Exam1 Review 3315

Table 2-1 Basic properties of the sine and cosine functions.

Property	Equation
Equivalence	$\sin\theta = \cos(\theta - \pi/2)$ or $\cos(\theta) = \sin(\theta + \pi/2)$
Periodicity	$\cos(\theta + 2\pi k) = \cos\theta$, when k is an integer
Evenness of cosine	$\cos(-\theta) = \cos\theta$
Oddness of sine	$\sin(-\theta) = -\sin\theta$
Zeros of sine	$\sin(\pi k) = 0$, when k is an integer
Ones of cosine	$\cos(2\pi k) = 1$, when k is an integer
Minus ones of cosine	$\cos[2\pi(k + \frac{1}{2})] = -1$, when k is an integer

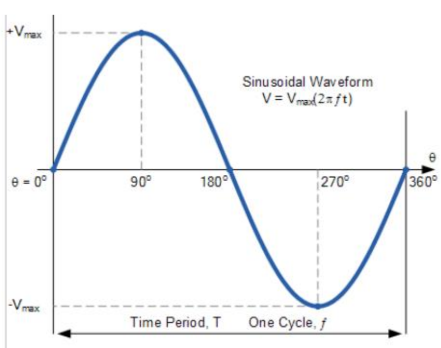
[1] It is a good idea to memorize the form of these plots and be able to sketch them accurately.

Table 2-2 Some basic trigonometric identities.

Number	Equation
1	$\sin^2\theta + \cos^2\theta = 1$
2	$\cos 2\theta = \cos^2\theta - \sin^2\theta$
3	$\sin 2\theta = 2\sin\theta\cos\theta$
4	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
5	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

1. **Show that cos(wt -pi/2) = sin(wt)**

2. **Let $E(t) = V_{max}(2\pi f t)$, f=50 Hz, Vmax = 240 v**



2a.Compute the period of the wave in ms and the equation for the ac wave fundamental.

2b. Write the equation for E(t) for the kth harmonic and list the first 5 harmonics

2c. What is the radian frequency of the fundamental?

3. **Time Shift**

$$x_0(t - t_1) = A\cos(\omega_0(t - t_1)) = A\cos(\omega_0 t + \varphi)$$
$$\Rightarrow \cos(\omega_0 t - \omega_0 t_1) = \cos(\omega_0 t + \varphi)$$

For the signal f(t) 10cos(2pi100t-pi/2), Determine

a. The period
b. The time delay in seconds
c. The form of the signal f(t) as Acos(w [t -tshift])

4.Add Sinusoids using phasors.

EXERCISE 2.8: Consider the two sinusoids,

$$x_1(t) = 5\cos(2\pi(100)t + \pi/3)$$
$$x_2(t) = 4\cos(2\pi(100)t - \pi/4)$$

Obtain the phasor representations of these two signals, add the phasors, plot the two phasors and their sum in the complex plane, and show that the sum of the two signals is

$$x_3(t) = 5.536\cos(2\pi(100)t + 0.2747)$$

In degrees the phase should be 15.74°. Examine the plots in Fig. 2-16 to see whether you can identify the cosine waves $x_1(t)$, $x_2(t)$, and $x_3(t) = x_1(t) + x_2(t)$.

Your answer may differ slightly depending on the round off error.

a. Write answer as a Complex Exponential Signal and a real solution as in x3(t). Show that the example is correct!
b. Draw the vectors from the complex exponentials to confirm your results.

5_ P2.5 ---

Prove

$$\cos(\theta_1 + \theta_2) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$$

By writing and expanding $\Re\left\{e^{j(\theta_1+\theta_2)}\right\}$

6. Determine the Roots of $z^5 = 1$. Write as complex exponentials and state the angles in degrees.

(Help from my Chapter 2.)

Let a be a nonzero complex number. Then if a complex number z satisfies the equation

$$z^N = a,$$

z is called an Nth *root* of a. The determination of the Nth roots of a uses the theorem of De Moivre

$$(\cos\theta + i\sin\theta)^N = \cos N\theta + i\sin N\theta \qquad \textbf{(2.20)}$$

which follows from Equation 2.16. Writing a in polar coordinates as

$$a = r(\cos\theta + i\sin\theta),$$

the roots of a are

$$a^{1/N} = \sqrt[N]{r}\left[\cos\left(\frac{\theta}{N} + p\frac{2\pi}{N}\right) + i\sin\left(\frac{\theta}{N} + p\frac{2\pi}{N}\right)\right] \qquad \textbf{(2.21)}$$

where p takes the values $0, 1, \ldots, N-1$. These are N distinct values, and it can be shown that there are no others. The term $\sqrt[N]{r}$ is the positive real Nth root of r and for the specific case $|a| = 1$ this value is 1.

7. Complex Numbers and polar form- Write in degrees, complex exponential form (Polar), complex trig form and x_jy form. Draw the vectors.

a. $2\angle 90°$

b.

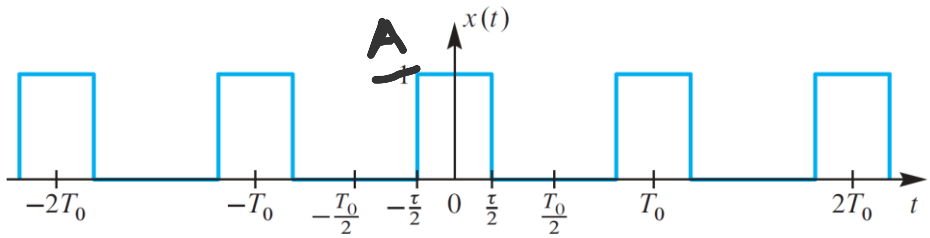
$2\underbrace{\angle 135°}_{\theta}$

8. Add two Sinusoids.

$$x_1(t) = 1.7\cos(20\pi t + 70\pi/180)$$
$$x_2(t) = 1.9\cos(20\pi t + 200\pi/180)$$

a. Compute the Complex Amplitudes
b. Convert to Cartesian form and add the amplitudes
c. Write the result as Acos (wt +phi) and define phi in radians and degrees
d. Determine the time shift in seconds

9. Fourier Series



a. Tell me what you can about this pulse train – Even/odd?, dc value, frequencies in Hertz for the Fourier Series for the first 8 terms

b. Without numerical values just using T0, tau, A as Amplitude, compute the coefficients as

Fourier Analysis Integral

$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j\omega_0 k t}\, dt$$

Henceforth, when we mention sums of complex exponentials, we assume that N can be either finite or infinite as needed. This is expressed in general by using what we call the ***Fourier synthesis summation***:[11]

Fourier Synthesis Summation

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/T_0)kt} \qquad (3.26)$$

C. If A=1, and f0=100 Hz, what are the first 5 frequencies in the Fourier Series?

10. Review our Fourier Series Lectures and examples on the Course Website.

GOOD LUCK