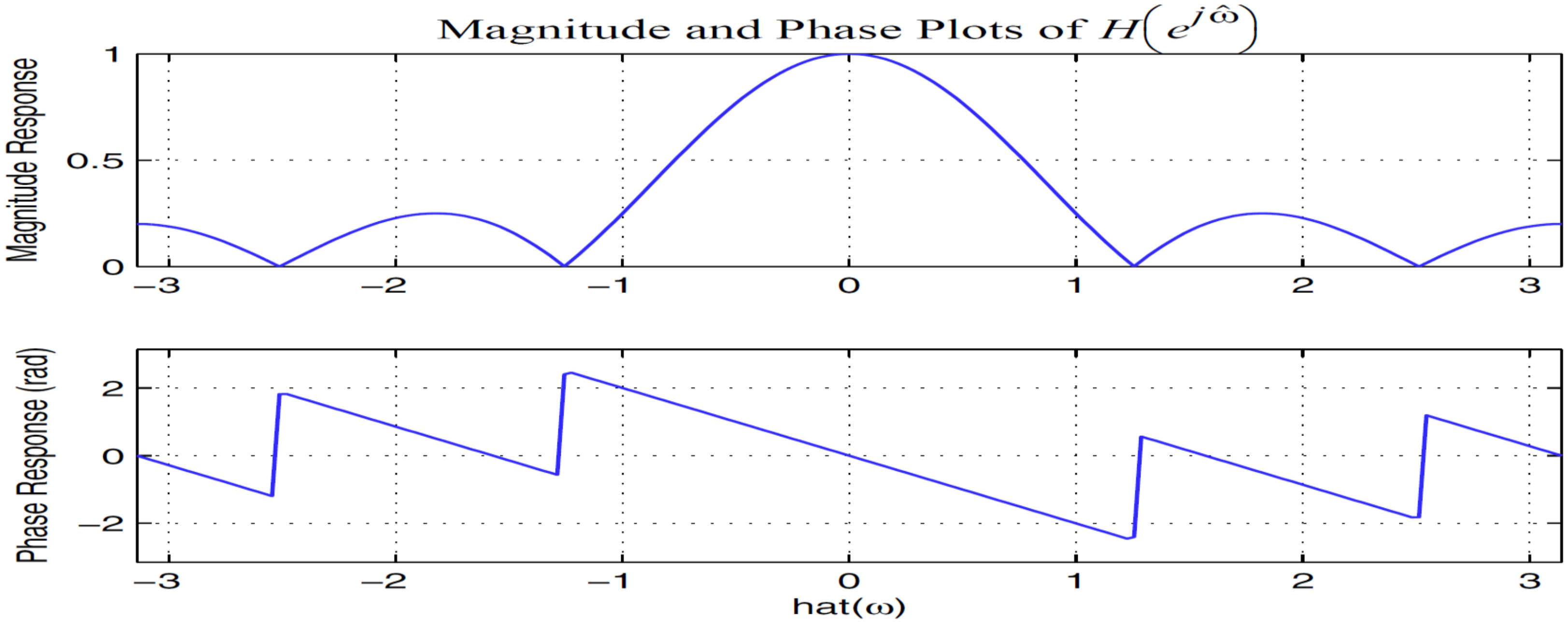
Range of $\hat{\omega}$ $\quad -\frac{500}{1000} \times 2\pi < \hat{\omega} < \frac{500}{1000} 2\pi$

Digital PLOT $\quad -\pi < \hat{\omega} < \pi$

Analog PLOT $\quad -500\text{ Hz} < f < 500\text{ Hz}$

```
>> plot(w,abs(H))
>> axis([-pi pi 0 1]); grid
>> ylabel('Magnitude Response')
>> subplot(212)
>> plot(w,angle(H))
>> axis([-pi pi -pi pi]); grid
>> ylabel('Phase Response (rad)')
>> xlabel('hat(\omega)')
```

