MATLAB COMMANDS

DSP USES

conv Convolution and polynomial multiplication.

C = conv(A, B) convolves vectors A and B.

Syntax

w = conv(u,v)

Description

example

w = conv(u,v) returns the convolution of vectors u and v.
If u and v are vectors of polynomial coefficients,
convolving them is equivalent to multiplying the two polynomials.

r .

% Create vectors u and v containing the **coefficients of the**
% polynomials X^2 +1 and 2X+7 .

r .

u = [1 0 1]; v = [2 7]; % Coefficients

Use convolution to multiply the polynomials.

w = conv(u,v)

w = 2 7 2 7 % 3^{th} order Product

w contains the polynomial coefficients for

2*X^3 + 7*x^2 + 2*x + 7

.

```
u = [1 1 1];
v = [1 1 0 0 0 1 1];
w = conv(u,v)
```

1 2 2 1 0 1 2 2 1

The length of w is length(u)+length(v)-1, which in this example is 9.

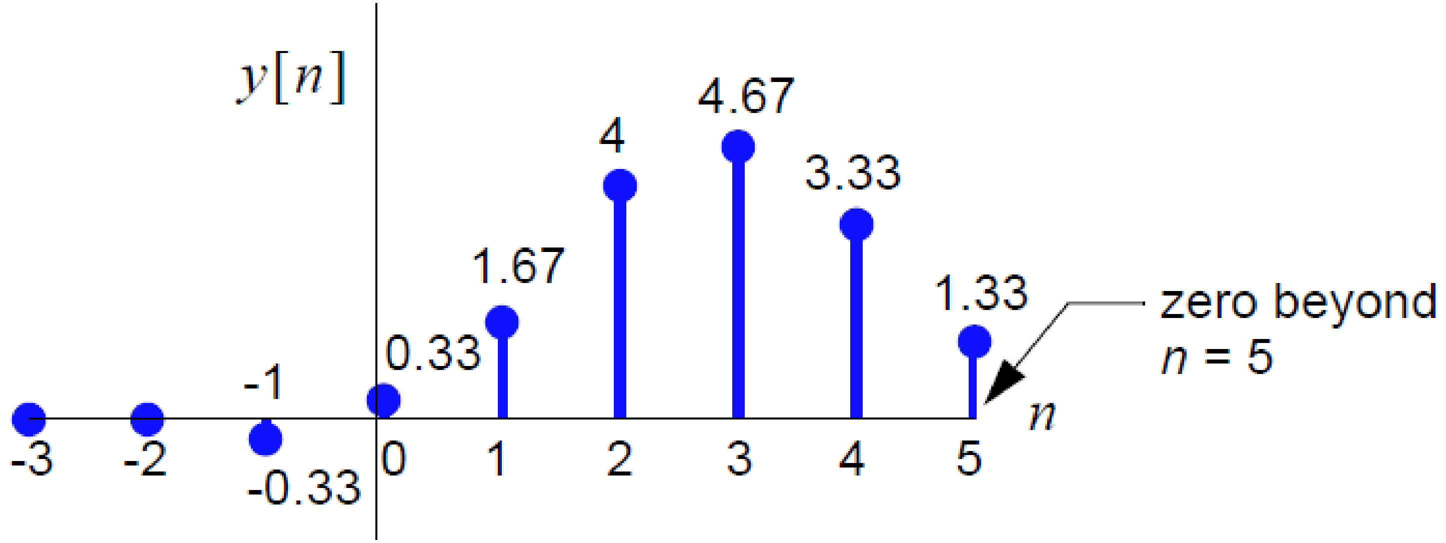
filter One-dimensional digital filter.

