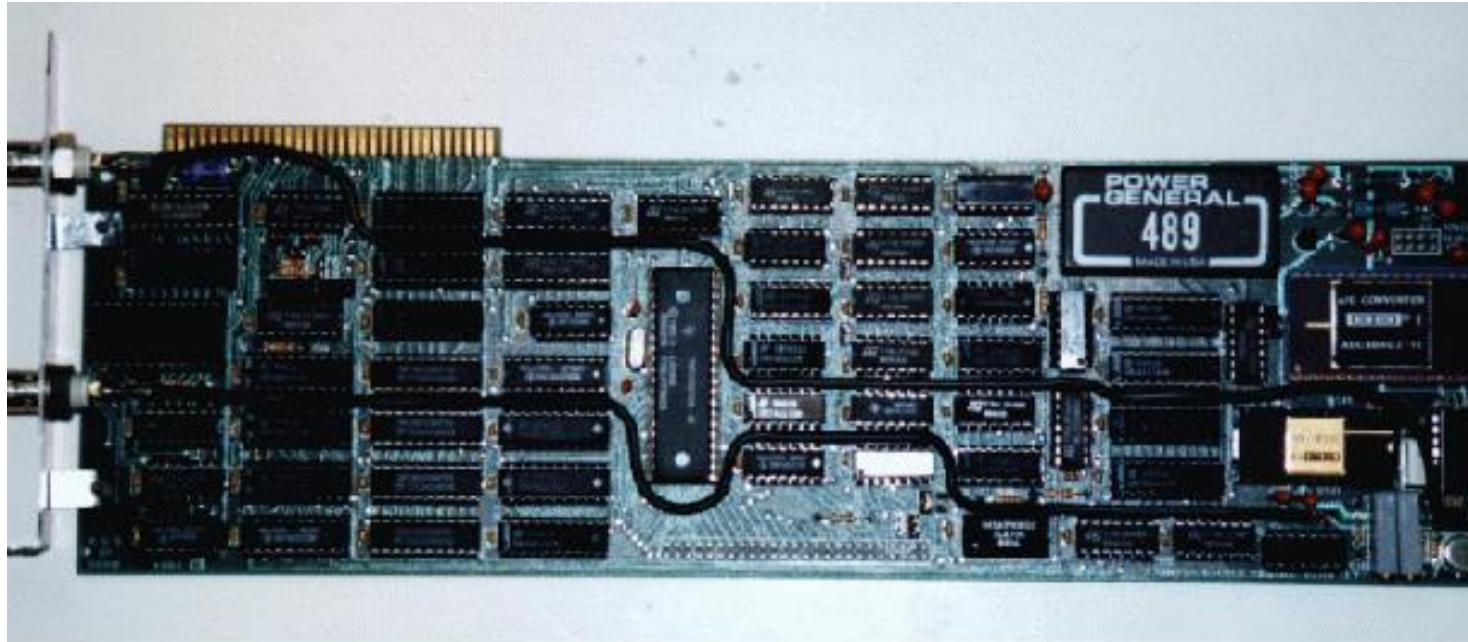


# Frequency Response of FIR Filters

C H A P T E R

6

# The TMS32010, 1983



First PC plug-in board from Atlanta Signal Processors Inc.

# Rockland Digital Filter, 1971



The image shows a photograph of the Rockland Model 4136 Programmable Digital Filter. The device is a rectangular metal box with a front panel featuring three large rotary knobs and several smaller buttons. The text on the front panel reads "ROCKLAND PROGRAMMABLE DIGITAL FILTER - MODEL 4136".

**Model 4136  
PROGRAMMABLE  
DIGITAL  
FILTER**

**Variable-Order Digital Filter for Realizing All Classical Designs**

The Rockland Model 4136 Programmable Digital Filter consists of a second-order digital filter section which is multiplexed four ways to achieve eighth-order filtering. Each of the four sections has fully-programmable coefficients which are stored internally in a read-write memory.

