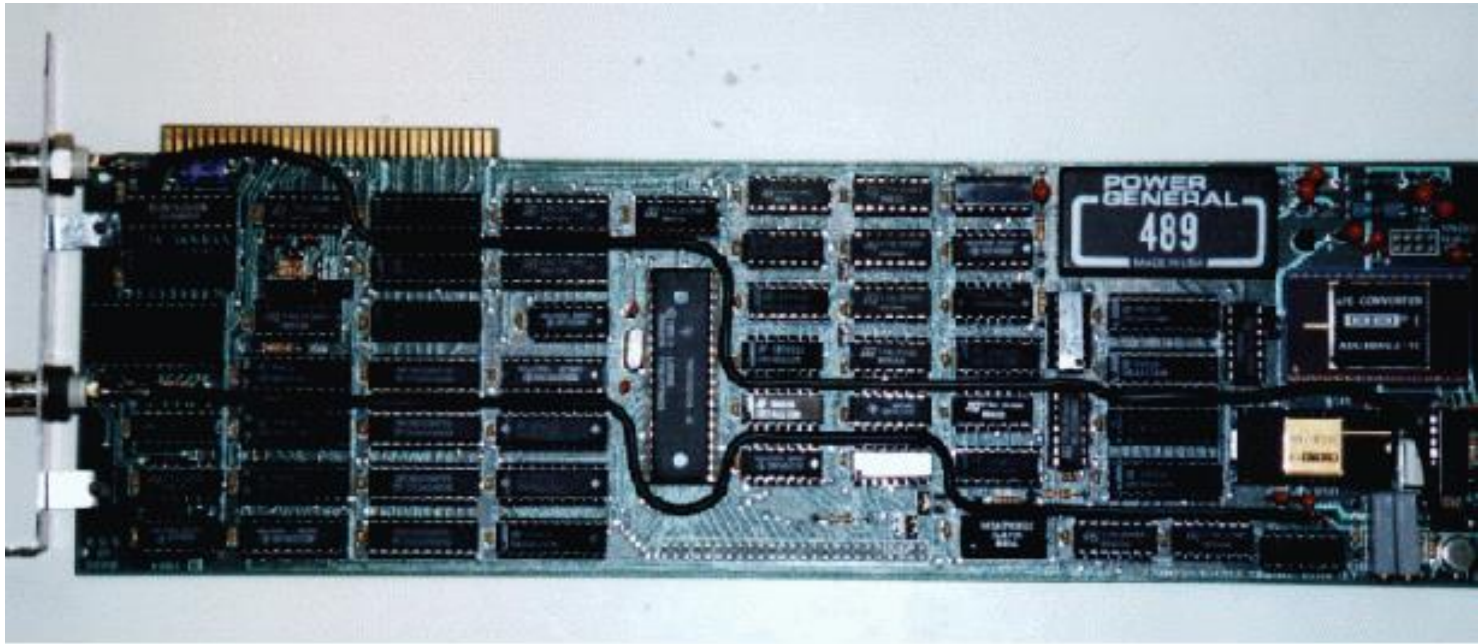
Frequency Response of FIR Filters

CHAPTER

6

The TMS32010, 1983

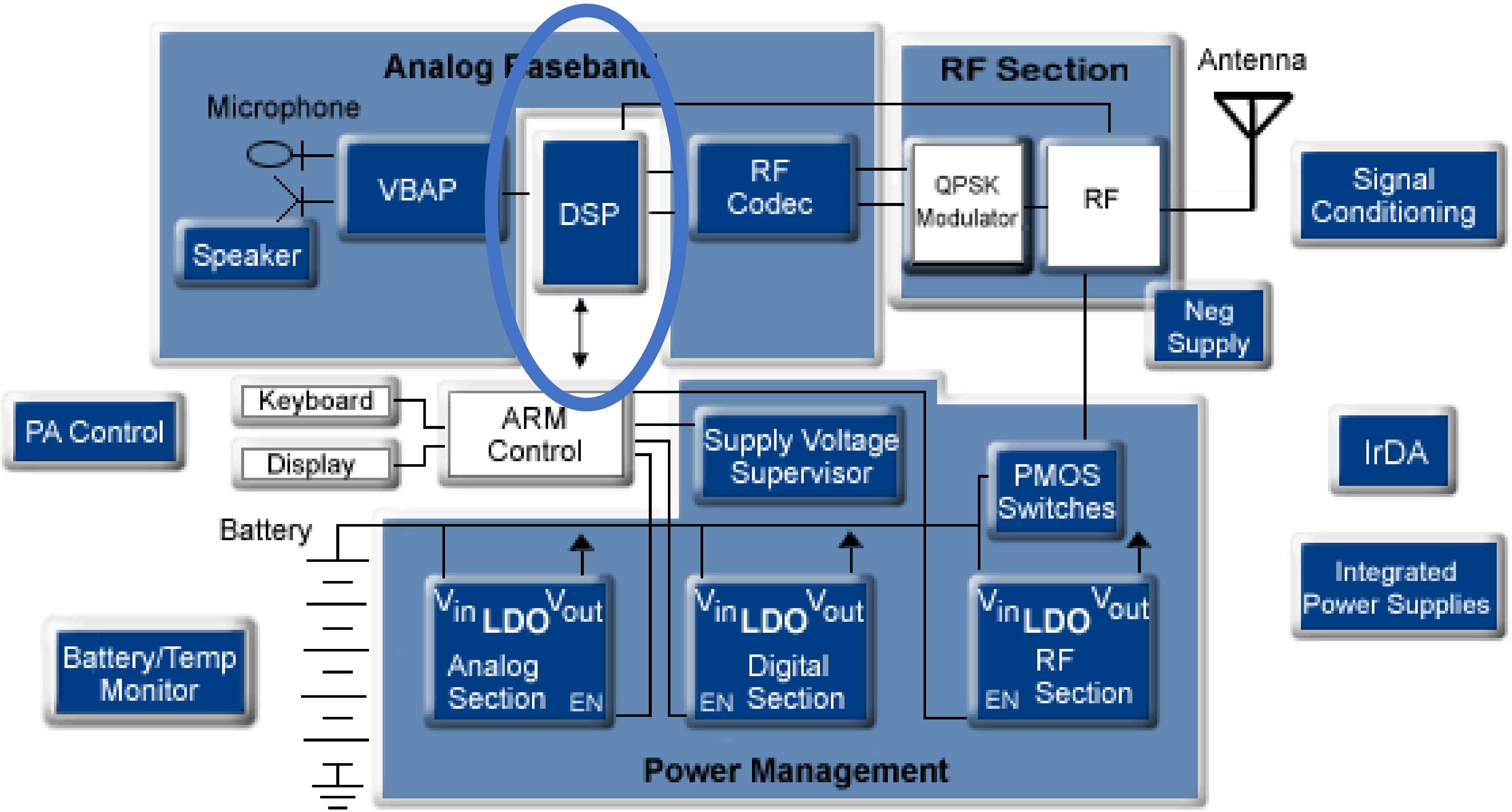


First PC plug-in board from Atlanta Signal Processors Inc.

Rockland Digital Filter, 1971

Cost was about the same as the price of a small house.

Digital Cell Phone (ca. 2000)



Now, digital cameras and video streaming rely on DSP algorithms

Frequency Response of an FIR System

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k} \quad (6.4$$

Relationship of digital frequency to analog w/(Sampling Frequency) Page 105

$\hat{w} = w T_s = w/f_s$ (Radians = radians/sec * Seconds)

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

FILTER OUTPUT

INPUT A SINUSOID (EXPONENTIAL DIGITAL FORM)

$$x[n] = Ae^{j\varphi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} \qquad (6.3)$$

TRANSFER FUNCTION

$$y[n] = \left(|H(e^{j\hat{\omega}})|e^{j\angle H(e^{j\hat{\omega}})}\right) Ae^{j\varphi}e^{j\hat{\omega}n}$$

