MODIFIED TLH

DSP First, 2/e

Lecture Chapter 5

Linearity & Time-Invariance

Convolution

Chapter 5 Presentation 3

OVERVIEW

- IMPULSE RESPONSE, $h[n]$

  - FIR case: same as $\{b_k\}$

- CONVOLUTION

  - GENERAL: $y[n] = h[n] * x[n]$

  - GENERAL CLASS of SYSTEMS

  - LINEAR   and   TIME-INVARIANT **LTI**

- ALL **LTI** systems have $h[n]$ & use convolution

Convolution is the process by which an input interacts with an LTI system to produce an output:

$$y[n] = h[n] * x[n]$$

LINEAR AND TIME-INVARIANT SYSTEMS

1. **Let's pretend we are speaking in church or some large hall.**

David Dorran Interesting Audio Example using MATLAB to simulate talking in church! An audio example showing where convolution is used in practice. A 'dry' speech recording is altered to sound as if were recorded in a church. Rather than modelling a complex real world system (the church) using a difference equation the impulse response is more readily available. 7:30 – Start at about 3:30 for "Speaking in a Church" DEMO

https://www.youtube.com/watch?v=8jCva6KHlYI

Using the conv function, Convolution is a mathematical process in which the output of a system can be determined if you have the systems impulse response y = conv(x,h);This technique is particularly useful for the situation where it is a relatively easy to determine a systems impulse response, but rather more difficult to try an model the system in question i.e. determine a set of b and a coeffients that model the behaviour of the system. Perhaps one good example of this for audio processing, whereby a recording is changed by adding 'reverberation' effect which makes the recording sound as if it was recorded in a different environment.

2. DSP: Using an FIR filter to remove 50/60Hz from an ECG (MATLAB/OCTAVE) 6:31

https://www.youtube.com/watch?v=r7ypfE5TQK0&feature=youtu.be

An electrocardiogram (ECG) is a medical test that detects heart problems by measuring the electrical activity generated by the heart as it contracts. ECGs from healthy hearts have a characteristic shape. If the ECG shows a different shape it could suggest a heart problem.

