DSP First, 2/e

Frequency Response

of FIR Filters

Overview: In chapter 6 the frequency response function for FIR filters is introduced.

When a pure sinusoid passes through a linear time-invariant filter, the output is a sinusoid at the same frequency, but its magnitude and phase might be changed.

In this chapter, we derive the frequency response formulas for several common FIR filters. Plots of the magnitude and phase versus frequency summarize how the filter treats sinusoidal inputs over the entire range of possible input frequencies.

Finally, the concept of filtering is introduced. Since all signals can be decomposed into sinusoidal components, the frequency response function characterizes frequency regions called stop bands and pass bands, where the FIR filter will reject signal components or pass them nearly undistorted.

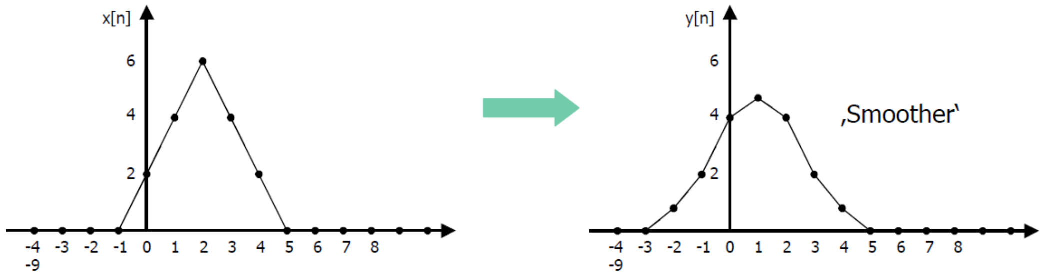
https://courses.cs.washington.edu/courses/cse466/13au/pdfs/lectures/Intro%20to%20DSP.pdf

Embedded DSP: Moving Average Filters

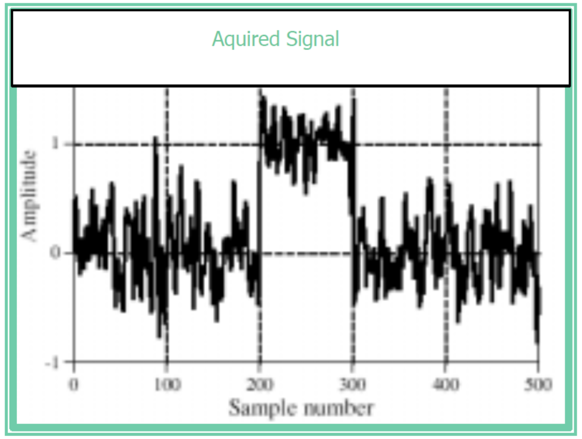
- Example: A three point averager:
- y[n] = 1/3 [x(n) + x(n+1) + x(n+2)]

NOT CAUSAL

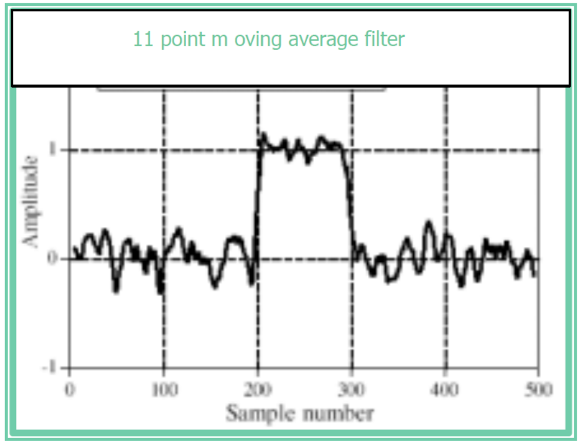
n	n < -2	-2	-1	0	1	2	3	4	5	n > 5
x[n]	0	0	0	2	4	6	4	2	0	n > 5
y[n]	0	2 / 3	2	4	14/3	2	3	5	0	n > 5



