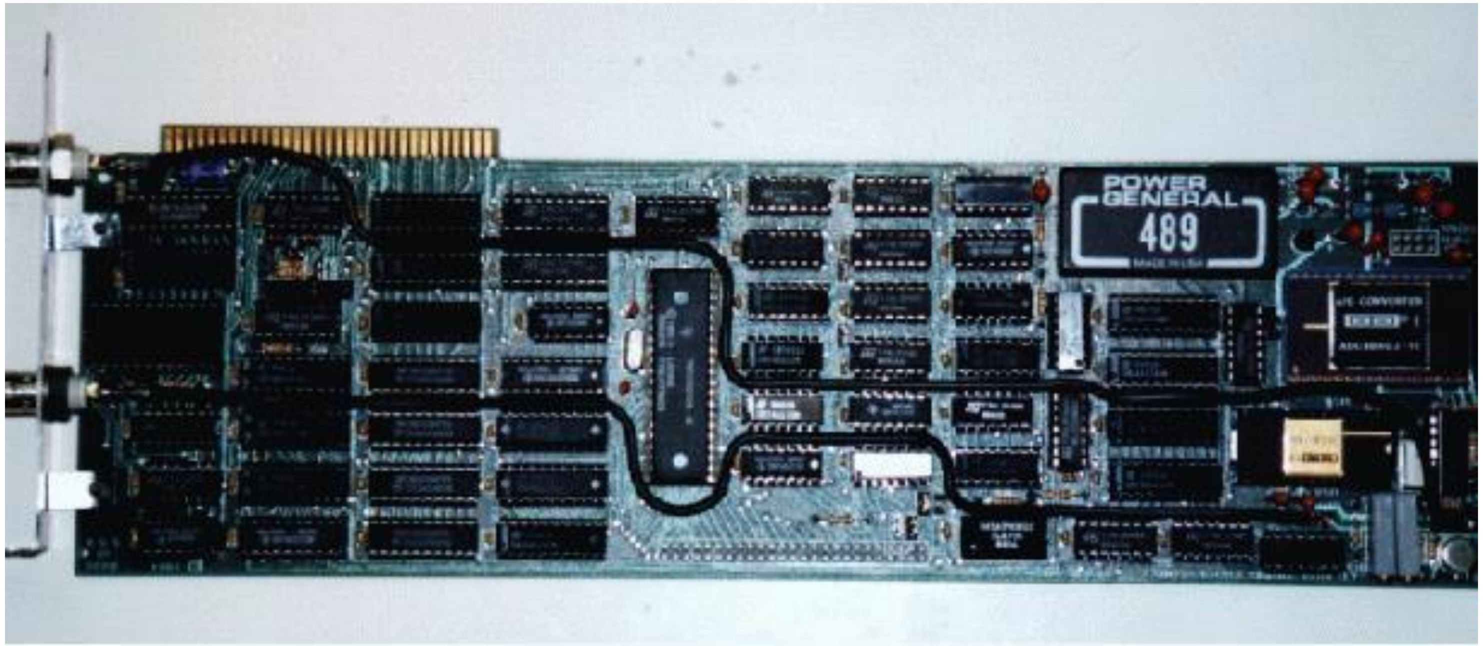
FIR FILTERS IN FREQUENCY

CHAPTER

6

The TMS32010, 1983

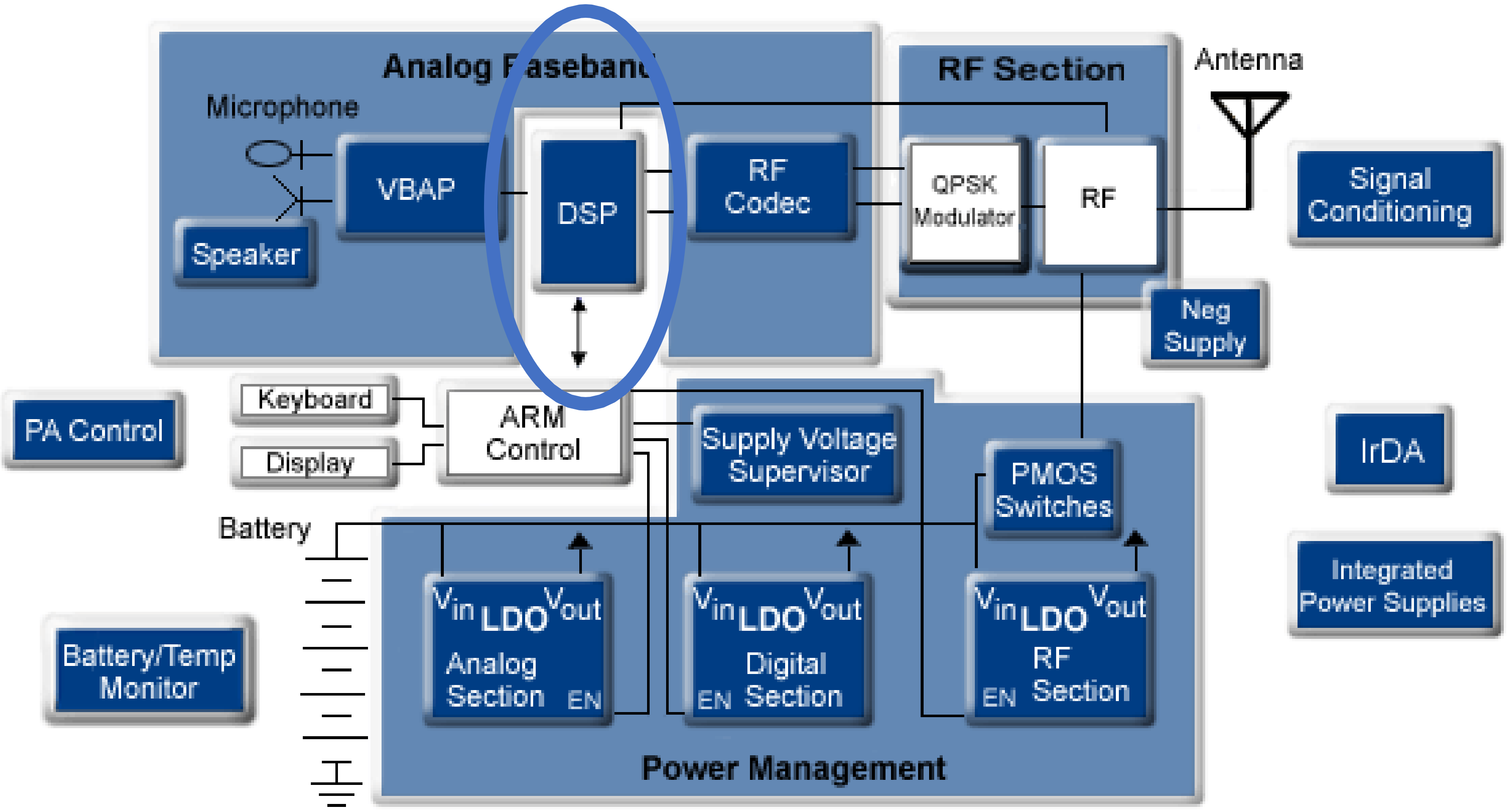


First PC plug-in board from Atlanta Signal Processors Inc.

Rockland Digital Filter, 1971

Cost was about the same as the price of a small house.

Digital Cell Phone (ca. 2000)



Now, digital cameras and video streaming rely on DSP algorithms

Frequency Response of an FIR System

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k} \qquad (6.4$$

Relationship of digital frequency to analog w/(Sampling Frequency) Page 105

$\hat{w} = w T_s = w/f_s$ (Radians = radians/sec * Seconds)

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

FILTER OUTPUT

INPUT A SINUSOID (EXPONENTIAL DIGITAL FORM)

$$x[n] = Ae^{j\varphi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} \tag{6.3}$$

TRANSFER FUNCTION

TRANSFER FUNCTION

$$y[n] = \left(|H(e^{j\hat{\omega}})|e^{j\angle H(e^{j\hat{\omega}})}\right) Ae^{j\varphi}e^{j\hat{\omega}n}$$

$$= (|H(e^{j\hat{\omega}})| \cdot A)e^{j(\angle H(e^{j\hat{\omega}})+\varphi)}e^{j\hat{\omega}n} \quad (6.5)$$