Filter input and output words are in 16-bit parallel form at a maximum sampling rate of 80 KHz while internal computations are made with 24-bit ac-

**TRANSFER FUNCTION**

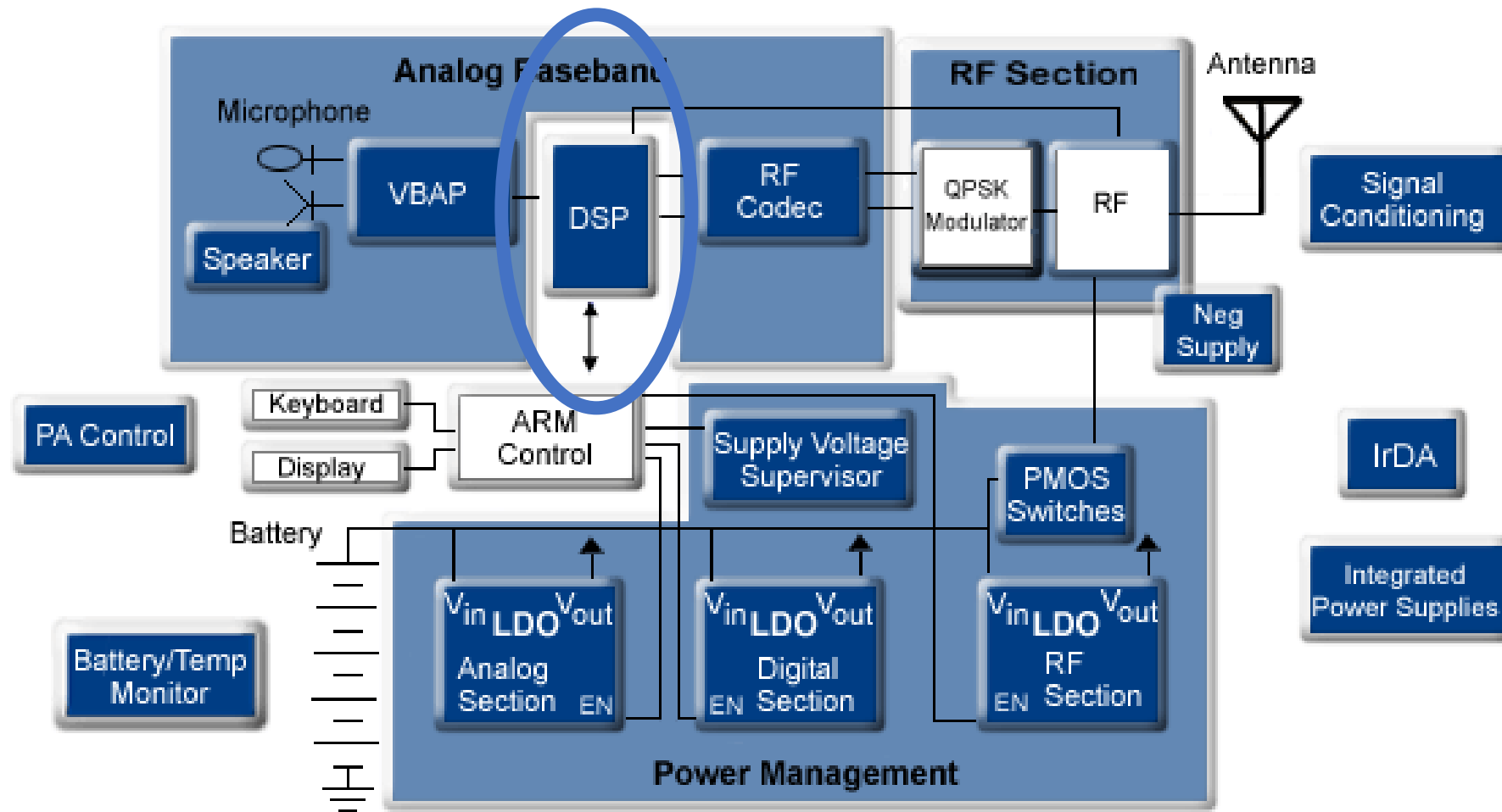
The transfer function from filter input to filter output in z-transform notation is given by

$$H_N(z) = \prod_{n=1}^N \frac{K_n(1+z^{-1}A1,+z^{-2}A2)}{1-z^{-1}B1,-z^{-2}B2}, \quad (1)$$

where  $N=0,1,2,3,4$  is one-half the filter order se-

Cost was about the same as the price of a small house.

