

Modified TLH

**DSP First, 2/e**  
**Lecture Chapter 5**  
**FIR Filtering Intro**  
**Impulse response**  
**PRESENTATION 2**

# SIGNALS USED TO CHARACTERIZE A SYSTEM

- Signals
  - ▶ Sinuoidal signals
  - ▶ Exponential signals
  - ▶ Complex exponential signals
  - ▶ Unit step and unit ramp
  - ▶ Impulse functions

# SPECIAL INPUT SIGNALS

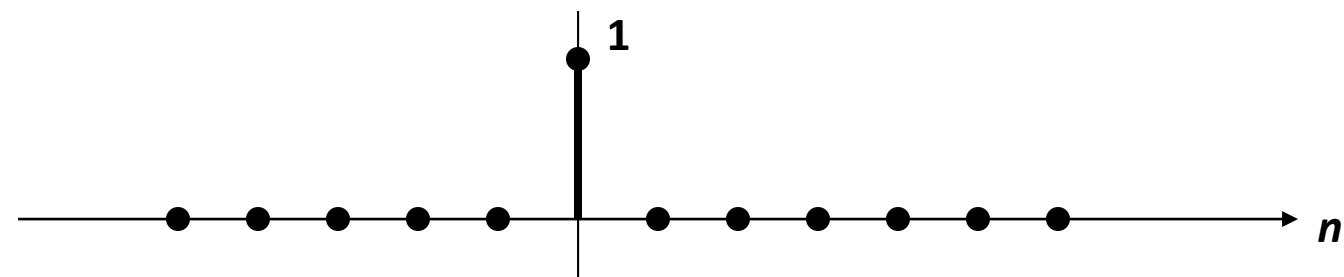
Another Test Signal

- $x[n] = \text{SINUSOID}$
- $x[n]$  has only one NON-ZERO VALUE

**FREQUENCY RESPONSE (LATER)**

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

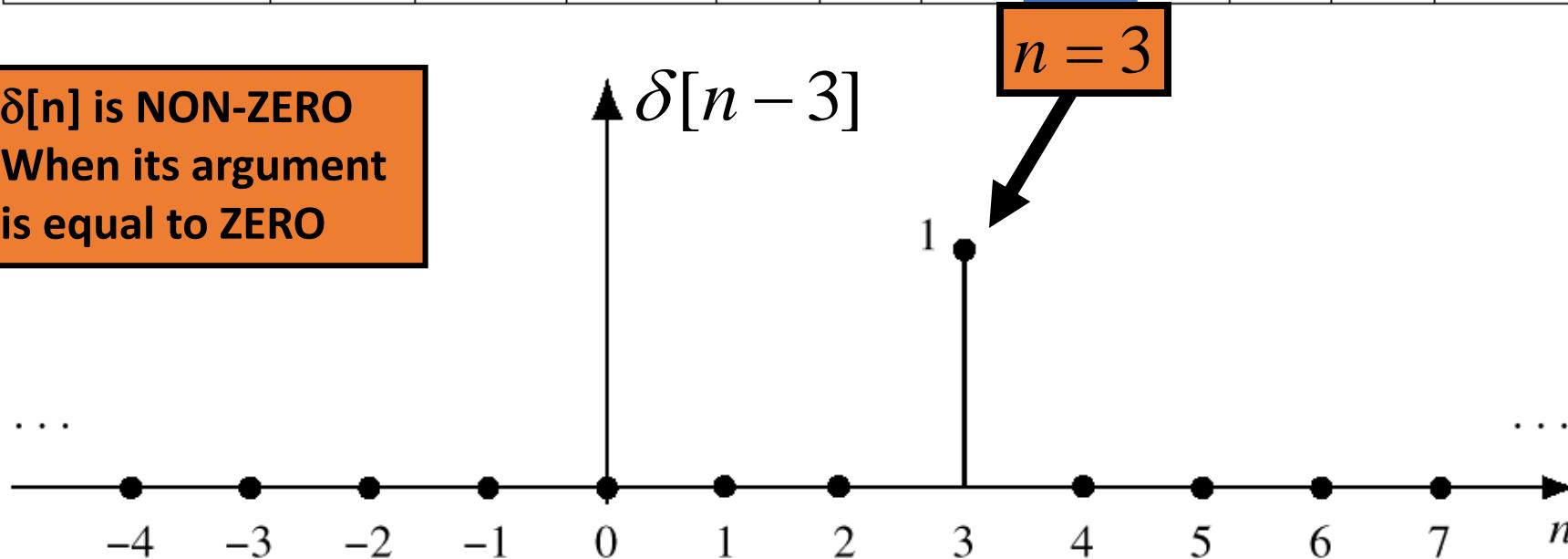
**UNIT-IMPULSE**



# UNIT IMPULSE SIGNAL $\delta[n]$

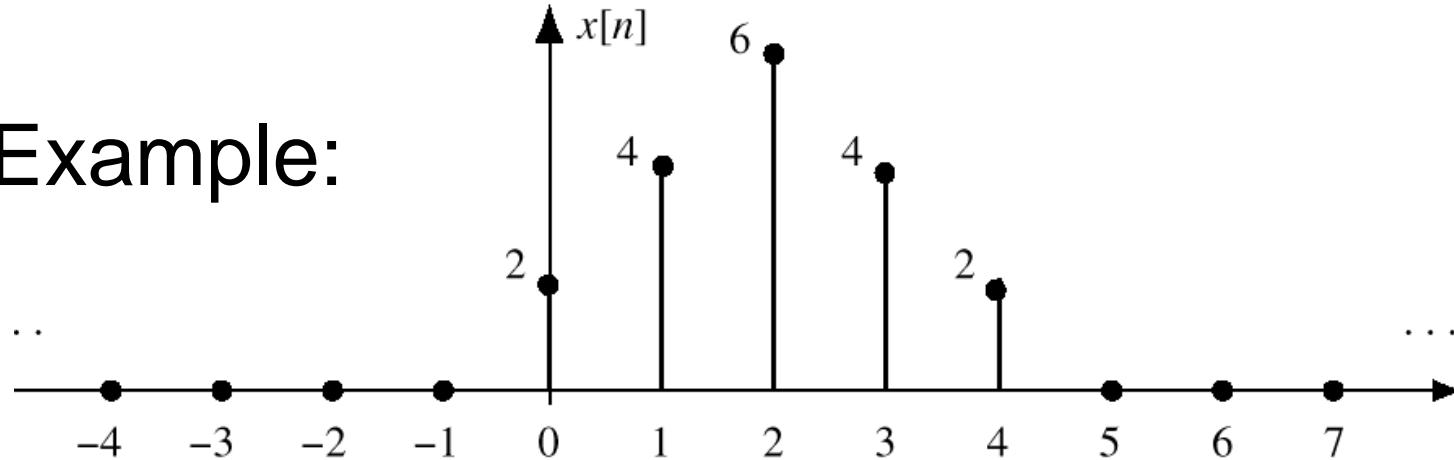
|                 |     |    |    |   |   |   |   |   |   |   |     |
|-----------------|-----|----|----|---|---|---|---|---|---|---|-----|
| $n$             | ... | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | ... |
| $\delta[n]$     | 0   | 0  | 0  | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0   |
| $\delta[n - 3]$ | 0   | 0  | 0  | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0   |

$\delta[n]$  is NON-ZERO  
When its argument  
is equal to ZERO



# Sequence Representation TO GIVE MATHEMATICAL FORMULA FOR AMPLITUDE VALUES

Example:



$$x[n=0] = x[0] = 2$$

$$x[n=1] = x[1] = 4$$

JUST NUMBERS

$$x[n=2] = x[2] = 6$$

$$x[n=3] = x[3] = 4$$

$$x[n] = \dots + \color{blue}{0} \delta[n+1] + \color{blue}{2} \delta[n] + \color{blue}{4} \delta[n-1]$$