PHASE CHANGE

AMPLITUDE CHANGE

*The term "**transfer function**" is also used in the frequency domain analysis of systems using transform methods such as the Laplace transform; here it means the amplitude of the output as a function of the frequency of the input signal.* ***For example, the transfer function of an electronic filter is the voltage amplitude at the output as a function of the frequency of a constant amplitude sine wave applied to the input.***

https://web.stanford.edu/~boyd/ee102/freq.pdf

Measuring frequency response

for $\omega = \omega_1, \ldots, \omega_N$,

- apply sinusoid at frequency ω, with phasor $\mathbf{U}$
- wait for output to converge to SSS
- measure $\mathbf{Y}_{\rm ss}$
  ($i.e.$, magnitude and phase shift of $y_{\rm ss}$)

N can be a few tens (for hand measurements) to several thousand

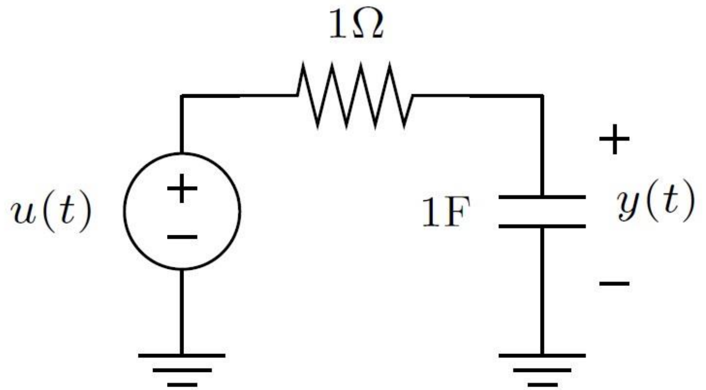
Frequency response plots

- $|H(j\omega)|$ & $\angle H(j\omega)$ versus ω (called *Bode plot*)

the most common format is a Bode plot

https://www.tutorialspoint.com/low-pass-and-high-pass-filter-bode-plot

example: RC circuit

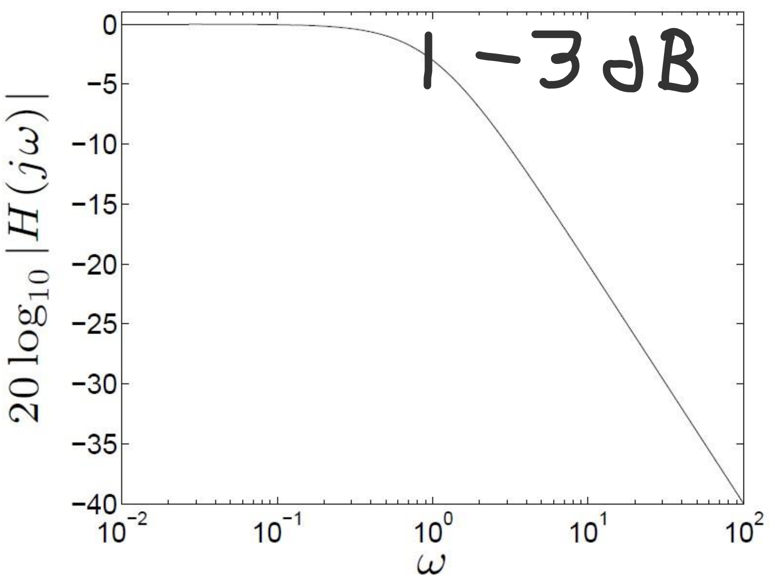


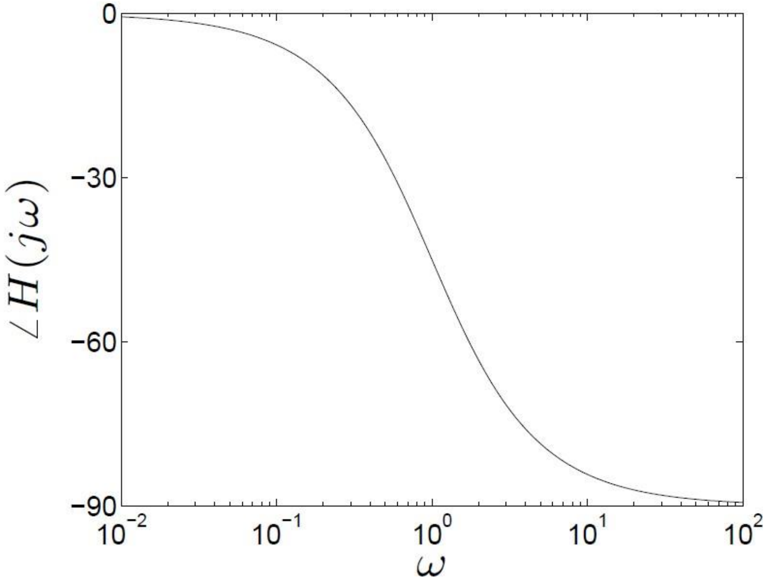
$$Y(s) = \frac{1}{1+s}U(s)$$

$$H(j\omega) = \frac{1}{1+j\omega}$$

R ohm =
C farad =1

A really BIG capacitor





WHY -90°

Log scale in radian frequency

Low Pass Filter Bode Plot

The frequency response function or transfer function of the RC low pass filter is given by,