# Digital Cell Phone (ca. 2000)



Now, digital cameras and video streaming rely on DSP algorithms

## *Frequency Response of an FIR System*

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k} = \sum_{k=0}^M h[k] e^{-j\hat{\omega}k}$$

(6.4)

Relationship of digital frequency to analog  $\omega$ /(Sampling Frequency) Page 105

$$\hat{\omega} = \omega T_s = \omega / f_s \quad (\text{Radians} = \text{radians/sec} * \text{Seconds})$$

$$y[n] = \sum_{k=0}^M b_k x[n - k]$$

**FILTER OUTPUT**

INPUT A SINUSOID (EXPONENTIAL DIGITAL FORM)

$$x[n] = A e^{j\varphi} e^{j\hat{\omega}n} \quad -\infty < n < \infty$$

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k} \quad (6.3)$$

# TRANSFER FUNCTION

$$\begin{aligned} y[n] &= \left( |H(e^{j\hat{\omega}})| e^{j\angle H(e^{j\hat{\omega}})} \right) A e^{j\varphi} e^{j\hat{\omega}n} \\ &= (|H(e^{j\hat{\omega}})| \cdot A) e^{j(\angle H(e^{j\hat{\omega}}) + \varphi)} e^{j\hat{\omega}n} \end{aligned} \quad (6.5)$$

↑ AMPLITUDE CHANGE

↑ PHASE CHANGE

The term "**transfer function**" is also used in the frequency domain analysis of systems using transform methods such as the Laplace transform; here it means the amplitude of the output as a function of the frequency of the input signal. **For example, the transfer function of an electronic filter is the voltage amplitude at the output as a function of the frequency of a constant amplitude sine wave applied to the input.**

## Measuring frequency response

for  $\omega = \omega_1, \dots, \omega_N$ ,

- apply sinusoid at frequency  $\omega$ , with phasor  $\mathbf{U}$
- wait for output to converge to SSS
- measure  $\mathbf{Y}_{ss}$   
(*i.e.*, magnitude and phase shift of  $y_{ss}$ )

$N$  can be a few tens (for hand measurements) to several thousand

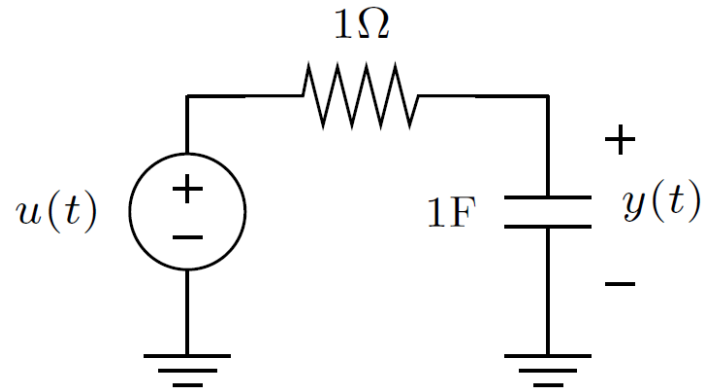


## Frequency response plots

- $|H(j\omega)|$  &  $\angle H(j\omega)$  versus  $\omega$  (called *Bode plot*)

