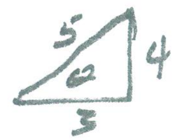
TRY PHASOR ADDITION

$a\cos\theta + b\sin\theta = c\cos(\theta - \alpha)$ ✓

$c = \sqrt{a^2+b^2} \qquad \tan\alpha = b/a$



Try $4\cos\omega t + 3\sin\omega t$

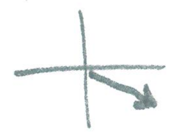
So $4\cos(\omega t + \phi) + 3\cos(\omega t - \pi/2)$

$X_1 = 4e^{j\phi} = 4\cos\phi \qquad X_1 = 4 + j0$

WATCH SIGN

$X_2 = 3e^{-j\pi/2} = \begin{matrix} 3\cos\pi/2 \\ +3j\sin\pi/2 \end{matrix} \qquad X_2 = 0 - j3$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad 4 - j3$



$\tan\alpha = 3/4$

So $X_3 = 4 - j3 = 5\tan^{-1}(-3/4) = 5\angle -36.86^\circ$

$X_3 = 5e^{j(-36.86\pi/180^\circ)} = 5e^{j(-0.2048\pi)}$

NOW BACK

$5\cos\left(\omega t - 36.86\,\pi/180\right)$

DO TEST CASES

$\omega t = 0 \qquad \underline{x(t) = 4.0} \qquad 5\cos\left(-\frac{36.86\pi}{180}\right) =$

$5\cos(\underbrace{-0.2048\pi}_{0.8}) = 4.0$ ✓

0.6433

Try 45° $(\pi/4)$ $\underline{\text{ok}}$!

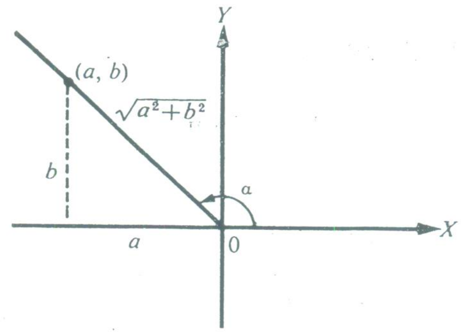


Figure 806

We multiply and divide the expression $a \cos \theta + b \sin \theta$ by $\sqrt{a^2 + b^2}$, substitute the values of $\sin \alpha$ and $\cos \alpha$ of (1) into the resulting expression, and apply Identity (17).

$$a \cos \theta + b \sin \theta = \sqrt{a^2 + b^2}\left[\frac{\overset{\cos\alpha}{a}}{\sqrt{a^2 + b^2}} \cos \theta + \frac{\overset{\sin\alpha}{b}}{\sqrt{a^2 + b^2}} \sin \theta\right]$$

$$= \sqrt{a^2 + b^2}\,[\cos \alpha \cos \theta + \sin \alpha \sin \theta]$$

p137

$$= \sqrt{a^2 + b^2}\,[\cos \theta \cos \alpha + \sin \theta \sin \alpha]$$

$$= \sqrt{a^2 + b^2} \cos (\theta - \alpha).$$

watch SIGN!

Hence:

If θ is any angle,

(35) $$\boldsymbol{a \cos \theta + b \sin \theta = c \cos(\theta - \alpha)},$$

where $$\boldsymbol{c = \sqrt{a^2 + b^2},\ \sin \alpha = \frac{b}{c},\ \cos \alpha = \frac{a}{c},\ \tan \alpha = \frac{b}{a}}.$$

Example 1. Transform $4 \cos \theta + 3 \sin \theta$ to the form $c \cos(\theta - \alpha)$, and find c and α.

Solution. Plot the point (4, 3) and place α in standard position with its terminal side passing through (4, 3). (*See* Fig. 807a.) Then $c = \sqrt{4^2 + 3^2} = 5$, $\sin \alpha = \frac{3}{5}$, $\cos \alpha = \frac{4}{5}$, and $\tan \alpha = \frac{3}{4}$. Therefore $\alpha = 36° 52'$ or any angle coterminal with $36° 52'$. By applying Formula (35) we obtain

$$4 \cos \theta + 3 \sin \theta = 5 \cos(\theta - 36° 52').$$

To further clarify what Formula (35) is doing for us in this problem, we show the following development. We plot the point (4, 3) and compute c; then

$$4 \cos \theta + 3 \sin \theta = 5(\tfrac{4}{5} \cos \theta + \tfrac{3}{5} \sin \theta)$$

$$= 5(\cos \alpha \cos \theta + \sin \alpha \sin \theta)$$

$$= 5 \cos(\theta - \alpha)$$

$$= 5 \cos(\theta - 36° 52').$$

$36° + \frac{52'}{60'} \rightarrow 36.8667°$

$\frac{36.8667\pi}{180°} = 0.2048\pi$

$\frac{52'}{60'} = 0.8667$

Exa
c ar
Solı
tern
c =
α =
As
and
Exa
find