$$= (|H(e^{j\hat{\omega}})| \cdot A)e^{j(\angle H(e^{j\hat{\omega}})+\varphi)}e^{j\hat{\omega}n} \quad (6.5)$$

PHASE CHANGE

AMPLITUDE CHANGE

*The term "**transfer function**" is also used in the frequency domain analysis of systems using transform methods such as the Laplace transform; here it means the amplitude of the output as a function of the frequency of the input signal.* ***For example, the transfer function of an electronic filter is the voltage amplitude at the output as a function of the frequency of a constant amplitude sine wave applied to the input.***

https://web.stanford.edu/~boyd/ee102/freq.pdf

Measuring frequency response

for $\omega = \omega_1, \ldots, \omega_N$,

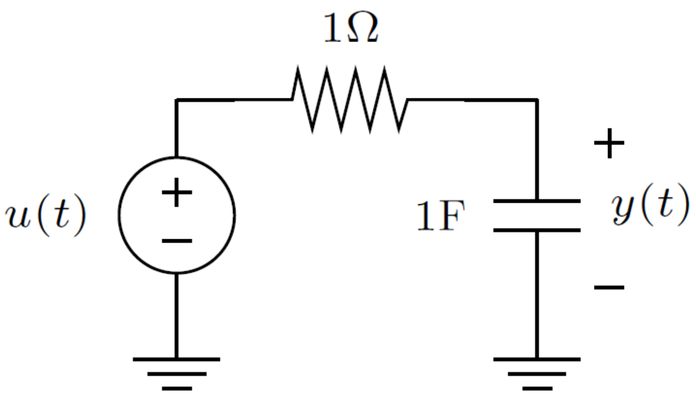
- apply sinusoid at frequency ω, with phasor $\mathbf{U}$
- wait for output to converge to SSS
- measure $\mathbf{Y}_{\rm ss}$
  ($i.e.$, magnitude and phase shift of $y_{\rm ss}$)

N can be a few tens (for hand measurements) to several thousand

- $|H(j\omega)|$ & $\angle H(j\omega)$ versus ω (called *Bode plot*)