Y = filter(B,A,X) filters the data in vector X with the filter described by vectors A and B to create the filtered data Y. The filter is a "Direct Form II Transposed" implementation of the standard difference equation:

```
a(1)*y(n) = b(1)*x(n) + b(2)*x(n-1) + ... + b(nb+1)*x(n-nb)
                      - a(2)*y(n-1) - ... - a(na+1)*y(n-na)
```

If a(1) is not equal to 1, filter normalizes the filter coefficients by a(1).

```
>> n= -3:5;
>> x = [0 0 -1 2 4 6 4 0 0]
>> % We will learn about the filter function later
>> y = filter(1/3*[1 1 1],1,x);
>> stem(n,y,'filled')
```



a = 1, bk =1/3*[1 1 1]

5-4 2610

Example: $\{b_k\} = \{1, 1, 3, 1, 1\}$

- The frequency response of this FIR filter is

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{4} b_k e^{-j\hat{\omega}k}$$

$$= 1 + e^{-j\hat{\omega}} + 3e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}[e^{j2\hat{\omega}} + e^{j\hat{\omega}} + 3 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}]$$

$$= e^{-j2\hat{\omega}}[2\cos(2\hat{\omega}) + 2\cos(\hat{\omega}) + 3]$$

- We have used the inverse Euler formula for cosine twice

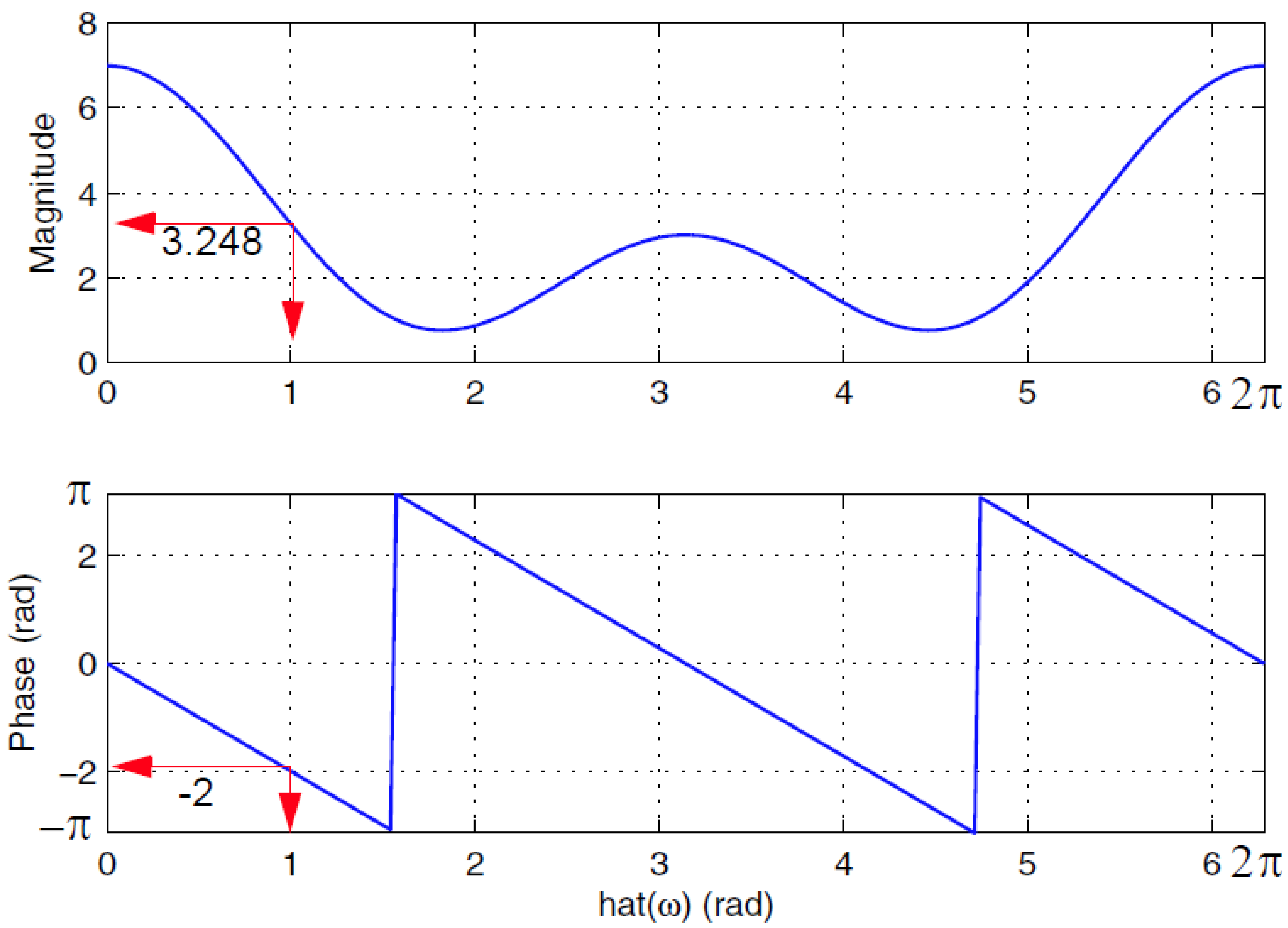
- For this particular filter we have that

