DSP First, 2/e

Modified TLH

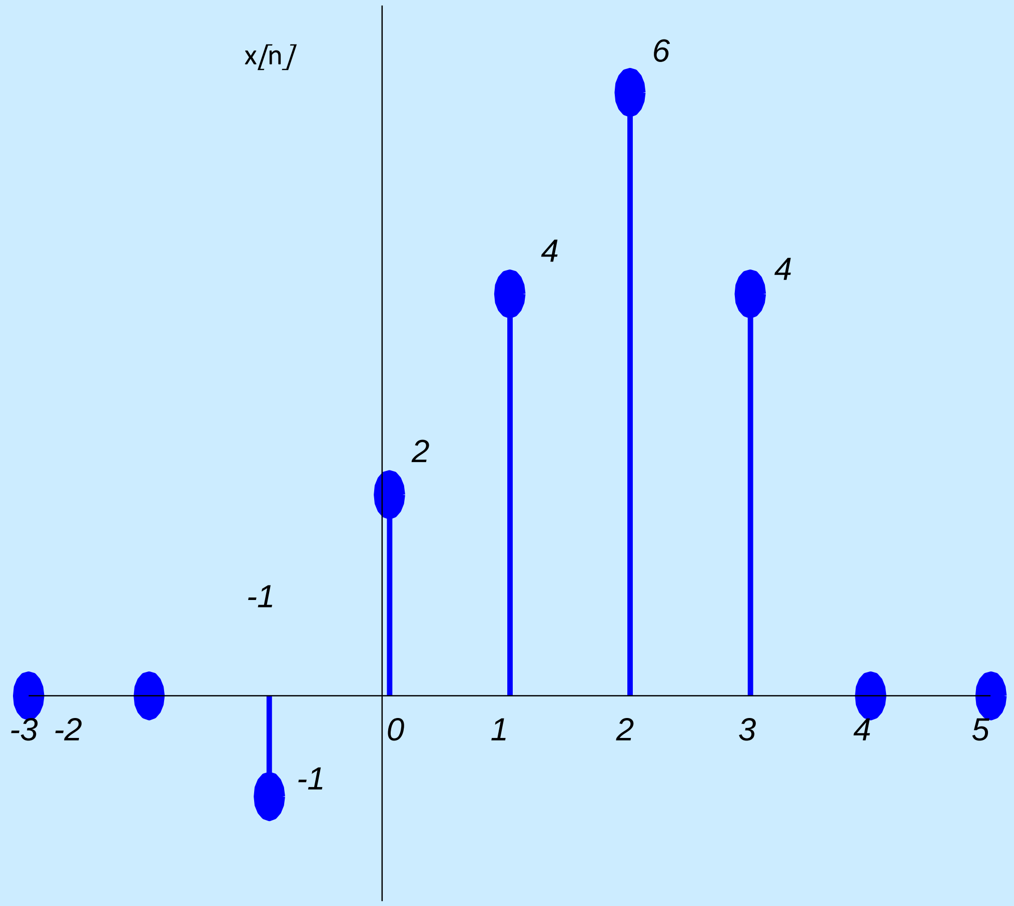
Lecture Chapter 5

FIR Filtering Intro

TOPICS

- RUNNING (MOVING) AVERAGE FILTER
- CAUSAL Filter >= 0
- Finite Impulse Response Description
- Unit Impulse Signal and Filter Response
- Compare 3-point and 7-point Averge

Consider the points



The Running (Moving) Average Filter

- A three-sample *causal* moving average filter is a special case of (5.1)

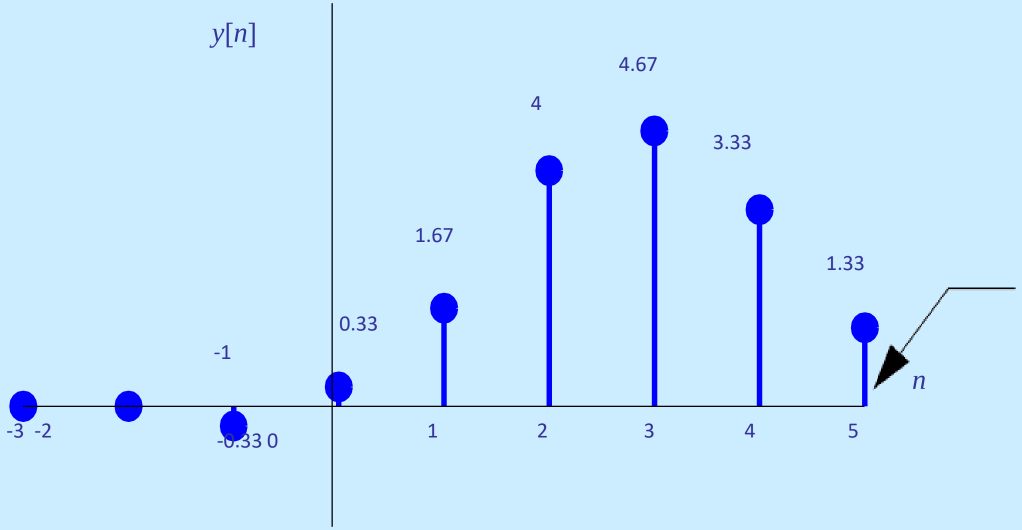
$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2]), \qquad (5.4)$$