50 Hz Notch Filter removes most of the 50 Hz noise from the ac lines.
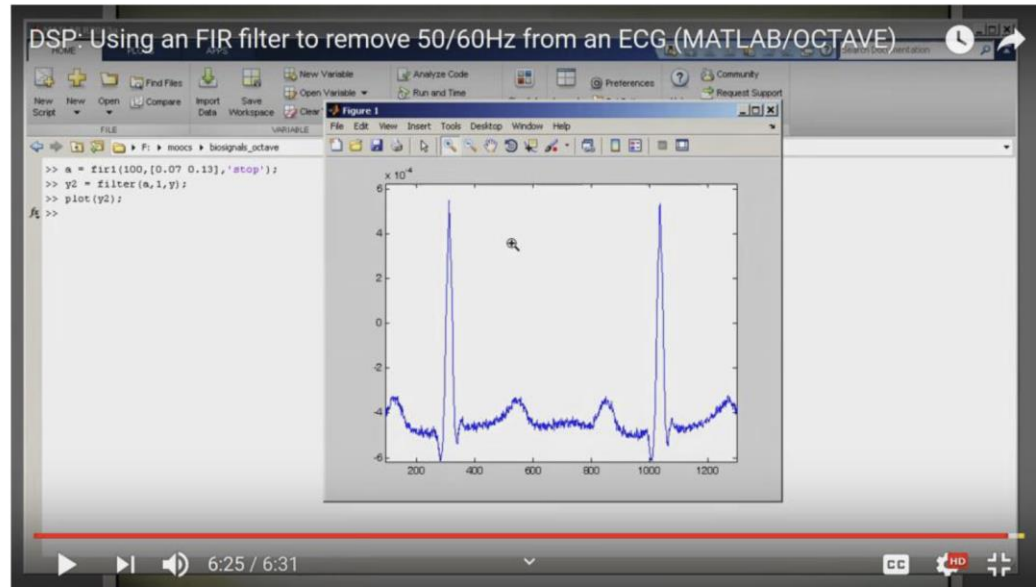
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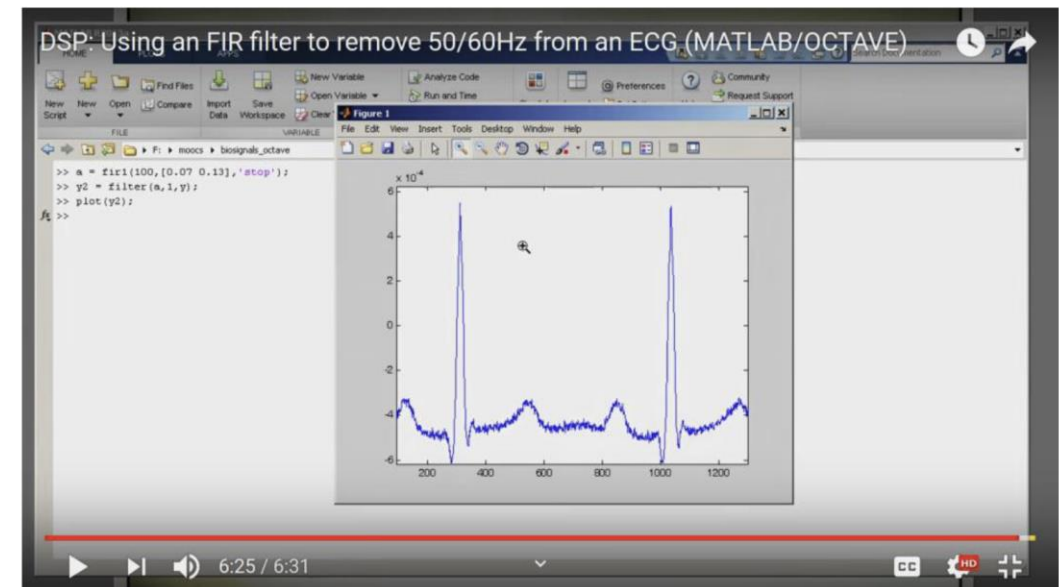
GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

  $$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$
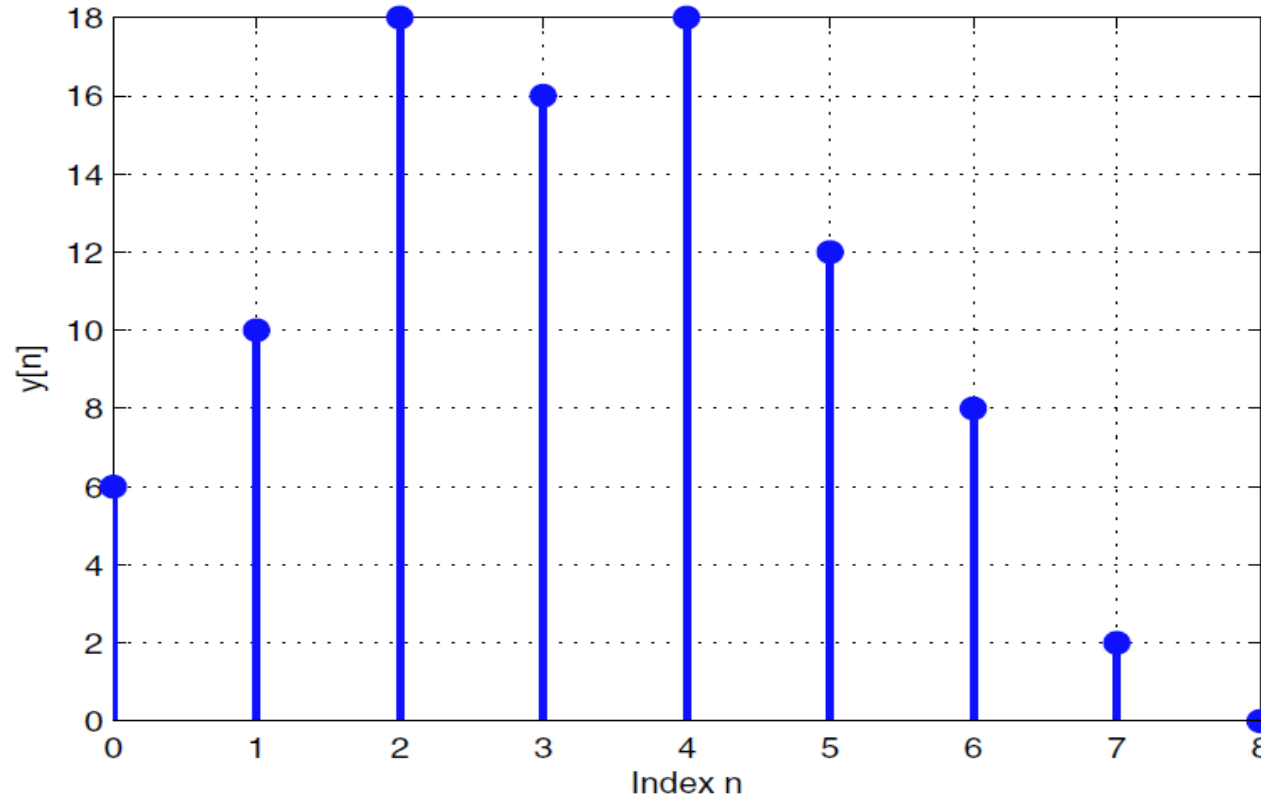
  - For example,

