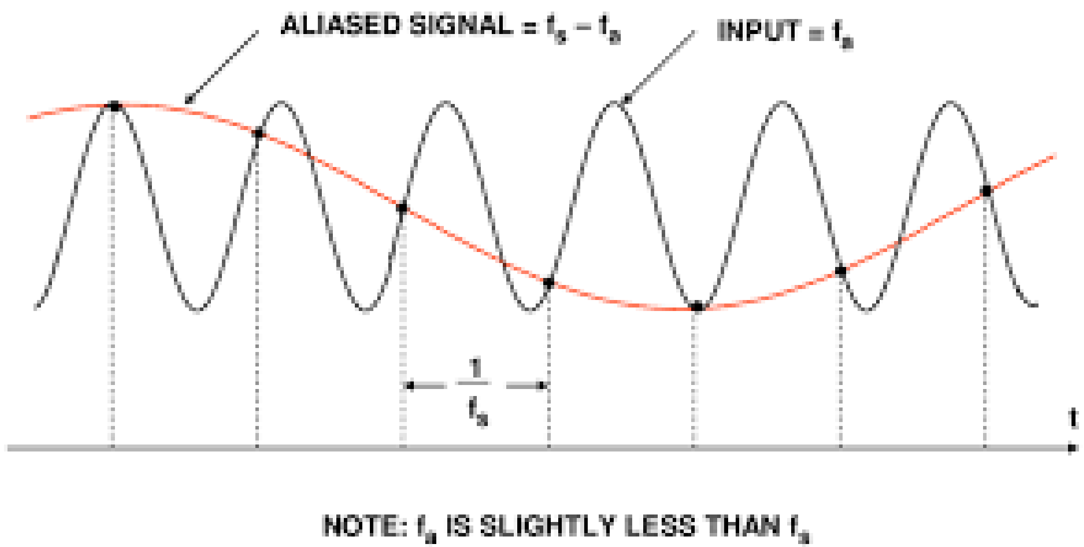


3315_Ch4_6_Presentation1

SAMPLING AND ALIASING

RECORD ON

WE SHALL MAKE HW5 A QUIZ

PLEASE DO IT CAREFULLY AND legibly

Pg 105

Sinusoidal Signals

Sinusoidal signals are examples of continuous-time signals that exist in the "real
outside the computer, and for which we can also write a simple mathematical f
Since the effects of sampling are easily understood for sinusoids, we use them as t
for our study of sampling.

If we sample a sinusoid of the form $A\cos(\omega t + \varphi)$, we obtain

$$
\begin{aligned}
x[n] &= x(nT_s) \\
&= A\cos(\omega n T_s + \varphi) \\
&= A\cos(\hat{\omega} n + \varphi)
\end{aligned}
$$

where we have defined $\hat{\omega}$ to be

etimes

$\Omega = \frac{\omega}{S}$

Normalized Radian Frequency

$$\hat{\omega} \stackrel{\text{def}}{=} \omega T_s = \frac{\omega}{f_s}$$

The signal $x[n]$ in (4.2) is a ***discrete-time cosine signal***, and $\hat{\omega}$ is its ***discre***

Ch 4 pg 165 $x[n] = x(nT_s)$ $\hat{\omega} = \omega T_s = \omega / f_s$

$-\pi \le \hat{\omega} \le \pi$

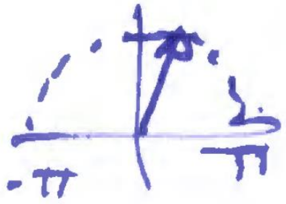
$= 2\pi \, f / f_s$

P166 $x(t) = \cos(200\pi t) = \cos(2\pi \cdot 100 t)$

$f_0 = 100$ Hz Let $f_s = 500$ Hz

Figure 4-3

$\hat{\omega} = \frac{2\pi 100}{500} = 0.4\pi$ 72°



PERIODIC EVERY 2 π

Alias Note

$$\cos(2.4\pi n) = \cos(0.4\pi n + 2\pi n)$$

✓ Proof

$$\cos(0.4\pi n + 2\pi n) = \cos(0.4\pi n)\,\overset{1}{\cos 2\pi n} - \sin(0.4\pi n)\,\overset{0}{\sin(2\pi n)}$$

So $\hat{\omega} = \dfrac{2\pi f}{500} = 2.4\pi \qquad f = \dfrac{2.4 \times 500}{2} = 1.2 \times 500 = 600\ \text{Hz}$

$$f_{alias} = \underset{f_0}{600\ \text{Hz}} - \underset{f_s}{500\ \text{Hz}} = 100\ \text{Hz}$$

Plog S_0

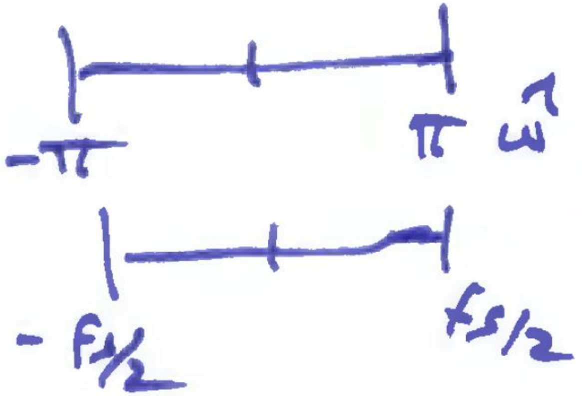
$$-\pi < \hat{\omega}_0 = \omega_0 T_s < \pi$$