$$\frac{V_{out}}{V_{in}} = \frac{A}{1+(j\omega T)} = \frac{A}{1+(j\omega/\omega_0)} = \frac{A}{\sqrt{1+(\omega/\omega_0)^2}} \angle -\tan^{-1}(\frac{\omega}{\omega_0})$$

Where,

- T = Time constant of the circuit = $1/\omega_0 = RC$
- A = Constant and
- ω_0 = Cut off frequency

Bode Magnitude Plot of LPF

The magnitude plot can be obtained from the absolute value of transfer function i.e.

$$|\frac{V_{out}}{V_{in}}| = 20\log_{10}\frac{|A|}{|1+j\omega/\omega_0|}$$

When $\omega < \omega_0$, then the imaginary part is much smaller than its real part, so $|1+j\omega/\omega_0| = 1$, thus,

$$|\frac{V_{out}}{V_{in}}|_{dB} = 20\log_{10}A - 20\log_{10}1 = 20\log_{10}A$$

Hence, at very low frequencies the frequency response function is approximated by a straight line of zero slope, which is the low frequency asymptote of the Bode plot.

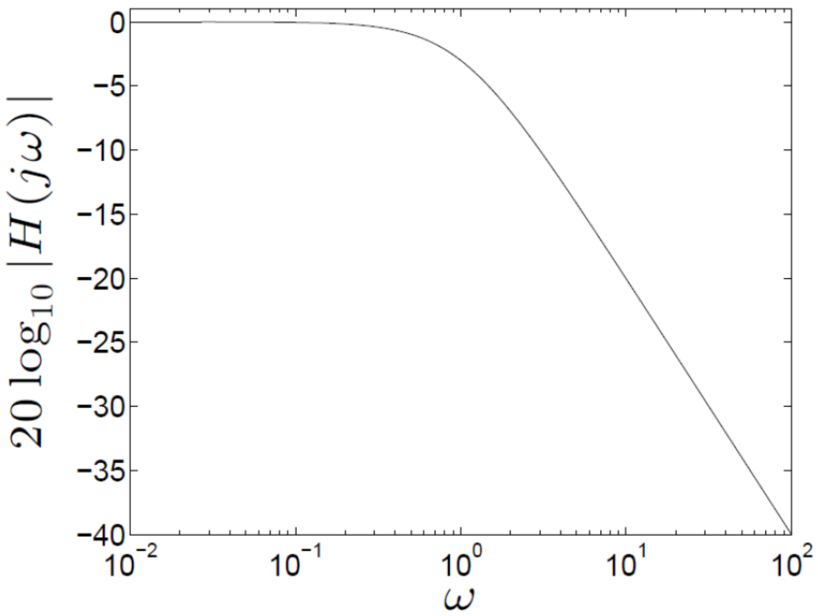
When $\omega > \omega_0$ then the imaginary part is much larger than its real part, so $|1 + j\omega / \omega_0| = |j\omega / \omega_0|$, thus,

$$|\frac{V_{out}}{V_{in}}|_{dB} = 20\log_{10} A - 20\log_{10}(\omega / \omega_0) = 20\log_{10} A - 20\log_{10} \omega - 20\log_{10} \omega_0$$

Hence, at very high frequencies the frequency response function is approximated by a straight line of (- 20 dB / Decade) slope that intercepts the $\log \omega$ at $\log \omega_0$. This line is the high frequency asymptote of the bode plot.

