DSP First, 2/e

Modified TLH

Lecture Chapter 5

FIR Filtering Intro

READING ASSIGNMENTS

- This Lecture:
  - Chapter 5, Sects. 5-1, 5-2, 5-3 & 5-4 (partial)
- Other Reading:
  - Next Lecture: Ch. 5, Sects 5-4, 5-6, 5-7 & 5-8
    - CONVOLUTION

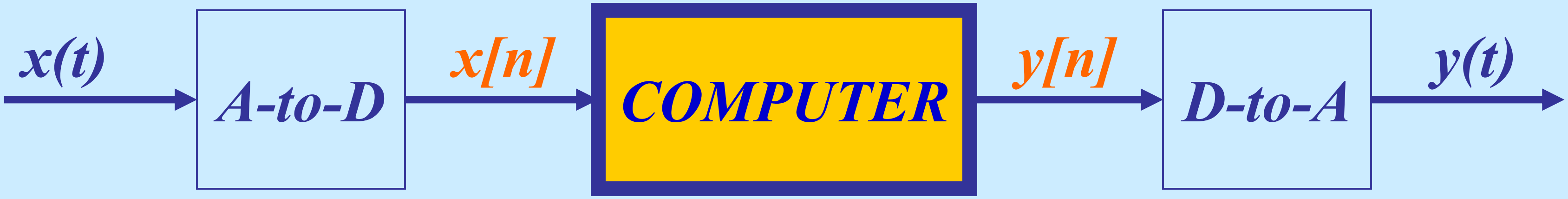
TOPICS

- RUNNING (MOVING) AVERAGE FILTER
- CAUSAL Filter   >= 0
- Finite Impulse Response Description
- Unit Impulse Signal and Filter Response
- Compare 3-point and 7-point Averge

LECTURE OBJECTIVES

- INTRODUCE FILTERING IDEA
  - Weighted Average
  - Running Average
- FINITE IMPULSE RESPONSE FILTERS
  - **FIR** Filters
  - Show how to **compute** the output y[n] from the input signal, x[n]

DIGITAL FILTERING



- Characterized SIGNALS (Fourier series)
- Converted to DIGITAL (sampling)
- Today: How to PROCESS them (DSP)?
- CONCENTRATE on the COMPUTER
  - ALGORITHMS, SOFTWARE (MATLAB) and HARDWARE (DSP chips, VLSI)

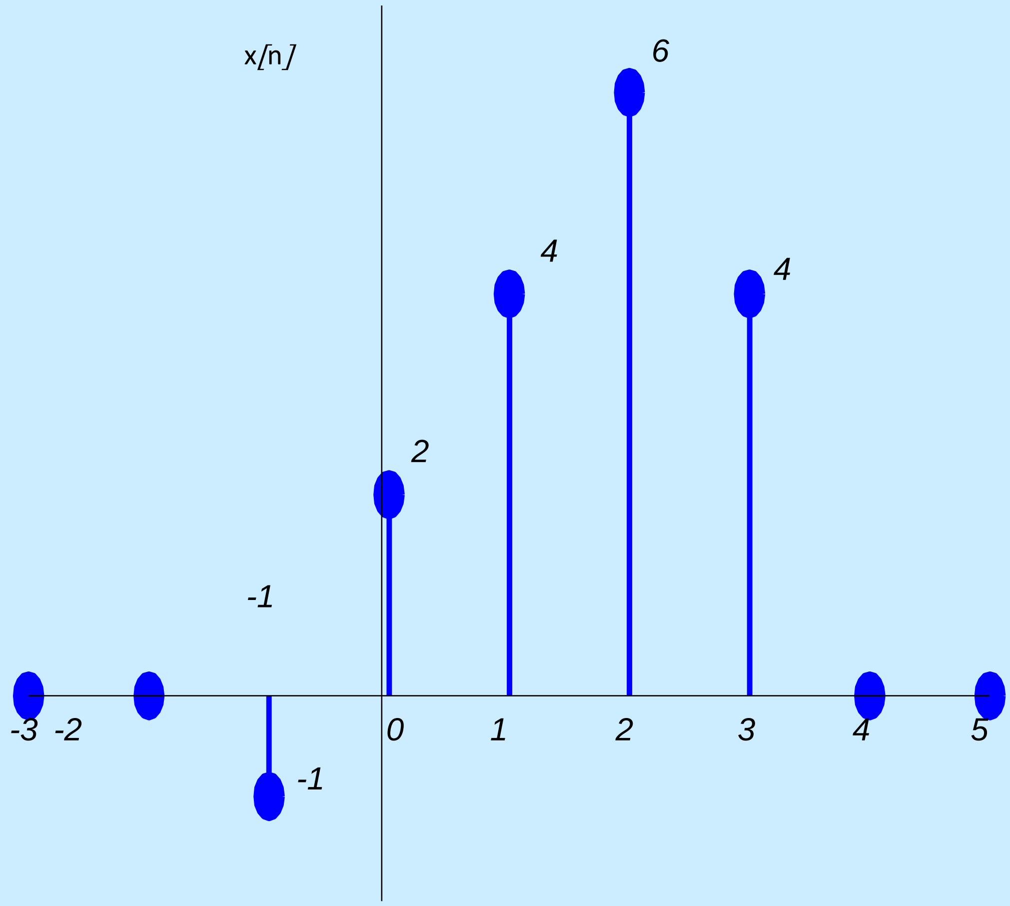
DISCRETE-TIME SYSTEM

- OPERATE on x[n] to get y[n]
- WANT a **GENERAL** CLASS of SYSTEMS
  - **ANALYZE** the SYSTEM
    - TOOLS: TIME-DOMAIN
      & FREQUENCY-DOMAIN
  - **SYNTHESIZE** the SYSTEM

D-T SYSTEM EXAMPLES

- EXAMPLES:
  - POINTWISE OPERATORS
    - SQUARING: $y[n] = (x[n])^2$
  - RUNNING AVERAGE
    - RULE: "the output at time n is the average of three consecutive input values"