AN EQUATION

$$+ \color{blue}{6} \delta[n-2] + \color{blue}{4} \delta[n-3] + \dots$$

# FIR IMPULSE RESPONSE

$h[n]$  CHARACTERIZES THE SYSTEM

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

$$h[n] = \sum_{k=0}^M b_k \delta[n-k]$$

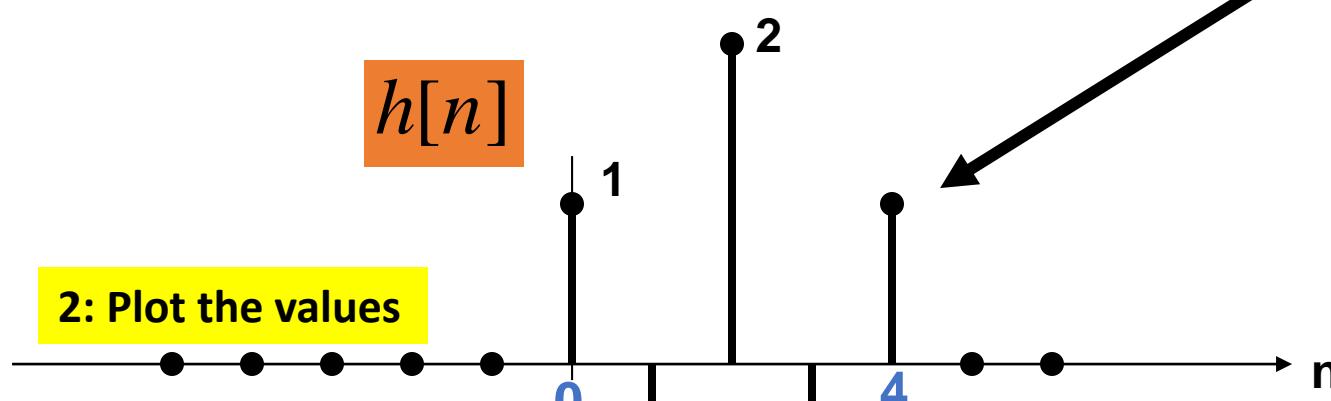
| $n$                | $n < 0$ | 0     | 1     | 2     | 3     | $\dots$ | $M$   | $M + 1$ | $n > M + 1$ |
|--------------------|---------|-------|-------|-------|-------|---------|-------|---------|-------------|
| $x[n] = \delta[n]$ | 0       | 1     | 0     | 0     | 0     | 0       | 0     | 0       | 0           |
| $y[n] = h[n]$      | 0       | $b_0$ | $b_1$ | $b_2$ | $b_3$ | $\dots$ | $b_M$ | 0       | 0           |

# 3 Ways to Represent the FIR filter

1

Use SHIFTED IMPULSES to write  $h[n]$

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$



3: List the values

$$b_k = \{ 1, -1, 2, -1, 1 \}$$

True for any signal,  $x[n]$

# FILTERING EXAMPLE

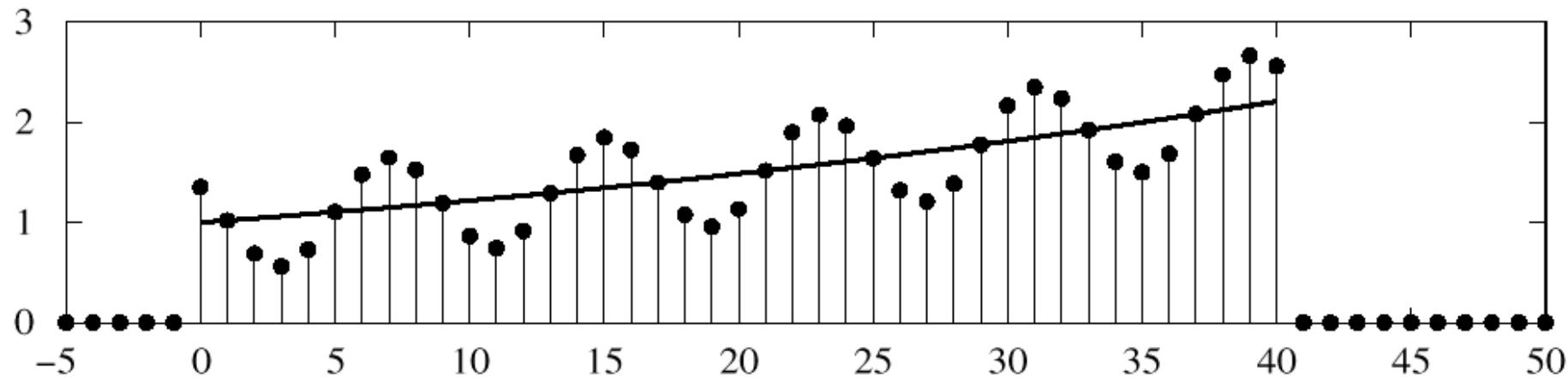
- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller
  