Embedded DSP: Moving Average Filters

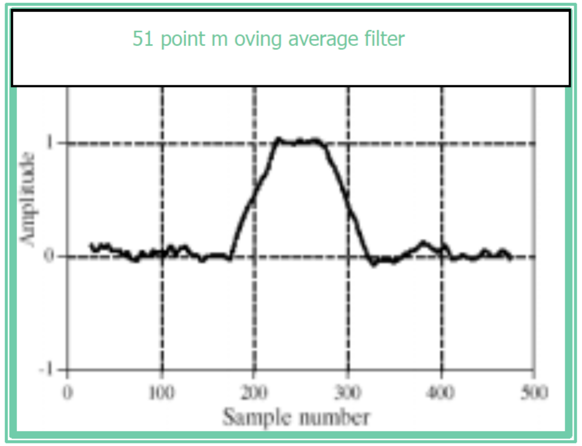


A rectangular pulse with noise



Filtering by a 11 point moving average filter

Processing time ++



Filtering by a 51 point moving average filter

Increasing the number of points in the filter leads to a better noise performance. But the edges are then less sharp. This filter is the best solution providing the lowest possible noise level for a given sharpness of the edges. The possible amount of noise reduction is equal to the square-root of the number of points in the average (a 16 point filter reduces the noise by a factor of 4)

Embedded DSP: Introduction to Digital Filters

- Every linear filter has an
  - Impulse response
  - Step response
  - Frequency response

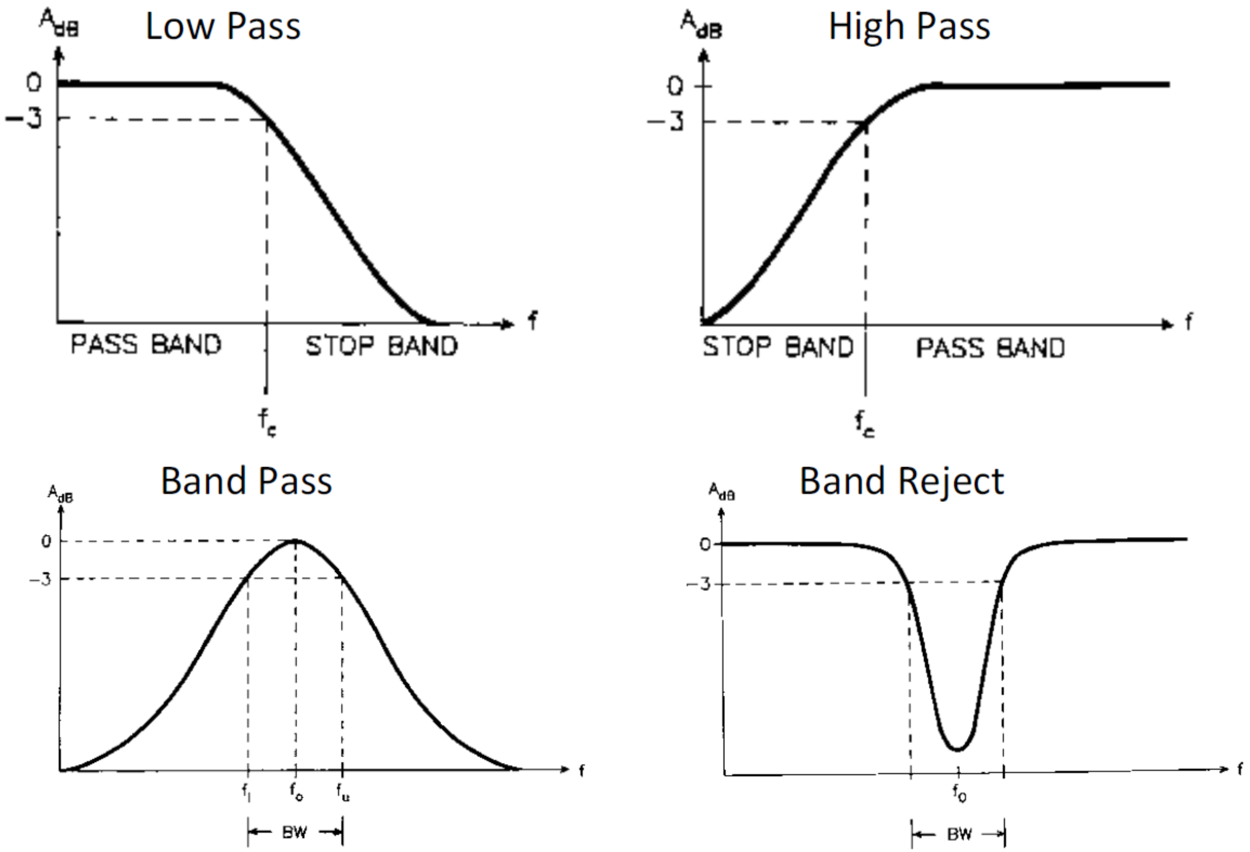
- Each of these responses contain the same information about the filter, but in different form.