Consider the points



The Running (Moving) Average Filter

- A three-sample *causal* moving average filter is a special case of (5.1)

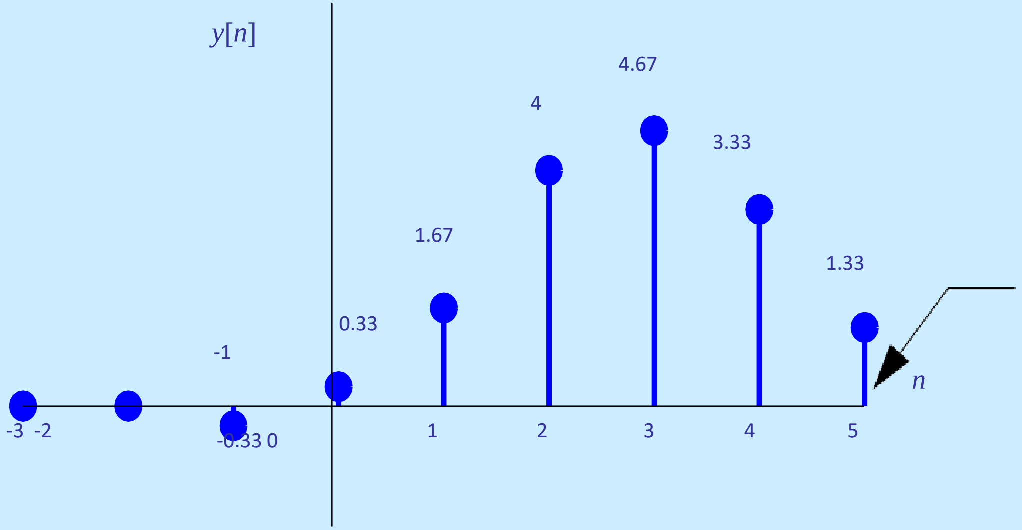
$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2]), \qquad (5.4)$$

which uses no future input values to compute the present output

From ECE 2601 Chapter 5
Causal is From The Past

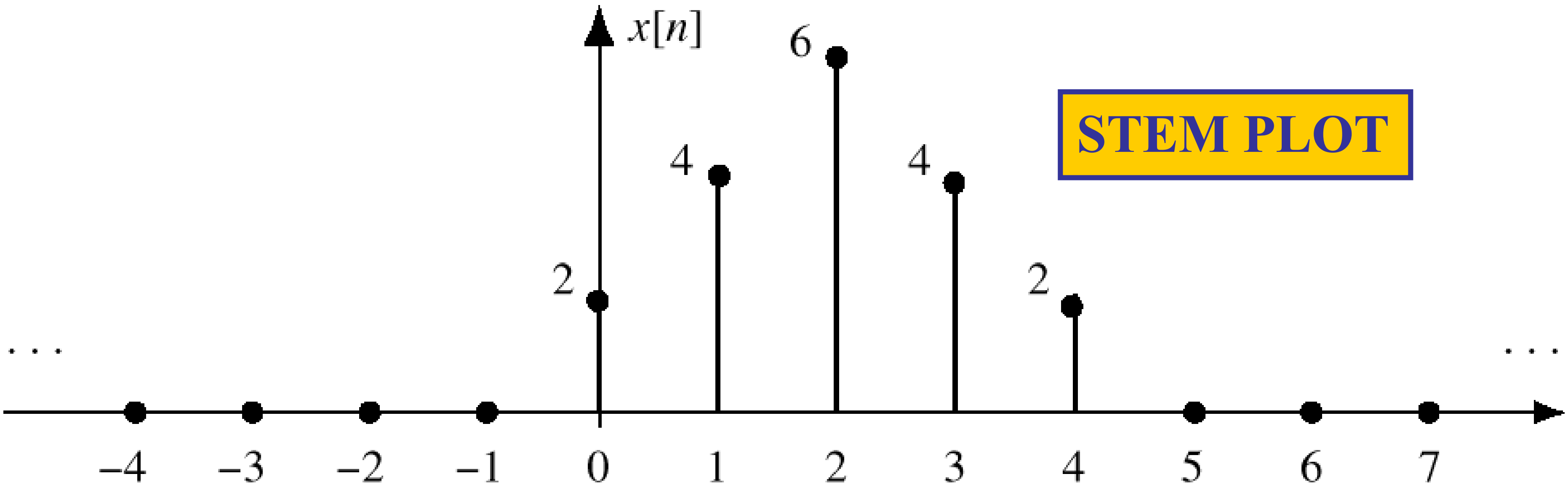
```
>> n= -3:5;
>> x = [0 0 -1 2 4 6 4 0 0]
>> % We will learn about the filter function later
>> y = filter(1/3*[1 1 1],1,x);
>> stem(n,y,'filled')
```

- The action of the moving average filter has resulted in the output being *smoother* than the input
- Since only past and present values of the input are being used to calculate the present output, this filtering operation can operate in *real-time*



DISCRETE-TIME SIGNAL

- x[n] is a LIST of NUMBERS
  - INDEXED by “n”



3-PT AVERAGE SYSTEM

- ADD 3 CONSECUTIVE NUMBERS
  - Do this for each "n"

the following input–output equation

Make a TABLE

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	$n > 5$
$x[n]$	0	0	0	2	4	**6**	**4**	**2**	0	0
$y[n]$	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	**4**	2	$\frac{2}{3}$	0	0