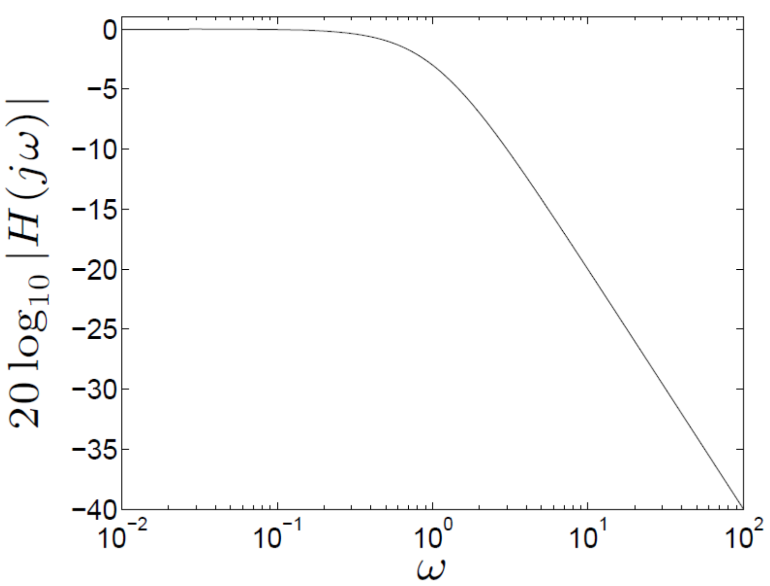
the most common format is a Bode plot

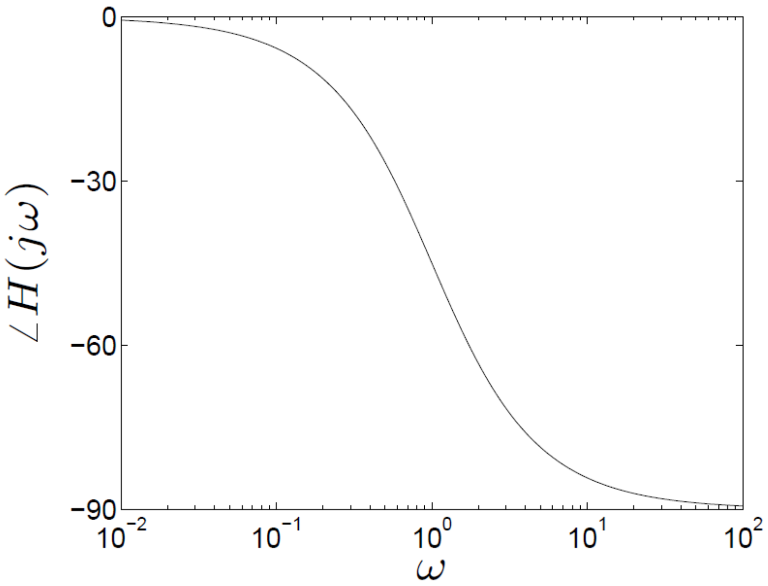
example: RC circuit



$$Y(s) = \frac{1}{1+s}U(s)$$

$$H(j\omega) = \frac{1}{1+j\omega}$$





WHY -90°

Log scale in radian frequency

Consider an LTI system for which the difference equation coefficients are $\{b_k\} = \{1,\ 2,\ 1\}$. Substituting into (6.3) gives

$$H(e^{j\hat{\omega}}) = 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2}$$

To obtain formulas for the magnitude and phase of the frequency response of this FIR filter, we can manipulate the equation as follows:

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2} \\ &= e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}\right) \\ &= e^{-j\hat{\omega}}\left(2 + 2\cos\hat{\omega}\right) \end{aligned}$$

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$\hat{w} = w\, T_s = w/f_s$

Suppose there is a dc component of 5 volts = A_0:

$H(e^{i\hat{w}}) = e^{-i\hat{w}}\,[2 + 2\cos(\hat{w})] = e^{0}\,(2 + 2) = 4$ **√ 0 Degrees** ;

Y[n] = H * 5 Volts = 4*5 = 20 volts √ 0 Degrees

DSPF
EXAMPLE 6.4
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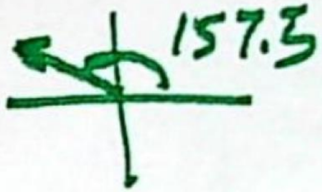
1. $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})$

2. $x[n] = 4 + A\cos(\frac{7\pi}{8}n)$

3. $H(e^{j0}) = 1 \cdot (2 + 2) = 4$

4. $H(e^{j7\pi/8}) = \overset{\to 157.5^\circ}{e^{-j7\pi/8}}\left[\underbrace{2 + 2\underbrace{\cos(\tfrac{7\pi}{8})}_{(-.9239)}}_{0.1522}\right]$



So $y[n] = 4 \cdot 4 + A(0.1522)\cos\left[\frac{7\pi}{8}n - \frac{7\pi}{8}\right]$