- All representations are important because they describe how the filter will react under various circumstances.

Filter Basics

- A filter is used to remove (or attenuate) unwanted frequencies in an audio signal
- "Stop Band" – the part of the frequency spectrum that is attenuated by a filter
- "Pass Band" – part of the frequency spectrum that is unaffected by a filter
- Filters are usually described in terms of their "frequency responses," e.g. *low pass, high pass, band pass, band reject (or notch)*

Frequency Response Curves

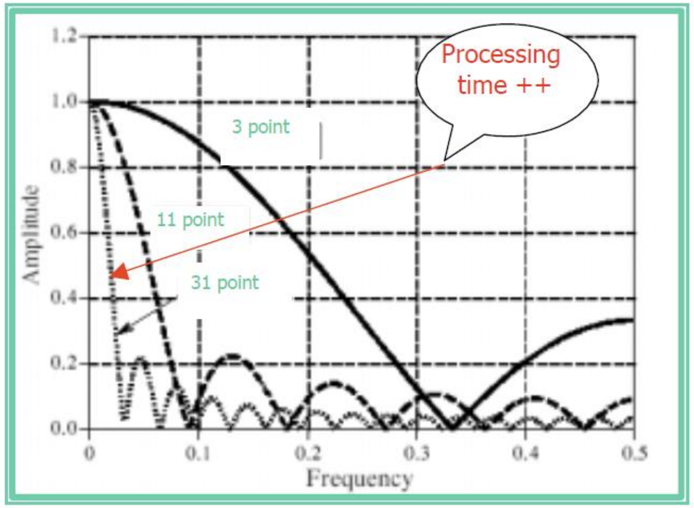


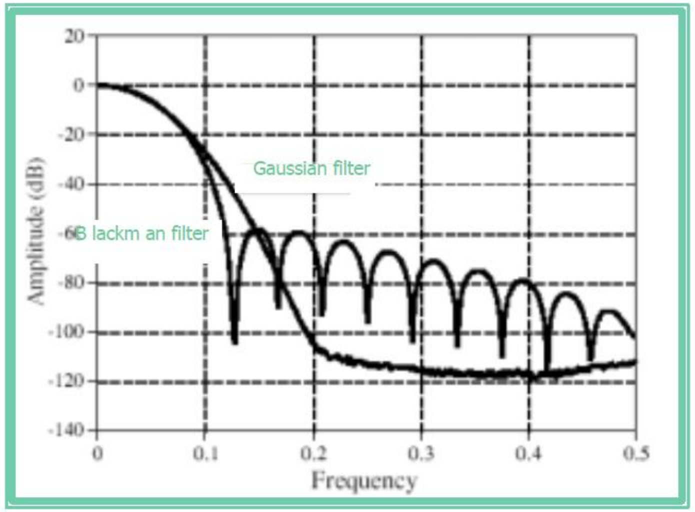
Embedded DSP: Moving Average Filters

- The frequency response is mathematically described by the Fourier Transform of the rectangular pulse.