When $\omega = \omega_0$,the real and imaginary part of the simple pole are equal, so $|1 + j\omega + \omega_0| = |1 + j| = \sqrt{2}$, thus,

$$|\frac{V_{out}}{V_{in}}|_{dB} = 20\log_{10} A - 20\log_{10} \sqrt{2} = 20\log_{10} A - 3dB$$



W0 = 1/RC = 1 rad/sec

Bode Phase Plot of LPF

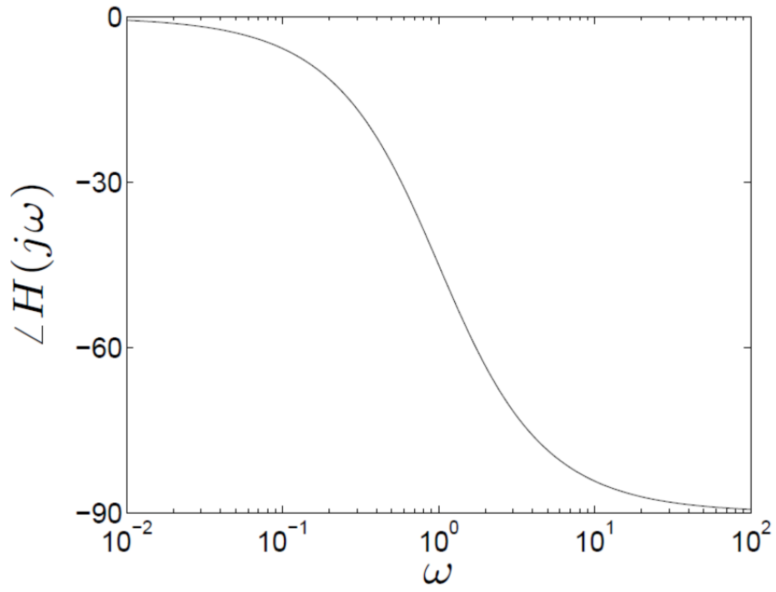
$$\angle(\frac{V_{out}}{V_{in}}) = -\tan^{-1}(\frac{\omega}{\omega_0})$$

When $\omega = 0$, Then

$$\angle(\frac{V_{out}}{V_{in}}) = -\tan^{-1}(\frac{\omega}{\omega_0}) = -\frac{\pi}{4}$$

When $\omega \to \infty$, Then

$$\angle(\frac{V_{out}}{V_{in}}) = -\tan^{-1}(\frac{\omega}{\omega_0}) = -\frac{\pi}{2}$$



FIRST ORDER LOW-PASS FILTER!

WHY -90°

Consider an LTI system for which the difference equation coefficients are $\{b_k\} = \{1,\ 2,\ 1\}$. Substituting into (6.3) gives

$$H(e^{j\hat{\omega}}) = 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2}$$

To obtain formulas for the magnitude and phase of the frequency response of this FIR filter, we can manipulate the equation as follows:

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2} \\ &= e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}\right) \\ &= e^{-j\hat{\omega}}\left(2 + 2\cos\hat{\omega}\right) \end{aligned}$$

Relationship of digital frequency to analog w/(Sampling Frequency) Page 105

$\hat{w} = w\, T_s = w/f_s$

Suppose there is a dc component of 5 volts = A_0:

$H(e^{i\hat{w}}) = e^{-i\hat{w}}\,[2 + 2\cos(\hat{w})] = \hat{e}^{0}\,(2 + 2) = 4$ **√ 0 Degrees** ;

