PROBLEM SESSION Aliasing

WEEK 5 P10 2.

$2\cos(2\pi 100t + \pi/3) + 2\cos(2\pi 600t + \pi/3)$

$F_s = 500$ samples/sec

$T_s = 1/500$ sec $= 0.0020 = 2$ ms

NOTE

$(2\cos 600t + \pi/3)$ is aliased

$$\hat{\omega} = \frac{2\pi 600}{500} = 0.8333 \cdot 2\pi$$

$$= 1.6667\pi$$

Need $-\pi \le \hat{\omega} \le \pi$

$$f_{aliased} = f_0 - f_s = 600 - 500 = +100 \text{ Hz}$$

NOTE:

$$x[n] = 2\cos(2\pi 600 n T_s)$$

$$= 2\cos(2\pi(100+500)nT_s)$$

$$= 2\cos(2\pi 100 n T_s + 2\pi 500 n T_s)$$ (f_s)

BUT $T_s = 1/f_s = 1/500$ SO

$$x[n] = 2\cos(2\pi 100 n T_s + 2\pi 500 n (1/500))$$

$$= 2\cos(2\pi 100 n T_s + 2\pi n)$$

So a 100 Hz wave

$f_s = 500$ $T_s = 0.002 = 2ms$

For the aliased cosine $T = 0.01 sec$

So in a period $N = \frac{0.01}{0.002} = 5$ samples

for a 100 Hz wave. ($T_{100} = 10ms$)

For 600 Hz, we would need

$f_{s_1} = 2 \times 600 Hz = 1200 Hz$

$T_s = .8333 ms$ between samples

✓ $T_{600} = 1.7ms$ so 2 samples/cycle

WEEKS PLOT

Fig 5.2a Both signals

5.2b samples spaced every 2ms

WEEKS PAGE 165
EXAMPLE 5.2

```
%
%  Example of aliasing with
%        2 cos(2 pi 100 t + pi/3)
%  and   2 cos(2 pi 600 t + pi/3)
%
%  From "DSP Using MATLAB and Wavelets", Michael Weeks, 2006
%
%  if these are sampled at 500 samples/sec,
%  then the sampled versions are identical!
%

freq = 100;     % example frequency
phase = pi/3;   % example phase
mag = 2;        % example magnitude
fs = 500;       % sampling frequency
Ts = 1/fs;      % sampling period
k = 1;          % number of repetitions

num_points = 200;    % How many points to use
                     %  200 makes it look smooth
num_samples = 11;    % How many samples to simulate reading
                     %  11 puts "sampled" under "analog"

step = 2/(freq*num_points);  % get a nice step size
t = 0:step:2*(1/freq);       % "time"
n = 0:num_samples-1;         % sample index

% x and y are simulated analog functions
x = mag*cos(2*pi*freq*t + phase);
y = mag*cos(2*pi*(freq+k*fs)*t + phase);

% x2 and y2 are simulated sampled version of x and y
x2(n+1) = mag*cos(2*pi*freq*n*Ts + phase);
y2(n+1) = mag*cos(2*pi*(freq+k*fs)*n*Ts + phase);

plot_simulated
```

T = 0.01 seconds

T = 1/600 Seconds

2 ms

$f_1 = 100$

$f_2 = 100 + 500$

x2 and y2 will have the same values due to Aliasing

```
%
% plot simulated analog / sampled signals
%
%  From "DSP Using MATLAB and Wavelets", Michael Weeks, 2006
%

% Plot the "analog" signals
subplot(2,1,1);
plot(t, x, 'r.-', t, y,'b-');
my_title = sprintf('Simulated analog signals, x=dots  y=solid');
title(my_title);
xlabel(' time ');
ylabel('Amplitude');

% Plot the "sampled" signals
subplot(2,1,2);
plot(n,x2,'rx', n,y2,'bo');
my_title = sprintf('Simulated digital signals, x=x  y=o');
title(my_title);
xlabel(' samples ');
ylabel('Amplitude');
```

Original signals

y2 - 600 Hz aliased to 100 Hz.

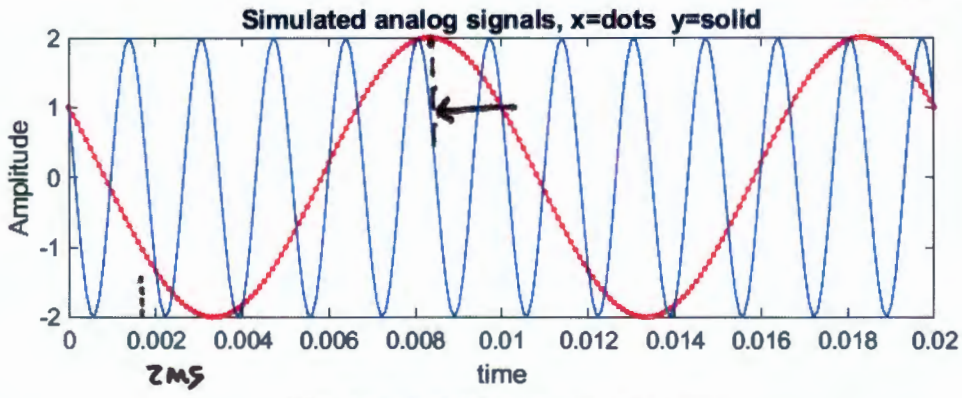
WEEKS EXAMPLE

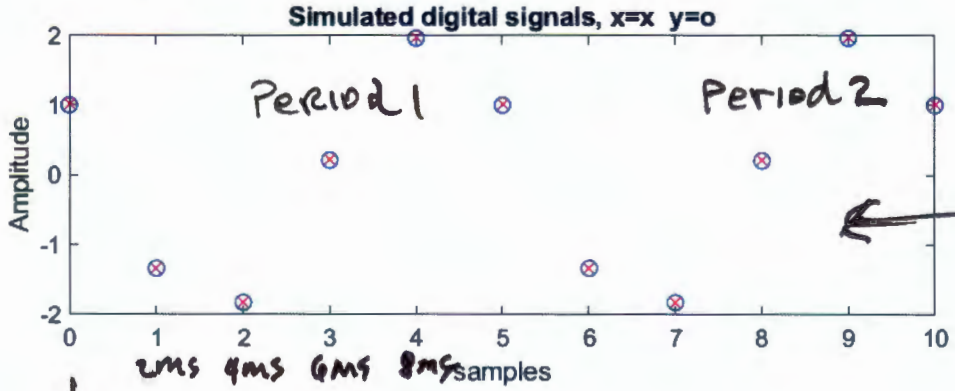
T_{100} = 0.01 Seconds

$T_{600} = 0.0017\ s$

$\Delta t_{600} = \frac{1}{6}(1.7)$

$\approx 0.28\ ms$





100, 600 Hz THE SAME POINTS

100 Hz WAVE OK!

5 x 2ms

10ms

f = 100 Hz

CONCLUSION. SAMPLING DETERMINES FREQUENCY IF ALIASED!