- H[f]=sin(Pi f M) / M sin(Pi f)

- The roll-off is very slow, the stopband attenuation is very weak !
- The moving average filter is a good smoothing filter but a bad low-pass-filter !





Example: Lowpass Averager

- Consider a 5-point moving average filter wrapped up between a C-to-D and D-to-C system
- We assume a sampling rate of 1000 Hz and an input composed of two sinusoids

$$x(t) = \cos[2\pi(100)t] + 3\cos[2\pi(300)t]$$

- Find the system frequency response in terms of the analog frequency variable f, and find the steady-state output $y(t)$
- We will use `freqz()` to obtain the frequency response

```
>> w = -pi:pi/100:pi;
>> H = freqz(ones(1,5)/5,1,w);
>> subplot(211)
```

SOLVE THE PROBLEM BEFORE MATLAB – IF POSSIBLE!
AT LEAST KNOW THE RANGES INVOLVED

$f_s = 1000\,Hz$ so $f_{max} = 500\,Hz$

$x(t) = \cos\{2\pi(100t)\} + 3\cos\{2\pi\,300t\}$

$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$ $\hat{\omega}_{100} = 2\pi\,\frac{100}{1000} = 0.2\pi$ $\hat{\omega}_{300} = 0.6\pi$

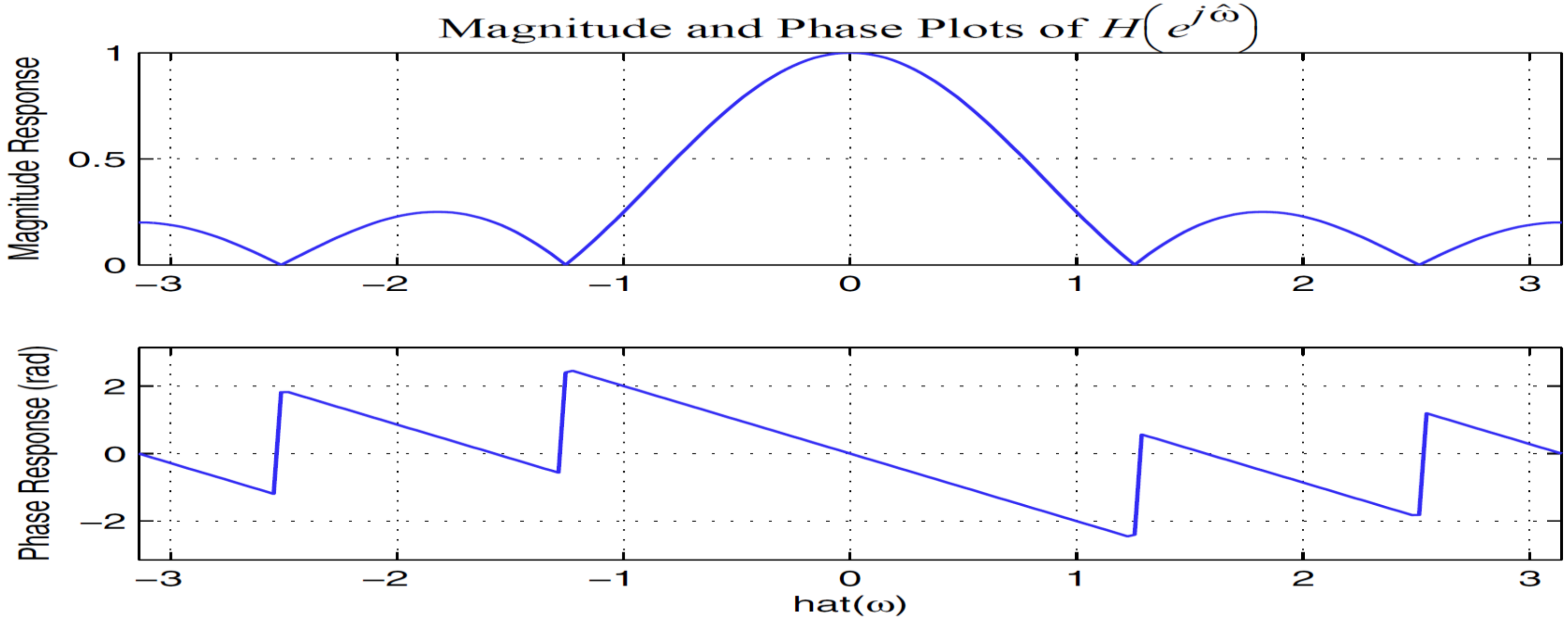
Range of $\hat{\omega}$ $-\frac{500}{1000} \times 2\pi < \hat{\omega} < \frac{500}{1000}\,2\pi$