$$|H(e^{j\hat{\omega}})| = 3 + 2\cos(\hat{\omega}) + 2\cos(2\hat{\omega})$$

$$\angle H(e^{j\hat{\omega}}) = -2\hat{\omega}$$

- Use MATLAB to plot the magnitude and phase response

```
>> w = 0:2*pi/200:2*pi;
>> H = exp(-j*2*w).*(3 + 2*cos(w) + 2*cos(2*w));
>> subplot(211)
>> plot(w,abs(H))
>> axis([0 2*pi 0 8])
>> grid
>> ylabel('Magnitude')
>> subplot(212)
>> plot(w,angle(H))
>> axis([0 2*pi -pi pi])
>> grid
>> ylabel('Phase (rad)')
>> xlabel('hat(\omega) (rad)')
```


Magnitude
8
6
4
2
0
3.248
0
1
2
3
4
5
6
2π
Phase (rad)
π
2
0
–2
–π
-2
0
1
2
3
4
5
6
2π
hat(ω) (rad)

Example: Find $y[n]$ for Input $x[n] = 5e^{j(1 \cdot n)}$

- The input frequency is $\hat{\omega}_0 = 1\,\text{rad}$, the amplitude is 5, and the phase is $\phi = 0$
- Assuming $x[n]$ is input to the 4-tap FIR filter in the previous example, the filter output is

$$y[n] = (3 + 2\cos(1) + 2\cos(2 \cdot 1))e^{-j(1 \cdot 2)} 5e^{j(1 \cdot n)}$$

$$= 16.2415 e^{-j2} \cdot e^{jn}$$

- The amplitude response or gain at $\hat{\omega}_0 = 1$ is $|H(e^{j1})| = 3.248$; why?

- For this particular filter we have that

$$\left|H(e^{j\hat{\omega}})\right| = 3 + 2\cos(\hat{\omega}) + 2\cos(2\hat{\omega})$$

Approximate this at theta = 1 rad; cos(1) about cos (60) ≈ ½
Cos(2) about cos(120) ≈ -1/2.

Expect 3 +2*.5 +2(-0.5) = about 3.

Exact 3+1.0806-0.8323 = 3.2843

Actually 1/pi = theta/180 = 1/(3.14)*180 = 1(3+ .14/3)*180 = 60 – 60(.05) ABOUT 57 DEGREES
1/(1+X) ≈ 1-X from Taylor X = 0.14/3≈ 0.05

Example: Three Inputs with $b_k = \{1, -1, 1\}$

- The input is

$$x[n] = 10 + 4\cos\left(\frac{\pi}{4}n + \frac{\pi}{8}\right) + 3\cos\left(\frac{\pi}{3}n - \frac{\pi}{4}\right)$$

- The frequency response is

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \\ &= e^{-j\hat{\omega}}[-1 + 2\cos(\hat{\omega})] \end{aligned}$$