Y[n] = H * 5 Volts = 4*5 = 20 volts √ 0 Degrees

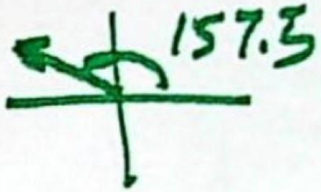
DSPF
EXAMPLE 6.4
Pg 200

1. $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})$

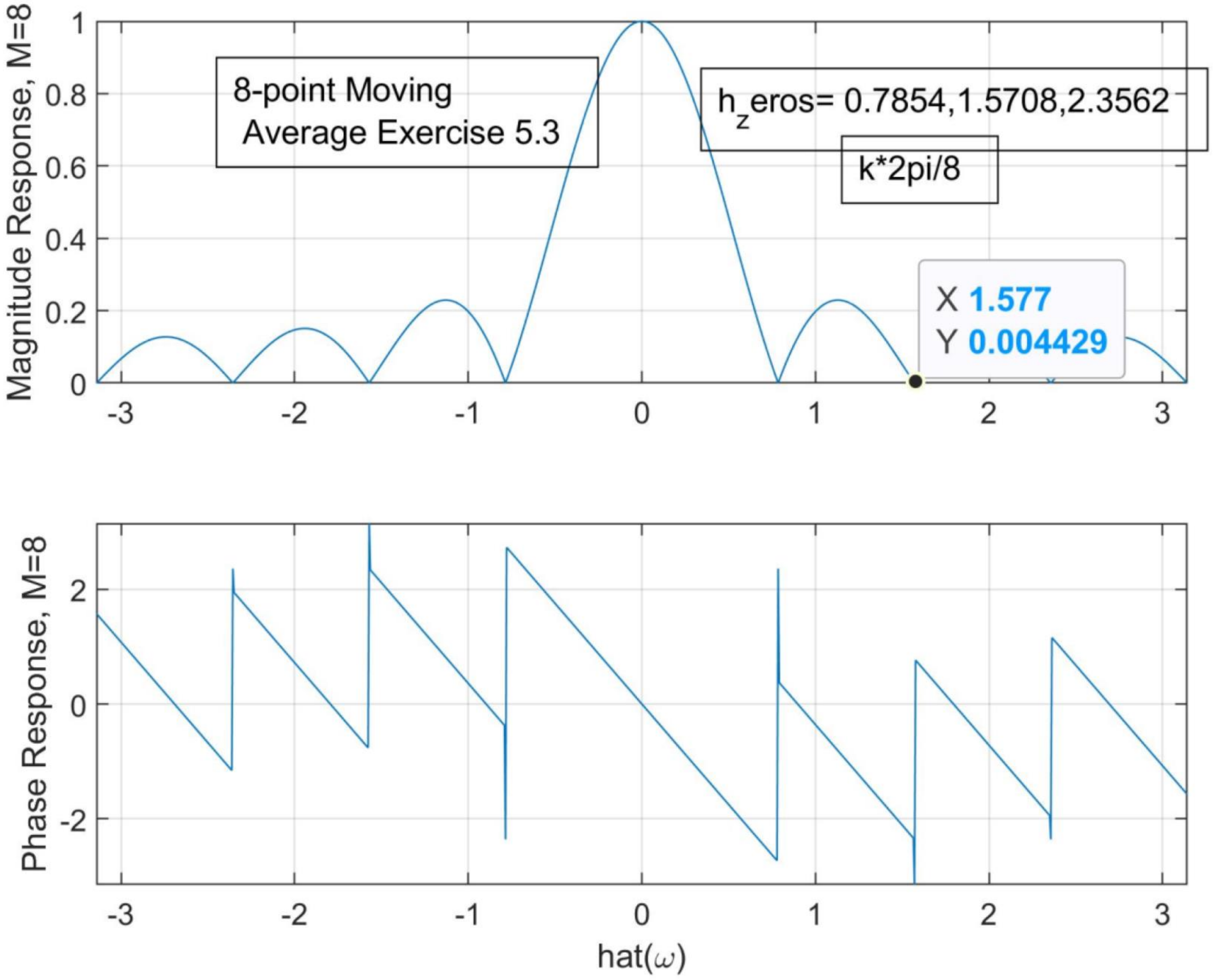
2. $x[n] = 4 + A\cos(\frac{7\pi}{8} n)$

3. $H(e^{j0}) = 1 \cdot (2+2) = 4$

4. $H(e^{j7\pi/8}) = e^{-j\overbrace{7\pi/8}^{\to 157.5^\circ}} \underbrace{\left[2 + 2\underbrace{\cos(7\pi/8)}_{(-.9239)}\right]}_{0.1522}$



So $\underline{y[n]} = 4 \cdot 4 + A(0.1522)\cos\left[\frac{7\pi}{8} n - \frac{7\pi}{8}\right]$



DSPF Pg 214 & 220

Zeros: k (2 pi/8)
Pi/4, pi/2, 3 pi/4

```matlab
% 8-point Moving Average Frequency Response
% Exercise 5.3
clc, clear all, clf
w_hat=-pi:pi/500:pi;
H_8=freqz(ones(1,8)/8,1,w_hat);
h_8zeros= (1/8)*[2*pi,4*pi,6*pi]
% h_8zeros = 0.7854    1.5708    2.3562
figure(1)
subplot(211)
plot(w_hat,abs(H_8))
grid;axis([-pi pi 0 1])
ylabel('Magnitude Response, M=8')
subplot(212)
plot(w_hat,angle(H_8))
grid;axis([-pi pi -pi pi]);
ylabel('Phase Response, M=8')
xlabel('hat(\omega)')
```