***n*=0** $y[0] = \tfrac{1}{3}(x[0] + x[1] + x[2])$

***n*=1** $y[1] = \tfrac{1}{3}(x[1] + x[2] + x[3])$

INPUT SIGNAL

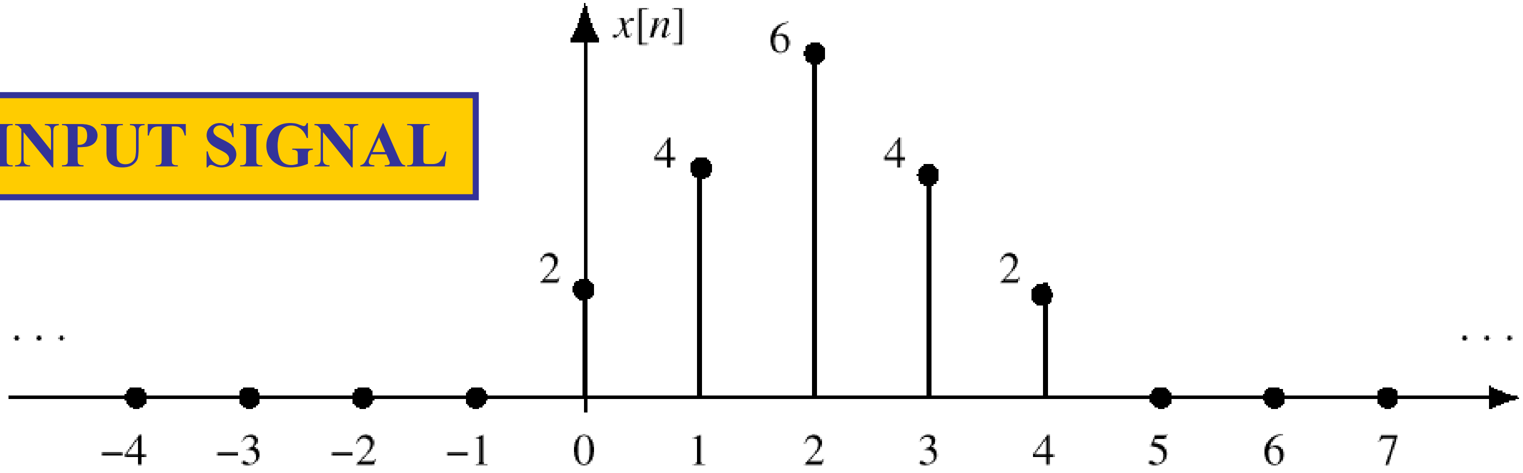


Figure 5.2 Finite-length input signal, $x[n]$.

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

OUTPUT SIGNAL

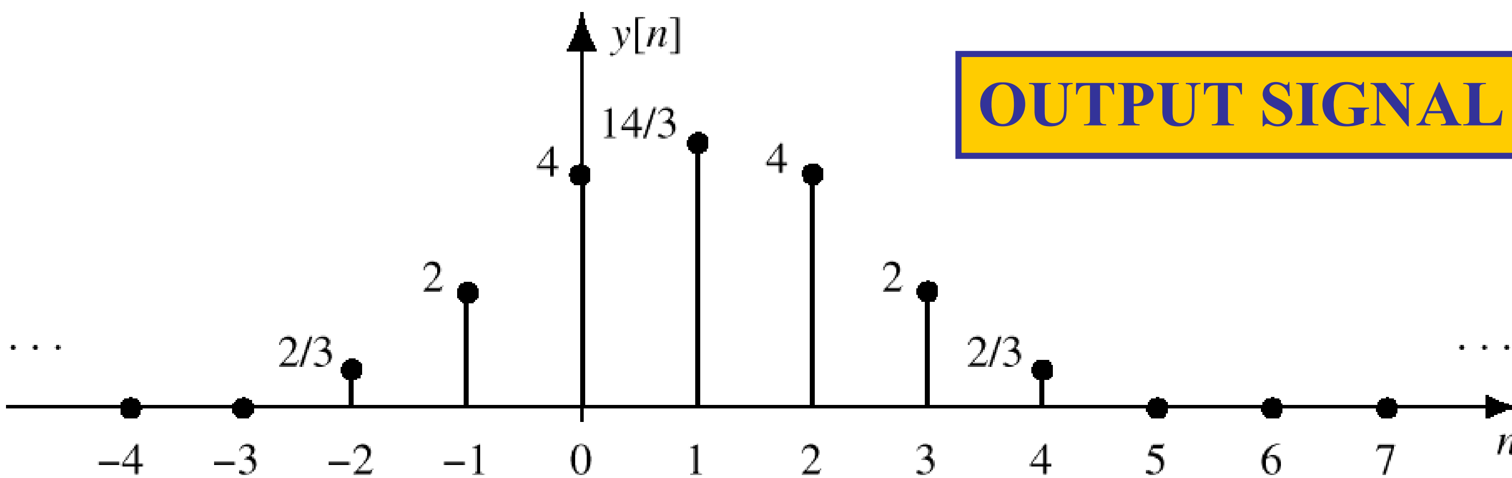


Figure 5.3 Output of running average, $y[n]$.

PAST, PRESENT, FUTURE

- SLIDE a WINDOW across x[n]