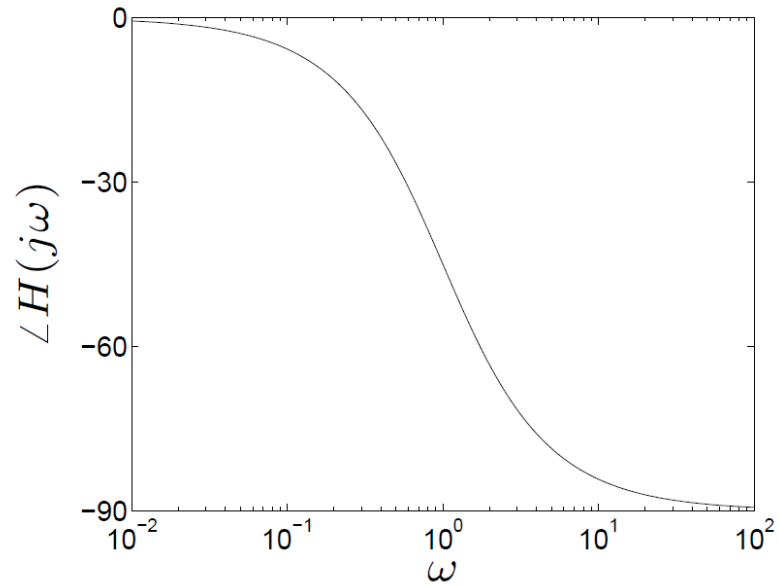
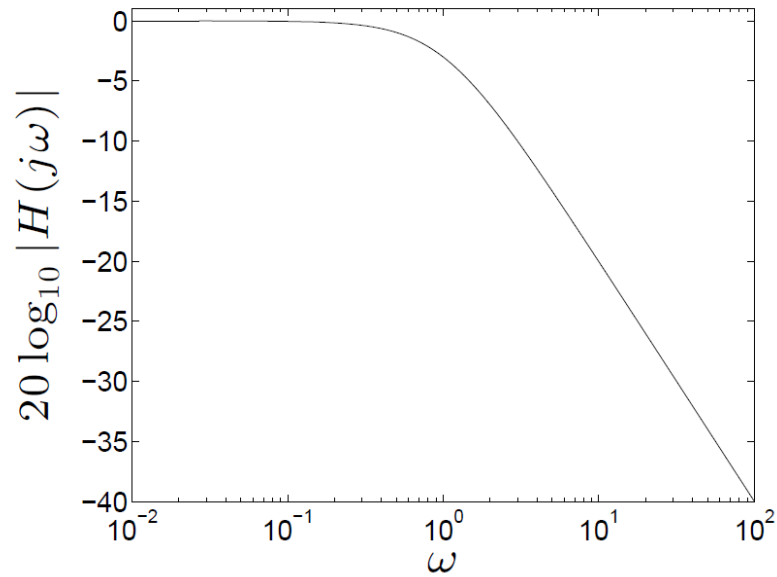
the most common format is a Bode plot

example: RC circuit



$$Y(s) = \frac{1}{1+s}U(s)$$

$$H(j\omega) = \frac{1}{1+j\omega}$$



Log scale in radian frequency

WHY -90°

Consider an LTI system for which the difference equation coefficients are  $\{b_k\} = \{1, 2, 1\}$ . Substituting into (6.3) gives

$$H(e^{j\hat{\omega}}) = 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2}$$

To obtain formulas for the magnitude and phase of the frequency response of this FIR filter, we can manipulate the equation as follows:

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2} \\ &= e^{-j\hat{\omega}} \left( e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}} \right) \\ &= e^{-j\hat{\omega}} (2 + 2 \cos \hat{\omega}) \end{aligned}$$

Relationship of digital frequency to analog  $\omega$ /(Sampling Frequency) Page 105

$$\hat{\omega} = \omega T_s = \omega / f_s$$

Suppose there is a dc component of 5 volts =  $A_0$ :

$$H(e^{i\hat{\omega}}) = e^{-i\hat{\omega}} [2 + 2\cos(\hat{\omega})] = e^0 (2 + 2) = 4 \angle 0 \text{ Degrees ;}$$

$$Y[n] = H * 5 \text{ Volts} = 4 * 5 = 20 \text{ volts } \angle 0 \text{ Degrees}$$

$$1. H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} (2 + 2\cos \hat{\omega})$$

DSP/F  
EXAMPLE 6.4  
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$$2. x[n] = 4 + A \cos\left(\frac{7\pi}{8}n\right)$$

$$3. H(e^{j0}) = 1 \cdot (2 + 2) = 4$$

$$4. H(e^{j\frac{7\pi}{8}}) = e^{-j\frac{7\pi}{8}} \left[ 2 + 2\cos\left(\frac{7\pi}{8}\right) \right]$$

$\rightarrow 157.5^\circ$   
 $\leftarrow 157.3^\circ$

$(-0.9239)$   
 $\underbrace{\hspace{10em}}_{0.1522}$

So  $y[n] = 4 \cdot 4 + A(0.1522) \cos\left[\frac{7\pi}{8}n - \frac{7\pi}{8}\right]$