Digital PLOT $-\pi < \hat{\omega} < \pi$

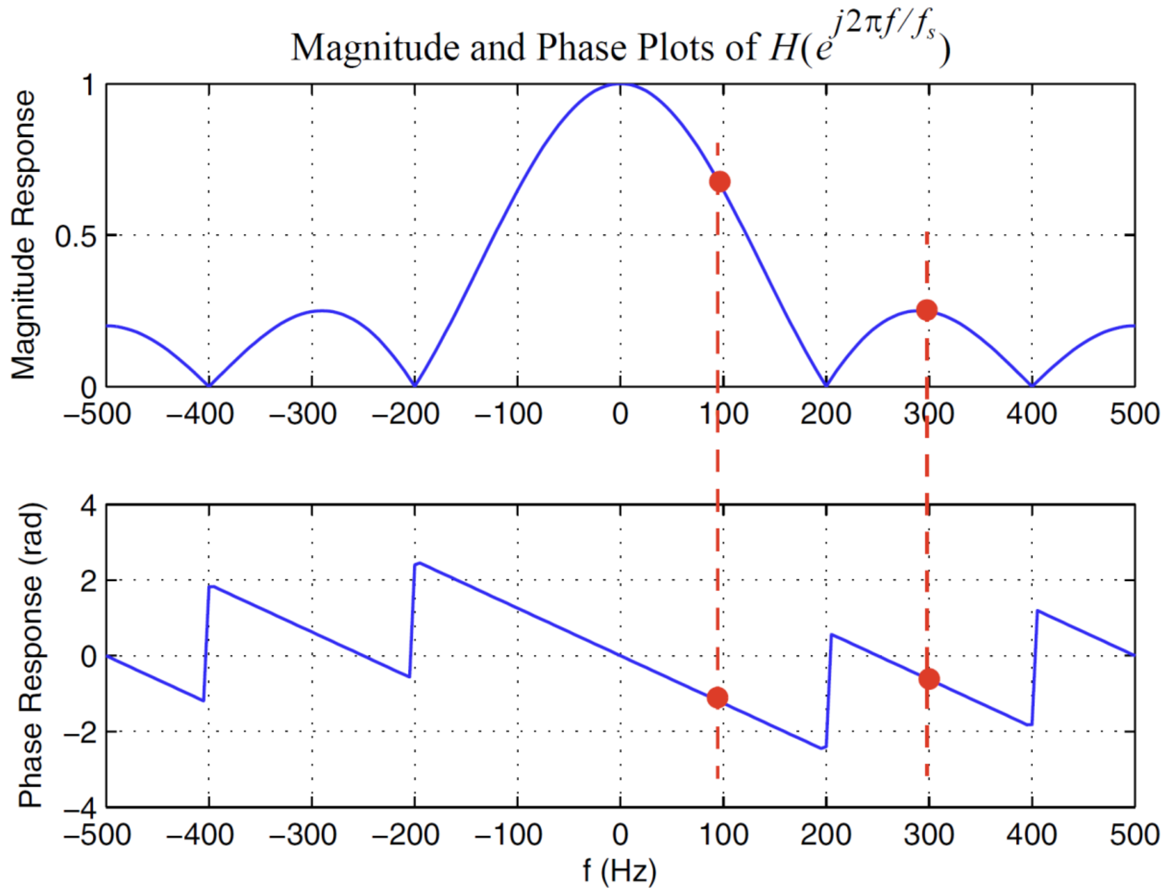
Analog PLOT $-500\,Hz < f < 500\,Hz$

```
>> plot(w,abs(H))
>> axis([-pi pi 0 1]); grid
>> ylabel('Magnitude Response')
>> subplot(212)
>> plot(w,angle(H))
>> axis([-pi pi -pi pi]); grid
>> ylabel('Phase Response (rad)')
>> xlabel('hat(\omega)')
```