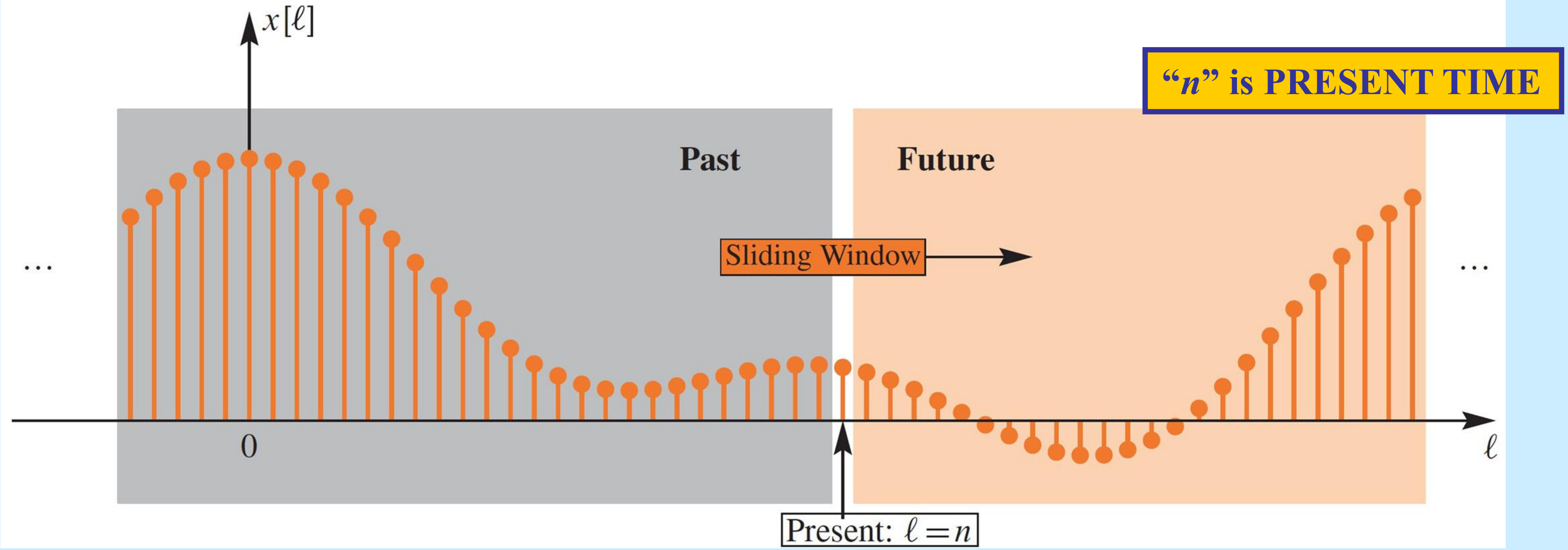


Figure 5-3 Filter calculation at the present time ($\ell = n$) uses values within a sliding window. Gray shading indicates the past ($\ell < n$); orange shading, the future ($\ell > n$). Here, the sliding window encompasses values from both the future and the past.

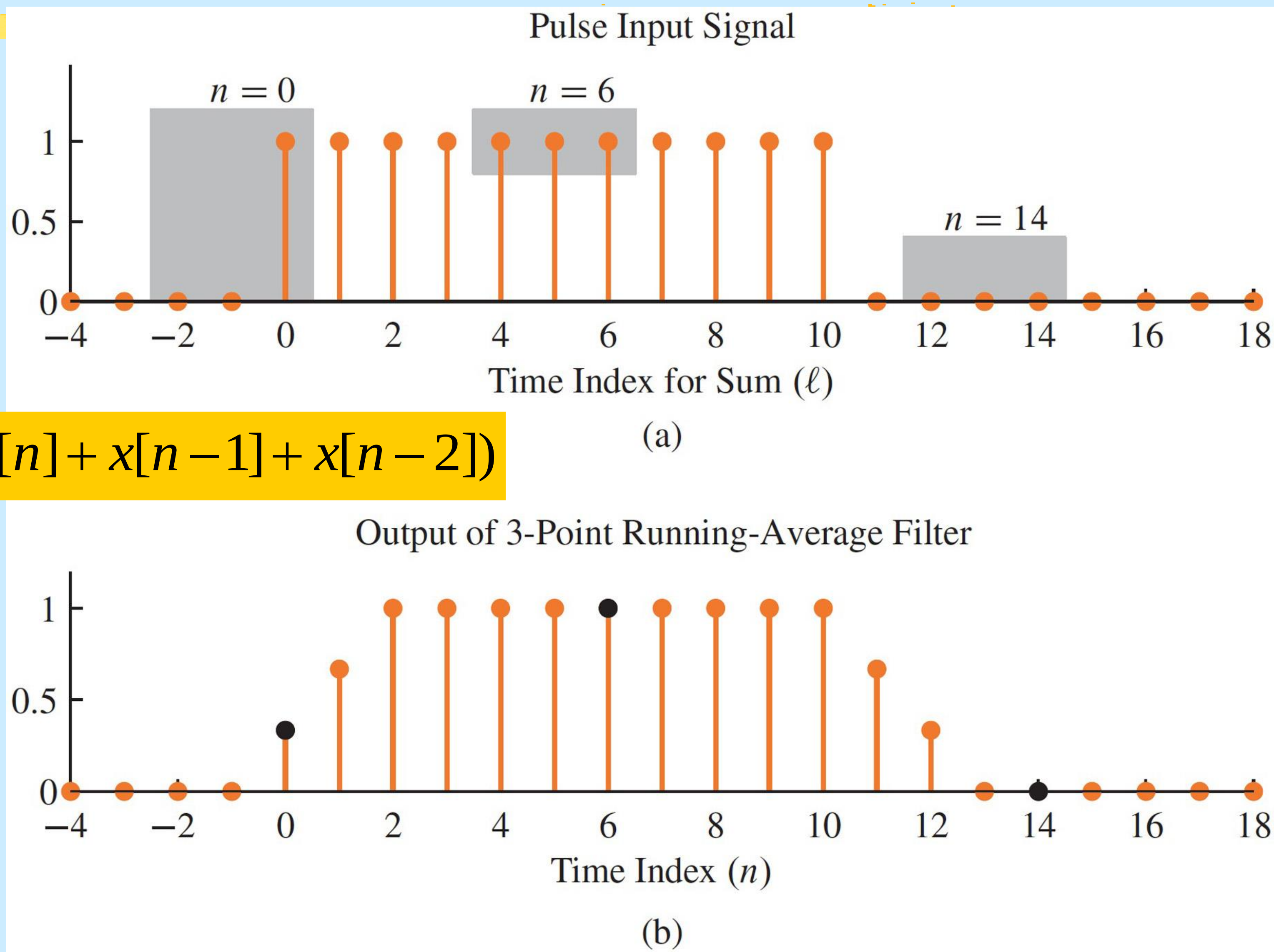
ANOTHER 3-pt AVERAGER

- Uses "PAST" VALUES of x[n]
  - IMPORTANT IF "n" represents REAL TIME
    - WHEN x[n] & y[n] ARE STREAMS

$$y[n] = \tfrac{1}{3}(x[n] + x[n-1] + x[n-2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	6	7	$n > 7$
$x[n]$	0	0	0	2	4	**6**	**4**	**2**	0	0	0	0
$y[n]$	0	0	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	**4**	2	$\frac{2}{3}$	0	0