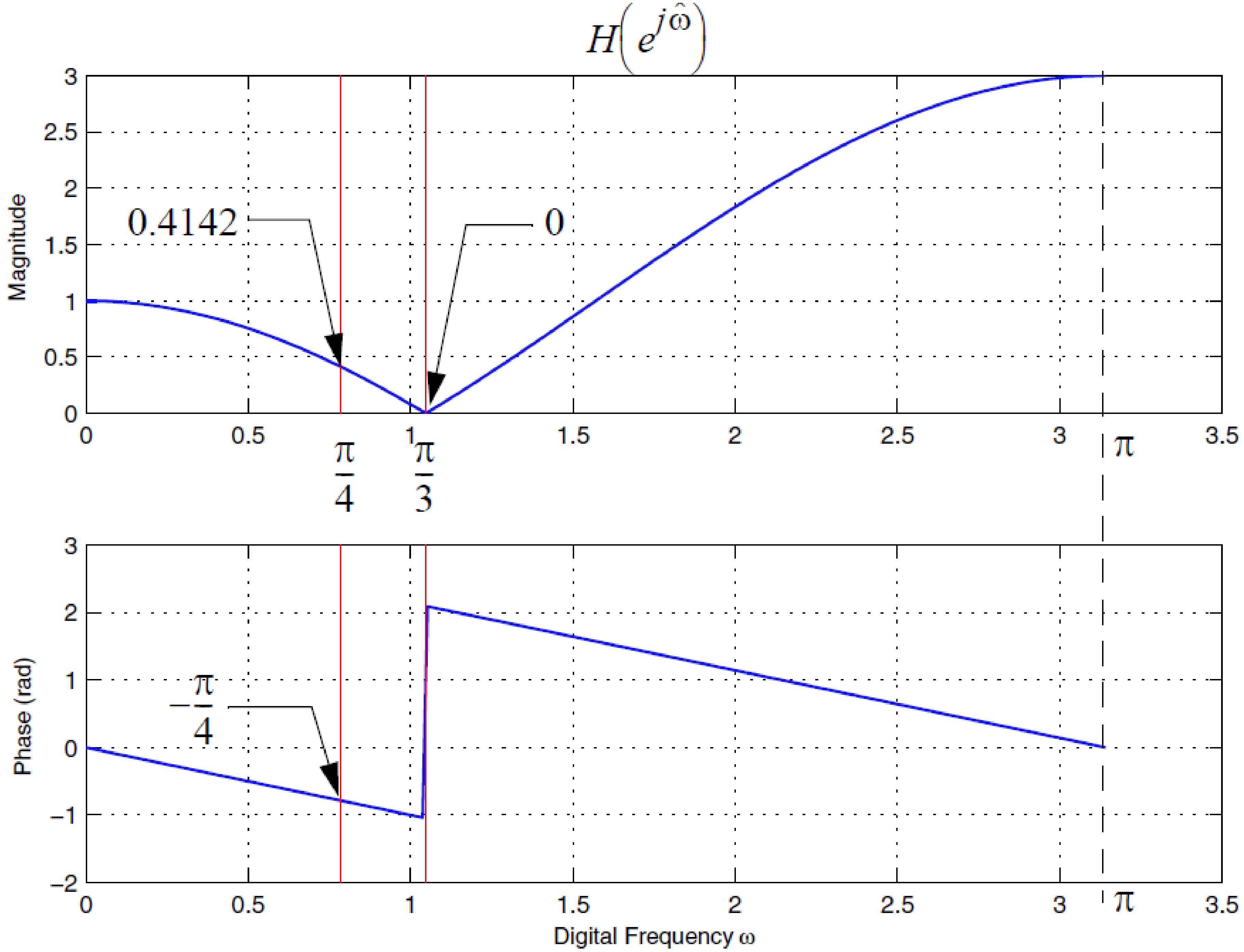
- The input frequencies are $\hat{\omega}_k = \{0, \pi/4, \pi/3\}$

$$\begin{aligned} H(e^{j0}) &= e^{-j0}[-1 + 2\cos(0)] = 1 \\ H(e^{j\pi/4}) &= e^{-j\pi/4}[-1 + 2\cos(\pi/4)] = 0.4142e^{-j\pi/4} \\ H(e^{j\pi/3}) &= e^{-j\pi/3}[-1 + 2\cos(\pi/3)] = 0 \end{aligned}$$