which uses no future input values to compute the present output

From ECE 2601 Chapter 5
Causal is From The Past

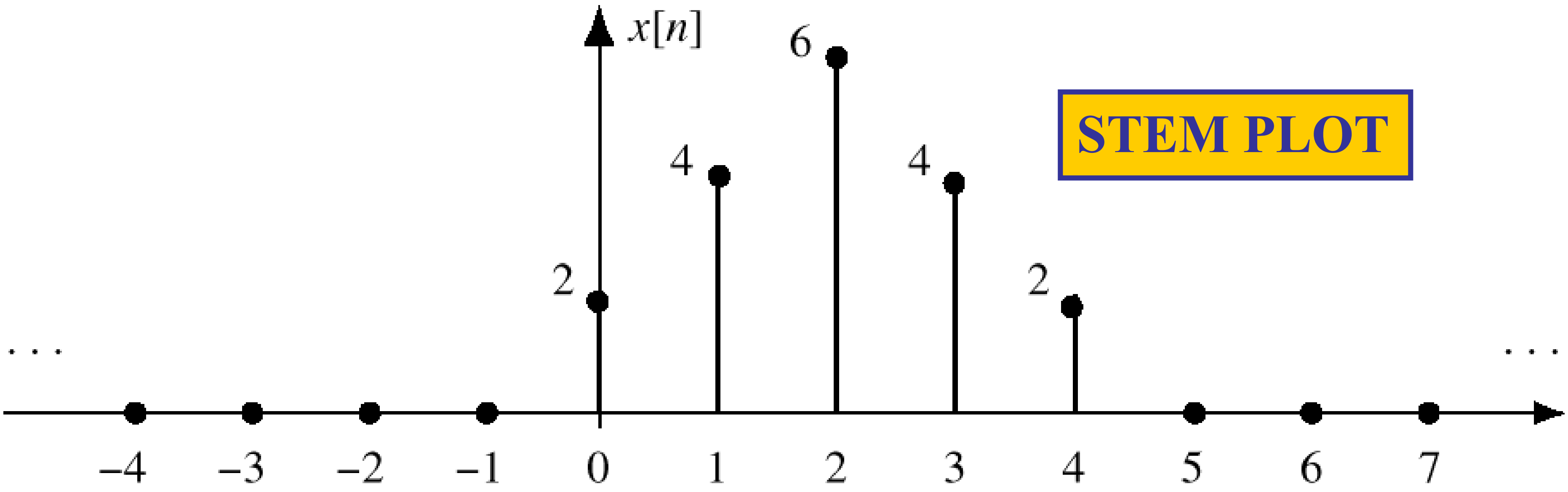
```
>> n= -3:5;
>> x = [0 0 -1 2 4 6 4 0 0]
>> % We will learn about the filter function later
>> y = filter(1/3*[1 1 1],1,x);
>> stem(n,y,'filled')
```

- The action of the moving average filter has resulted in the output being *smoother* than the input
- Since only past and present values of the input are being used to calculate the present output, this filtering operation can operate in *real-time*



DISCRETE-TIME SIGNAL

- x[n] is a LIST of NUMBERS
  - INDEXED by “n”