CAUSAL 3-pt AVERAGER



$$y[n] = \tfrac{1}{3}(x[n] + x[n-1] + x[n-2])$$

Finite Impulse Response

- Each output value y[n] is the some of a FINITE number of weighted values of the input sequence x[n]

- The FIR filter can be represented in various ways:
- By a difference Equation Page 150
- By the Impulse Response Page 158
- By the Convolution Sum Page 162

GENERAL CAUSAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

NOTE: Index k = 0, 1,2,…

  - For example, $\boxed{b_k = \{3, -1, 2, 1\}}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

DIFFERENCE EQUATION

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

GENERAL CAUSAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

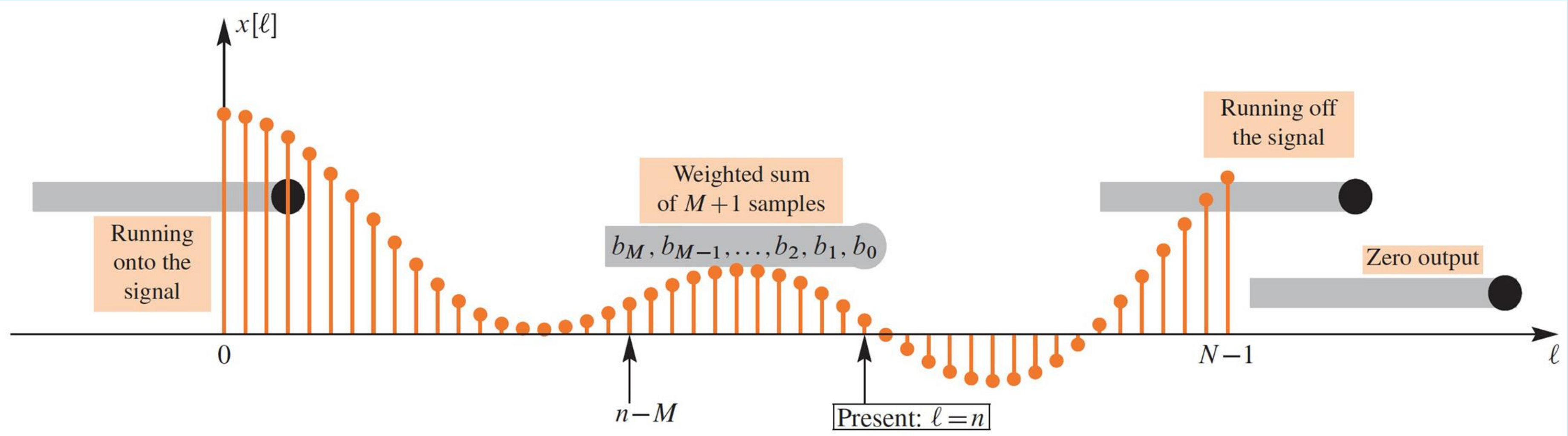
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- FILTER **ORDER** is M
- FILTER **"LENGTH"** is L = M+1
  - NUMBER of FILTER COEFFS is L

GENERAL CAUSAL FIR FILTER

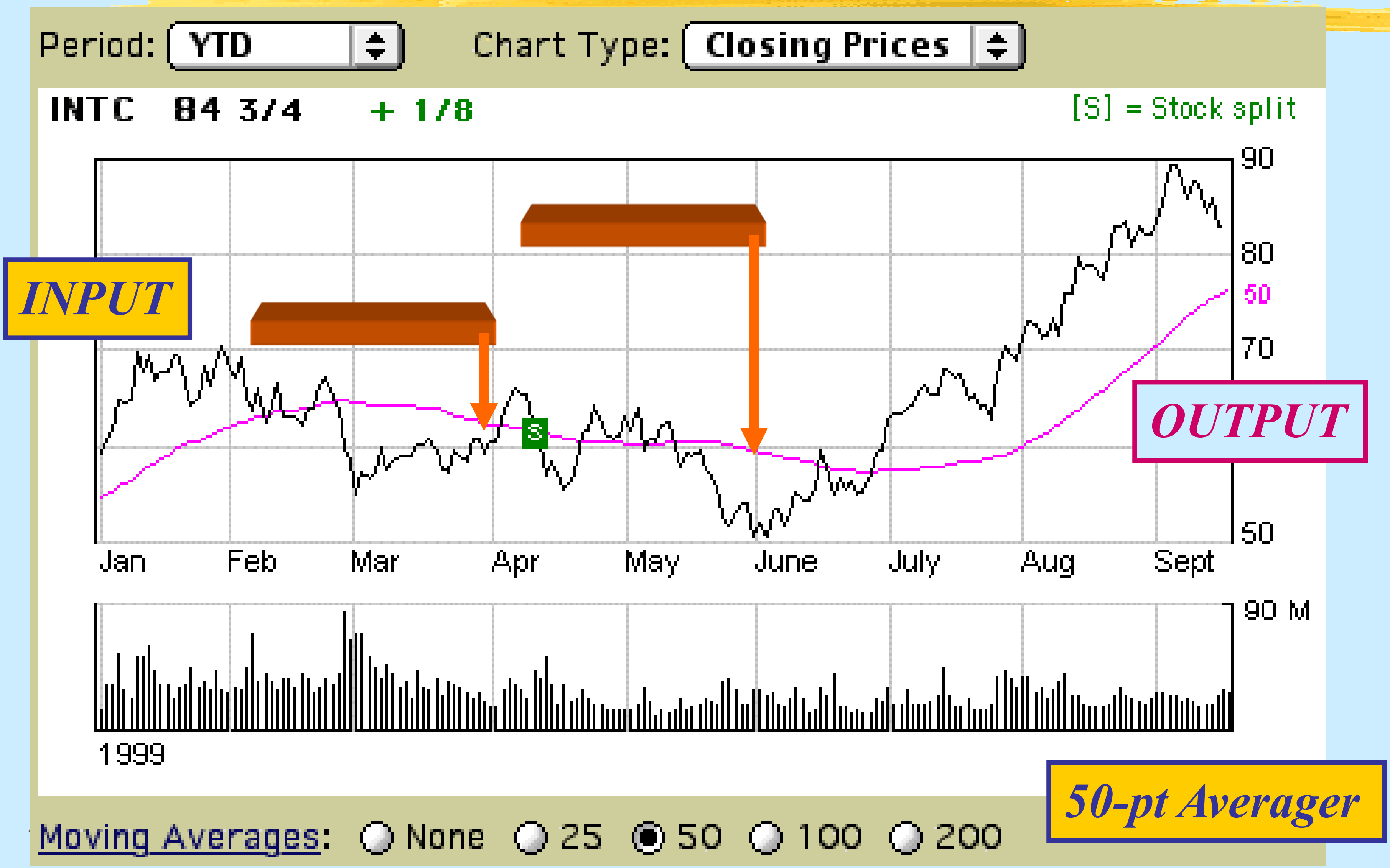
- SLIDE a WINDOW across x[n]