- Thus

$$\begin{aligned} y[n] &= 10 \cdot 1 + 4 \cdot 0.4142\cos\left(\frac{\pi}{4}n + \frac{\pi}{8} - \frac{\pi}{4}\right) \\ &= 10 + 1.6569\cos\left(\frac{\pi}{4}n - \frac{\pi}{8}\right) \end{aligned}$$

```
>> w = 0:pi/200:pi;
>> H = 1 - exp(-j*w) + exp(-j*2*w);
>> subplot(211)
>> plot(w,abs(H))
>> grid
>> hold
>> plot([pi/4 pi/4],[0 3],'r')
>> plot([pi/3 pi/3],[0 3],'r')
>> ylabel('Magnitude')
>> subplot(212)
>> plot(w,angle(H))
>> grid
>> hold
>> plot([pi/4 pi/4],[-2 3],'r')
>> ylabel('Phase (rad)')
>> xlabel('Digital Frequency \omega')
```



$H(e^{j\hat{\omega}})$
Magnitude
0.4142
0
$\frac{\pi}{4}$
$\frac{\pi}{3}$
π
Phase (rad)
$-\frac{\pi}{4}$
Digital Frequency ω

```
>> help freqz
 FREQZ Digital filter frequency response.
    [H,W] = FREQZ(B,A,N) returns the N-point complex frequency response
    vector H and the N-point frequency vector W in radians/sample of
    the filter:
                jw               -jw              -jmw
         jw  B(e)    b(1) + b(2)e + .... + b(m+1)e
      H(e) = ---- = ------------------------------------
                jw               -jw              -jnw
             A(e)    a(1) + a(2)e + .... + a(n+1)e
    given numerator and denominator coefficients in vectors B and A. The
    frequency response is evaluated at N points equally spaced around the
    upper half of the unit circle. If N isn't specified, it defaults to
    512.
```