Range –pi to pi

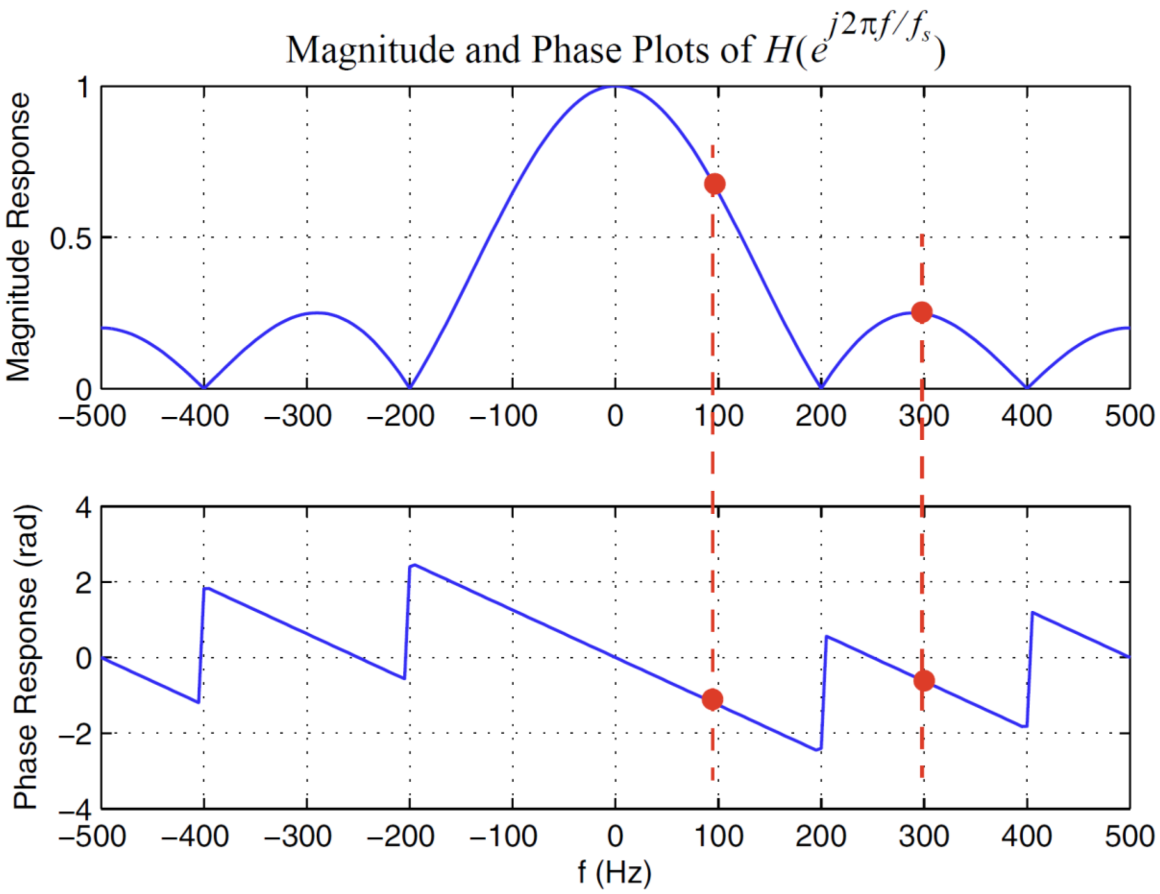



```
>> subplot(211)
>> plot(w*1000/(2*pi),abs(H))
>> grid
>> ylabel('Magnitude Response')
>> subplot(212)
>> plot(w*1000/(2*pi),angle(H))
>> grid
>> ylabel('Phase Response (rad)')
>> xlabel('f (Hz)')
```

IN THE DIGITAL FREQUENCY DOMAIN

IN THE ANALOG FREQUENCY DOMAIN

Filtering Sampled Continuous-Time Signals



EACH POINT IS A TRANSFER VALUE SHOWING THE CHANGE IN MAGNATUDE AND PHASE AT EACH FREQUENCY.

DO A GOOD JOB ON QUIZ = HW5

RECORD OFF