$$y[n]=\sum_{k=0}^{M} b_k x[n-k] = b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]$$



$x[n\text{-}M]$ $x[n]$

FILTERED STOCK SIGNAL



SPECIAL INPUT SIGNALS

- x[n] = SINUSOID

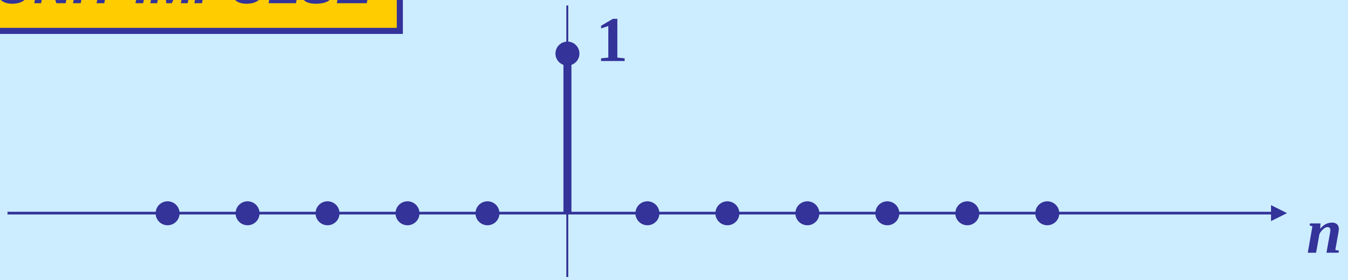
FREQUENCY RESPONSE (LATER)

- x[n] has only one NON-ZERO VALUE

Test Signal

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



UNIT IMPULSE SIGNAL δ[n]

n	...	−2	−1	0	1	2	3	4	5	6	...
$\delta[n]$	0	0	0	1	0	0	0	0	0	0	0
$\delta[n-3]$	0	0	0	0	0	0	1	0	0	0	0

$n = 3$

δ[n] is NON-ZERO
When its argument
is equal to ZERO

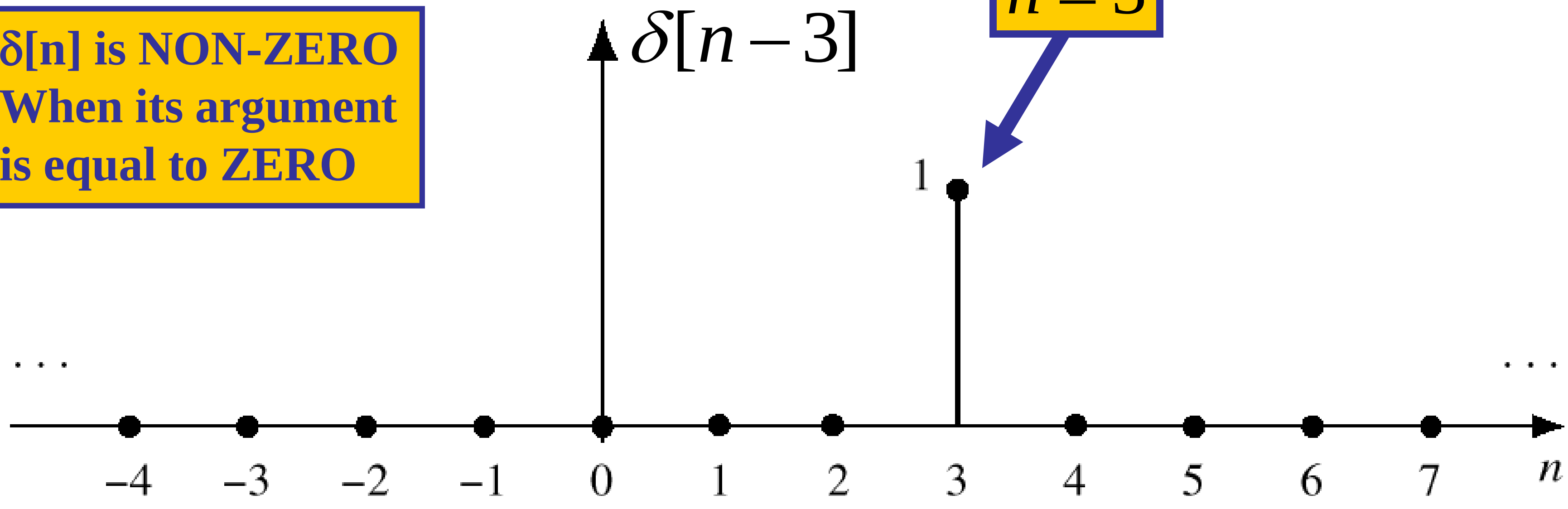
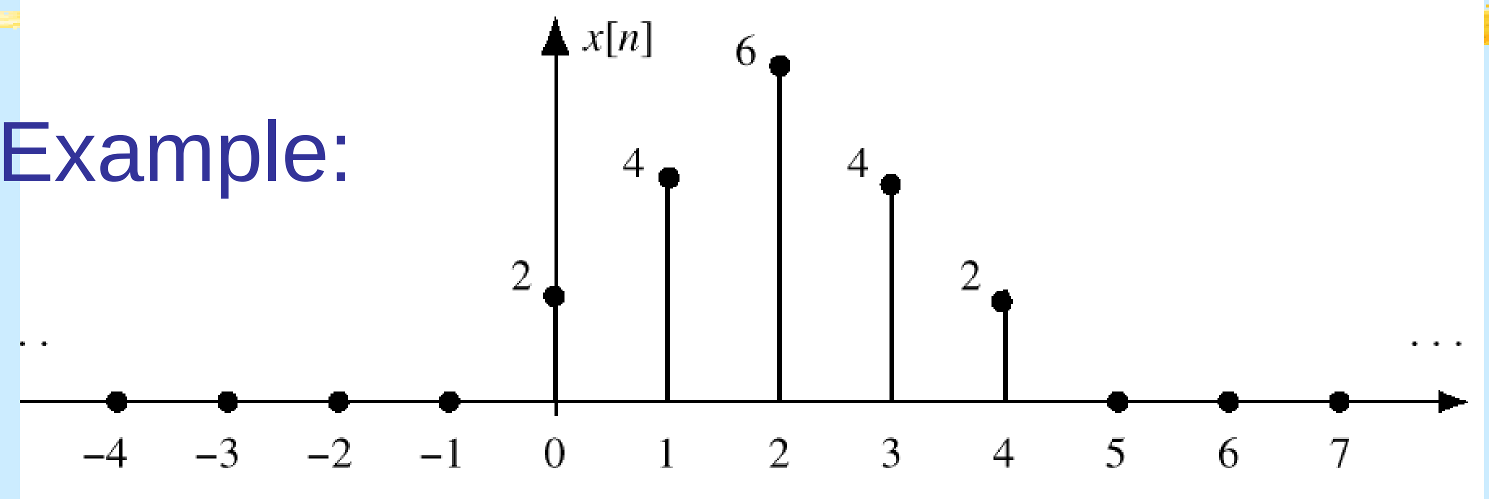