$\omega_0 = 2\pi f_0$

$T_s = 1/f_s$

$$-\pi < 2\pi \frac{f_0}{f_s} < \pi$$

S_0 $\quad -\frac{1}{2} < f_0/f_s < 1/2$



If result is <0; Negate Phase

DON'T ALIAS

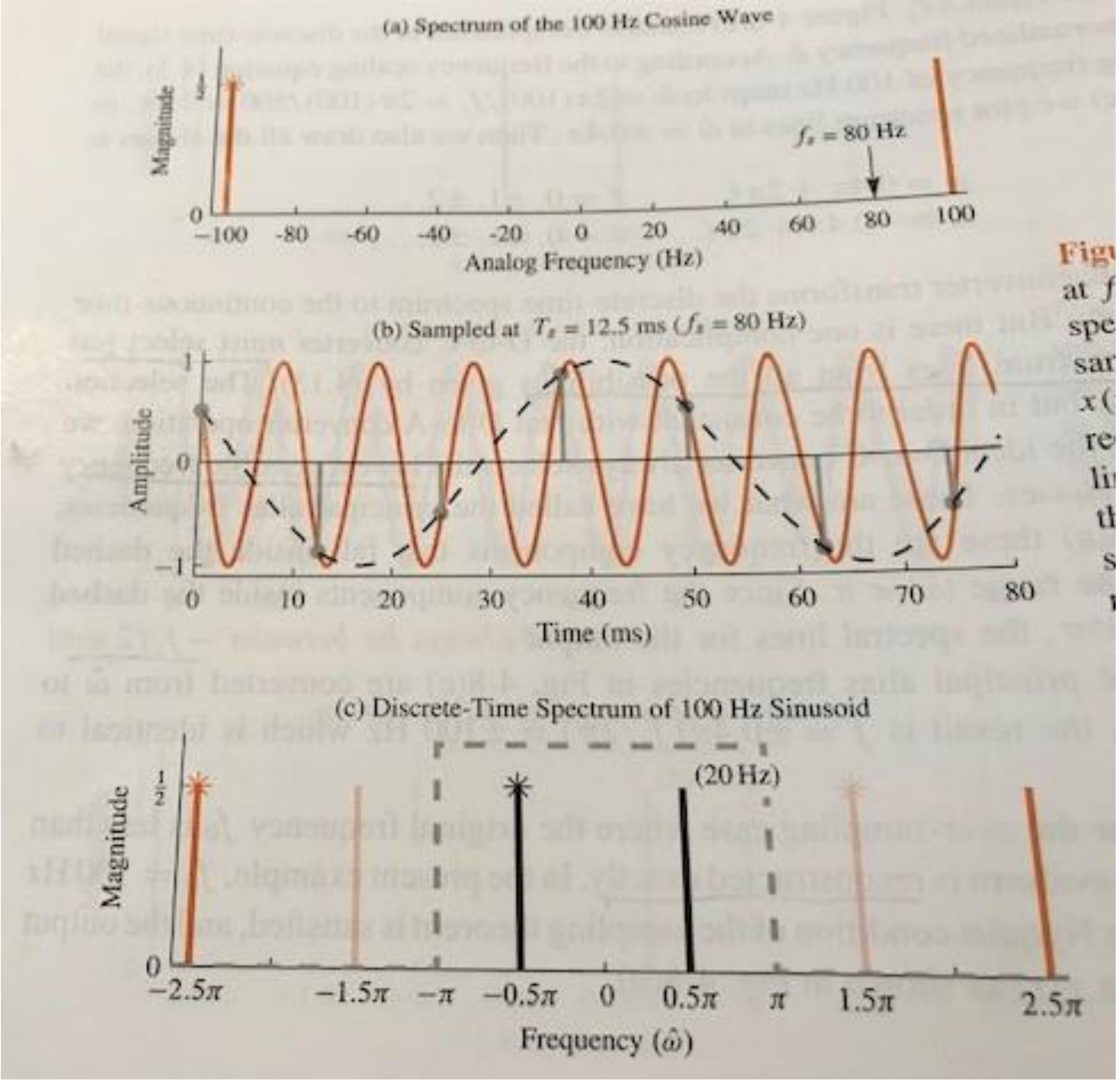


Figure 4-9 *Under-sampling* a 100 Hz sinusoid at $f_s = 80$ samples/s. (a) Continuous-time spectrum; (b) time-domain plot, showing the samples $x[n]$ as gray dots, the original signal $x(t)$ as a continuous orange line, and the reconstructed signal $y(t)$ as a dashed black line, which is a 20 Hz sinusoid passing through the same sample points; and (c) discrete-time spectrum plot, showing the positive and negative frequency components of the original sinusoid at $\hat{\omega} = \pm 2.5\pi$ rad, along with two sets of alias components.

100 Hz signal with fs =100 s/s == **20 Hz**

DON'T ALIAS

RECORD OFF