Range –pi to pi




```
>> subplot(211)
>> plot(w*1000/(2*pi),abs(H))
>> grid
>> ylabel('Magnitude Response')
>> subplot(212)
>> plot(w*1000/(2*pi),angle(H))
>> grid
>> ylabel('Phase Response (rad)')
>> xlabel('f (Hz)')
```



EACH POINT IS A TRANSFER VALUE SHOWING THE CHANGE IN MAGNATUDE AND PHASE AT EACH FREQUENCY.

$$\textbf{\textit{INPUT}}\ A\cos(2\pi(100)t)\ \text{TO A LINEAR SYSTEM}$$

$$\textbf{\textit{OUTPUT}}\ \ B\cos(2\pi(100)t + \varphi)$$

CHANGE IN AMPLITUDE AND PHASE (TYPICALLY A TIME DELAY)

From the Frequency response, estimate B = 0.6 and phase shift - 1.3 radians

- The output $y(t)$ will be of the same form as the input $x(t)$, except the sinusoids at 100 and 300 Hz need to have the filter frequency response applied
  - Note 100 Hz and 300 Hz < 1000/2 = 500 Hz (no aliasing)
- To properly apply the filter frequency response we need to convert the analog frequencies to the corresponding discrete-time frequencies

$$\begin{aligned} 100\,\text{Hz} &\rightarrow 2\pi \cdot \frac{100}{1000} = 2\pi \cdot 0.1 = 0.2\pi \\ 300\,\text{Hz} &\rightarrow 2\pi \cdot \frac{300}{1000} = 2\pi \cdot 0.3 = 0.6\pi \end{aligned} \tag{6.40}$$

- The frequency response for $L = 5$ in the general moving average filter is

$$H(e^{j\hat{\omega}}) = \frac{\sin(2.5\hat{\omega})}{5\sin(\hat{\omega}/2)} e^{-j2\hat{\omega}} \tag{6.41}$$

- The frequency response at these two frequencies is

$$\begin{aligned} H(e^{j0.2\pi}) &= \frac{\sin(2.5(0.2\pi))}{5\sin((0.2\pi)/2)} e^{-j2(0.2\pi)} = 0.6472 e^{-j0.4\pi} \\ H(e^{j0.6\pi}) &= \frac{\sin(2.5(0.6\pi))}{5\sin((0.6\pi)/2)} e^{-j2(0.6\pi)} = \underbrace{-0.2472 e^{-j1.2\pi}}_{0.2472 e^{-j0.2\pi}} \end{aligned} \tag{6.42}$$

```
>> diric(0.2*pi,5)

ans = 0.6472

>> diric(0.6*pi,5)

ans = -0.2472 % also = 0.2472 at angle +/- pi
```

- The filter output $y[n]$ is

$$y[n] = 0.6472\cos[0.2\pi n - 0.4\pi] + 0.7416\cos[0.2\pi n - 1.2\pi + \pi] \quad (6.43)$$

and the D-to-C output is

$$y(t) = 0.6472\cos[2\pi(100)t - 0.4\pi] + 0.7416\cos[2\pi(300)t - 0.2\pi] \quad (6.44)$$

Back to Analog

ece2610_chap6

http://www.eas.uccs.edu/~mwickert/ece2610/lecture_notes/ece2610_chap6.pdf