- 3-point AVERAGER
  - Changes A slightly

$$y_7[n] = \sum_{k=0}^6 \left(\frac{1}{7}\right) x[n - k]$$

$$y_3[n] = \sum_{k=0}^2 \left(\frac{1}{3}\right) x[n - k]$$

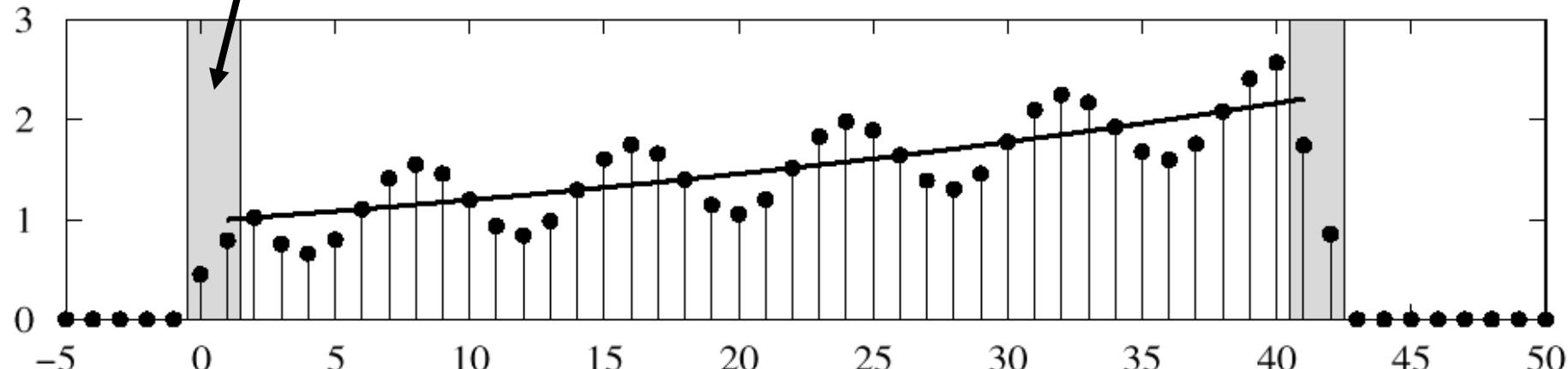
# 3-pt AVG EXAMPLE

Input :  $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$  for  $0 \leq n \leq 40$



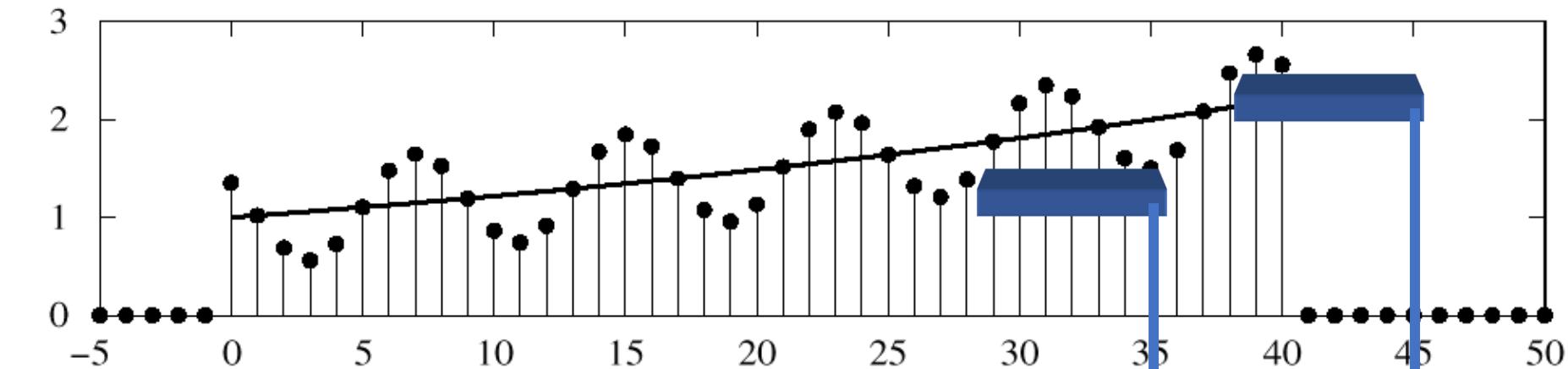
**USE PAST VALUES**

Output of 3-Point Running-Average Filter



# 7-pt FIR EXAMPLE (AVG)

Input :  $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$  for  $0 \leq n \leq 40$



**CAUSAL: Use Previous**

Output of 7-Point Running-Average Filter