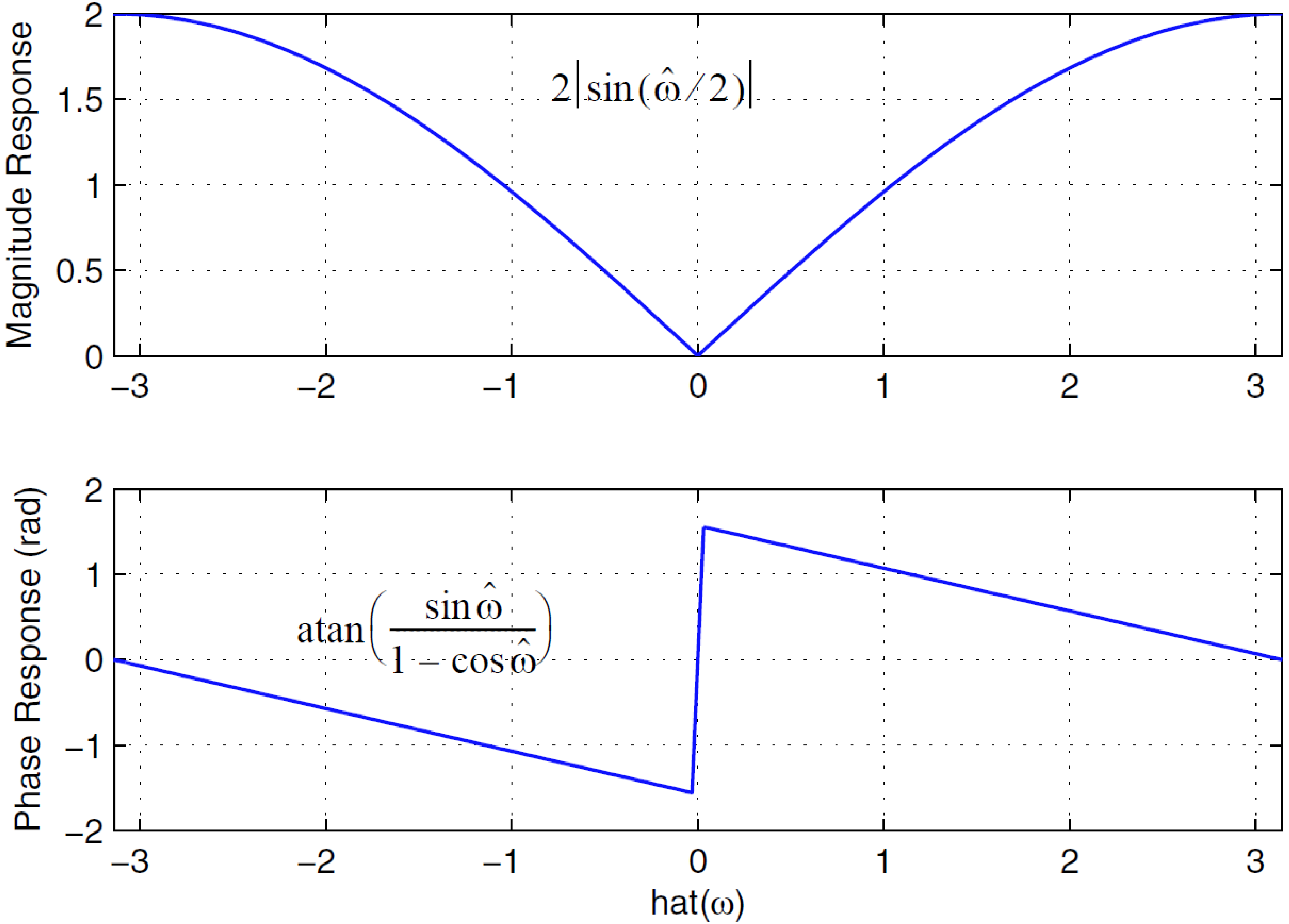
Example: First-Difference System $y[n] = x[n] - x[n-1]$

- The frequency response is

$$H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}} = (1 - \cos\hat{\omega}) + j\sin\hat{\omega}$$

```
>> w = -pi:pi/100:pi;
>> H = freqz([1 -1],1,w); %use a custom w axis
>> subplot(211)
>> plot(w,abs(H))
>> axis([-pi pi 0 2]); grid; ylabel('Magniude Response')
>> subplot(212)
>> plot(w,angle(H))
>> axis([-pi pi -2 2]); grid;
>> ylabel('Phase Response (rad)')
>> xlabel('hat(\omega)')
```

KILL THE dc component



2
1.5
1
0.5
0
Magnitude Response
−3
−2
−1
0
1
2
3
$2|\sin(\hat{\omega}/2)|$
2
1
0
−1
−2
Phase Response (rad)
$\mathrm{atan}\left(\frac{\sin\hat{\omega}}{1-\cos\hat{\omega}}\right)$
−3
−2
−1
0
1
2
3
hat(ω)

```
>> help diric
 diric    Dirichlet, or periodic sinc function
    Y = diric(X,N) returns a matrix the same size as X whose elements     are the Dirichlet
function of the elements of X.  Positive integer     N is the number of equally spaced extrema
of the function in the     interval 0 to 2*pi.

    The Dirichlet function is defined as

        d(x) = sin(N*x/2)./(N*sin(x/2))   for x not a multiple of 2*pi
                                +1 or -1 for x a multiple of 2*pi. (depending on limit)

            % Example 1:
        %   Plot the Dirichlet function over the range 0 to 4, for N = 7 and     %   N = 8.

            x = linspace(0,4*pi,300);
        subplot(211); plot(x,diric(x,7)); % Generate linearly spaced vectors
        title('Diric, N = 7'); axis tight;
        subplot(212); plot(x,diric(x,8));
        title('Diric, N = 8'); axis tight;
```