  $$b_k = \{3, -1, 2, 1\}$$

  $$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$
  $$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

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- We can check the answers using MATLAB's filter function

```
>> n = 0:8;
>> x = [2 4 6 4 2 0 0 0 0];
>> h = [3 -1 2 1];
>> y = filter(h,1,x);
>> y
```

y =   6   10   18   16   18   12   8   2   0

Y0=3*2 =6

Y1=3*4-1*2=10

...

`filter` can handle FIR and IIR systems, while **conv** takes two inputs and returns their convolution.

So `conv(h,x)` and `filter(h,1,x)` would give the same result. The 1 in filter indicates that the recursive coefficients of the filter are just `[1]`. (The denominator =1)

But if you have an IIR filter, you can't use `conv`. `filter` can also return the filter states, so that it can be used in subsequent calls without incurring filter transients.

See the [conv](#) and [filter](#) documentation for details.

SYSTEM PROPERTIES

- MATHEMATICAL DESCRIPTION

- TIME-INVARIANCE

- LINEARITY

- CAUSALITY
  - "No output prior to input"

TIME-INVARIANCE

- IDEA:
  - "Time-Shifting the input will cause the same time-shift in the output"
  - THE COEFFICIENTS OF THE SYSTEM ARE **CONSTANT!**

- EQUIVALENTLY,
  - **DO THE EXPERIMENT TODAY YIELDING Y(nT) FROM INPUT X(nT)**
  - **DO IT TOMORROW OR NEXT YEAR ...   SAME Y(nT) FROM SAME INPUT X(nT)**

    ADAPTIVE SYSTEMS HAVE COEFFICIENTS THAT CHANGE WITH THE TASK!

LINEAR SYSTEM
THINK OF Y=MX (NOT Y=MX+B)

- LINEARITY = Two Properties

- SCALING
  - "Doubling x[n] will double y[n]"

- SUPERPOSITION:
  - "Adding two inputs gives an output that is the sum of the individual outputs"

  - LINEAR SYSTEMS ARE WELL UNDERSTOOD AND HAVE PROPERTIES THAT CAN BE PROVEN MATHEMATICALLY.   NON-LINEAR SYSTEMS ARE DIFFICULT.

INPUT $A\cos(2\pi(440)t)$ TO A LINEAR SYSTEM

OUTPUT $B\cos(2\pi(440)t + \varphi)$

CHANGE IN AMPLITUDE AND PHASE (TYPICALLY A TIME DELAY)

NON LINEAR EXAMPLE   SYSTEM SQUARES INPUT

y(t) = x(t)^2 thus if x(t)= sin(wt) then y(t) = sin(wt)^2 BUT

sin(wt)^2 = (1- cos(2wt))/2   No sinusoid fidelity