Figure 5.7 Shifted impulse sequence, $\delta[n-3]$.

Sequence Representation

Example:



$$x[n=0]=x[0]=2 \qquad x[n=1]=x[1]=4$$

$$x[n=2]=x[2]=6 \qquad x[n=3]=x[3]=4$$

$$x[n]=\cdots+\ \mathbf{0}\ \delta[n+1]+\mathbf{2}\ \delta[n]+\mathbf{4}\ \delta[n-1]$$

$$+\ \mathbf{6}\ \delta[n-2]+\mathbf{4}\ \delta[n-3]+\cdots$$

UNIT IMPULSE RESPONSE

- FIR filter description usually given in terms of coefficients b_k

$$\boxed{y[n] = \sum_{k=0}^{M} b_k x[n-k]}$$

- Can we describe the filter using a **SIGNAL** instead?
- What happens if input is a unit impulse?

Example: 4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES

$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

- INPUT = UNIT IMPULSE SIGNAL = δ[n]

$$x[n] = \delta[n]$$

$$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$

- OUTPUT is called "**IMPULSE RESPONSE**"
  - Denoted h[n]=y[n] when x[n]=δ[n]

Unit Impulse Response

$$y[n] = \tfrac{1}{4}x[n] + \tfrac{1}{4}x[n-1] + \tfrac{1}{4}x[n-2] + \tfrac{1}{4}x[n-3]$$

n	-3	-2	-1	0	1	2	3	4	5
$x[n]$	0	0	0	1	0	0	0	0	0
$y[n]$	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0

$$x[n] = \delta[n]$$

$$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$

$$= h[n]$$

SUM of Shifted Impulses

$$h[n]=\tfrac{1}{4}\delta[n]+\tfrac{1}{4}\delta[n-1]+\tfrac{1}{4}\delta[n-2]+\tfrac{1}{4}\delta[n-3]+0\delta[n-4]$$

n	-1	0	1	2	3	4	5	6	7
$h[n]$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0	0
$h[0]\delta[n]$	0	$\frac{1}{4}$	0	0	0	0	0	0	0
$h[1]\delta[n-1]$	0	0	$\frac{1}{4}$	0	0	0	0	0	0
$h[2]\delta[n-2]$	0	0	0	$\frac{1}{4}$	0	0	0	0	0
$h[3]\delta[n-3]$	0	0	0	0	$\frac{1}{4}$	0	0	0	0
$h[4]\delta[n-4]$	0	0	0	0	0	0	0	0	0

FIR IMPULSE RESPONSE

$$y[n]=\sum_{k=0}^{M} b_k x[n-k]$$

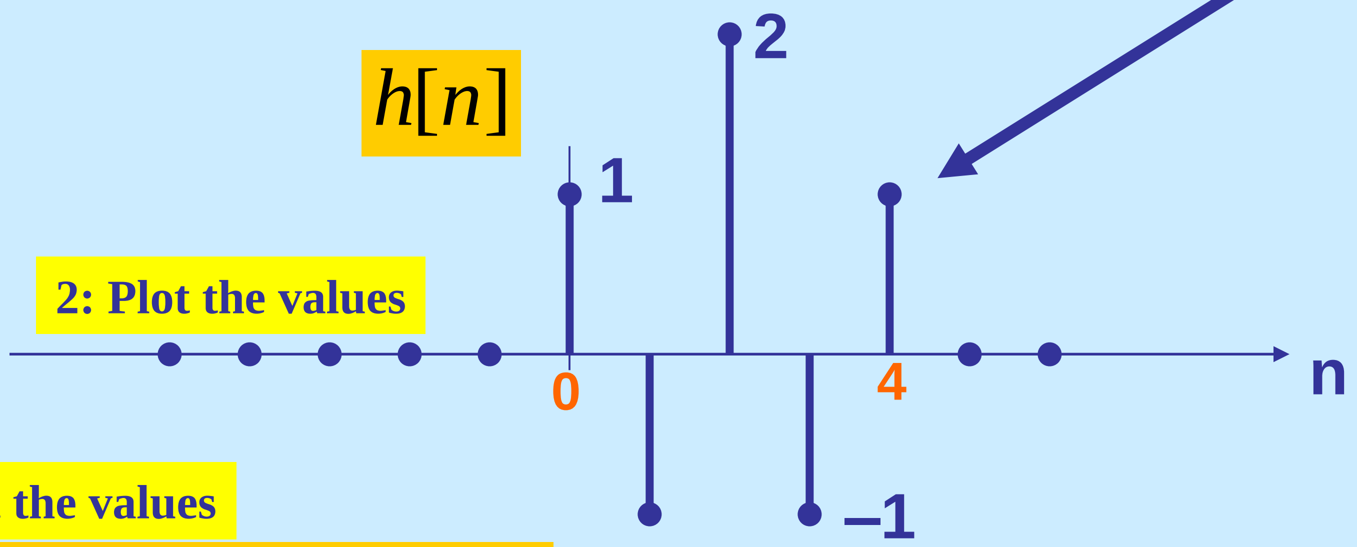
$$h[n]=\sum_{k=0}^{M} b_k \delta[n-k]$$

n	$n<0$	0	1	2	3	$\ldots$	M	$M+1$	$n>M+1$
$x[n]=\delta[n]$	0	1	0	0	0	0	0	0	0
$y[n]=h[n]$	0	b_0	b_1	b_2	b_3	$\ldots$	b_M	0	0

3 Ways to Represent the FIR filter

1 Use SHIFTED IMPULSES to write $h[n]$

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$



2: Plot the values

3: List the values

$$b_k = \{\,1, -1, 2, -1, 1\,\}$$

True for any signal, x[n]

FILTERING EXAMPLE

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

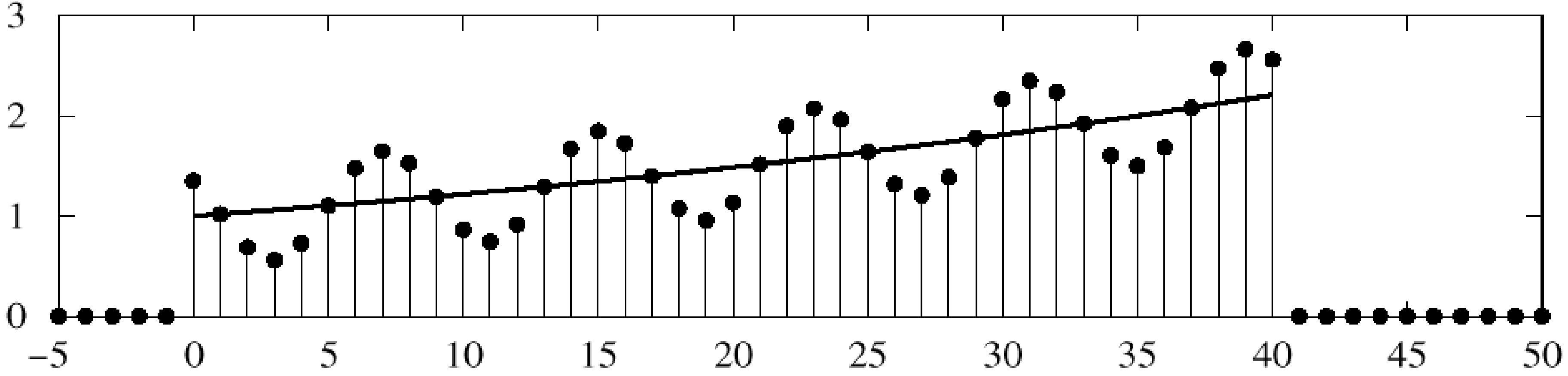
$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER
  - Changes A slightly

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

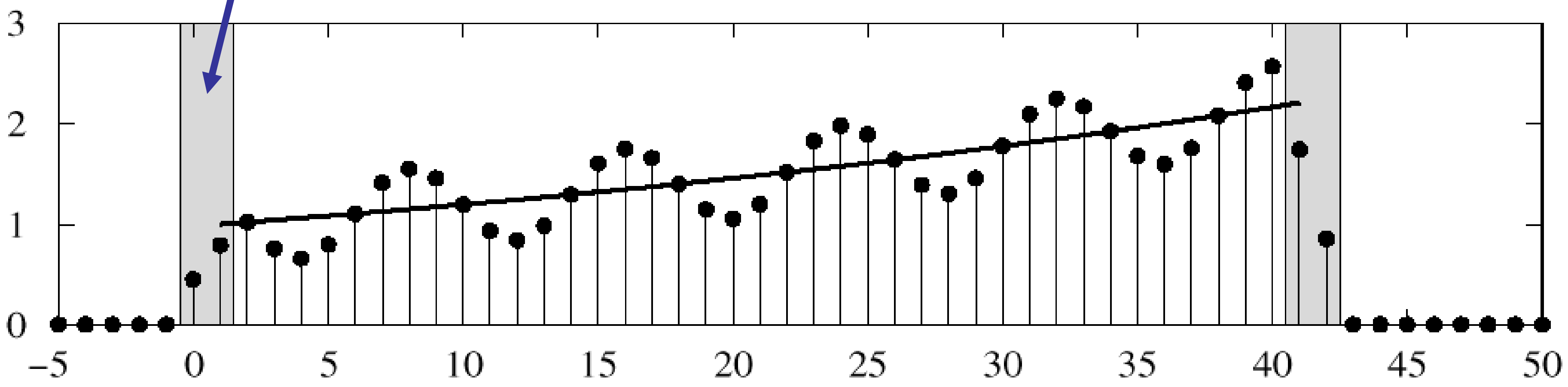
3-pt AVG EXAMPLE

$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$



USE PAST VALUES

Output of 3-Point Running-Average Filter



7-pt FIR EXAMPLE (AVG)

$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$