GENERAL CAUSAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

NOTE: Index k = 0, 1,2,…

  - For example, $b_k = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

DIFFERENCE EQUATION

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

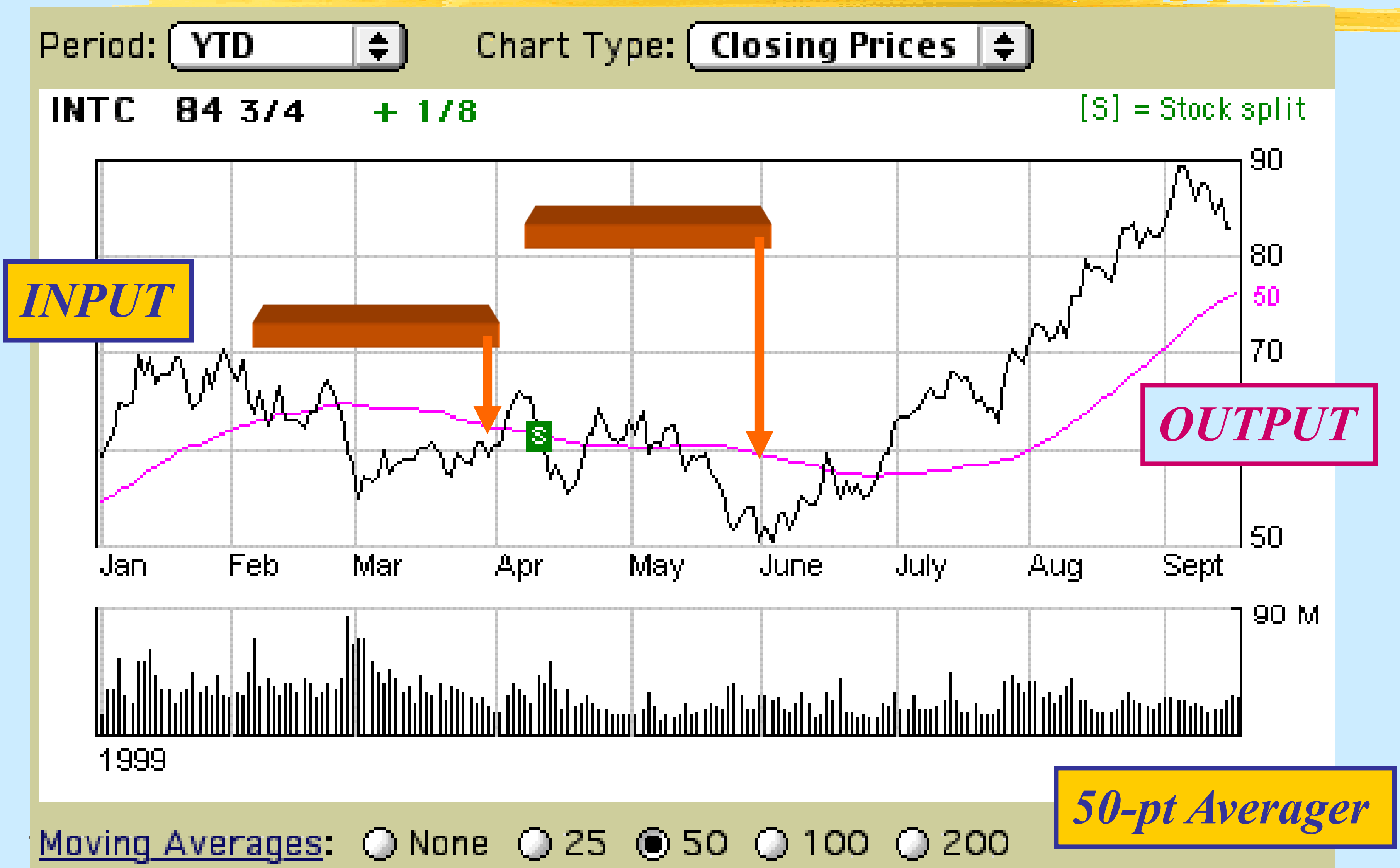
GENERAL CAUSAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- FILTER **ORDER** is M
- FILTER **"LENGTH"** is L = M+1
  - NUMBER of FILTER COEFFS is L

FILTERED STOCK SIGNAL



SPECIAL INPUT SIGNALS

- x[n] = SINUSOID

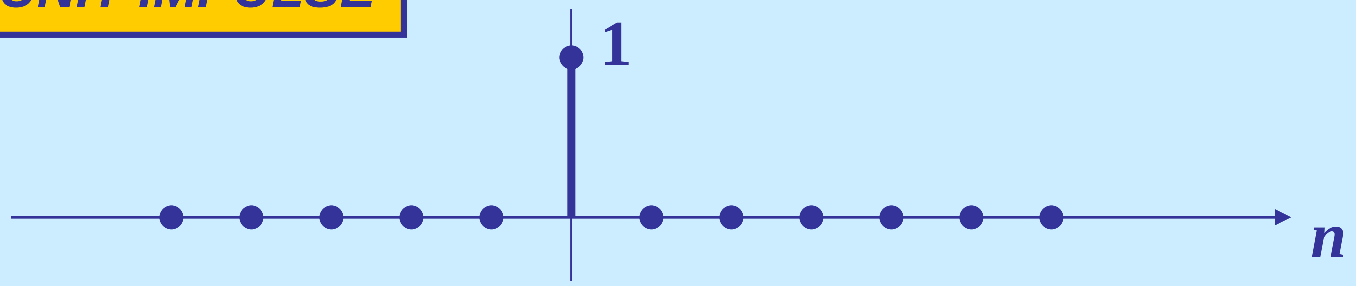
FREQUENCY RESPONSE (LATER)

- x[n] has only one NON-ZERO VALUE

Test Signal

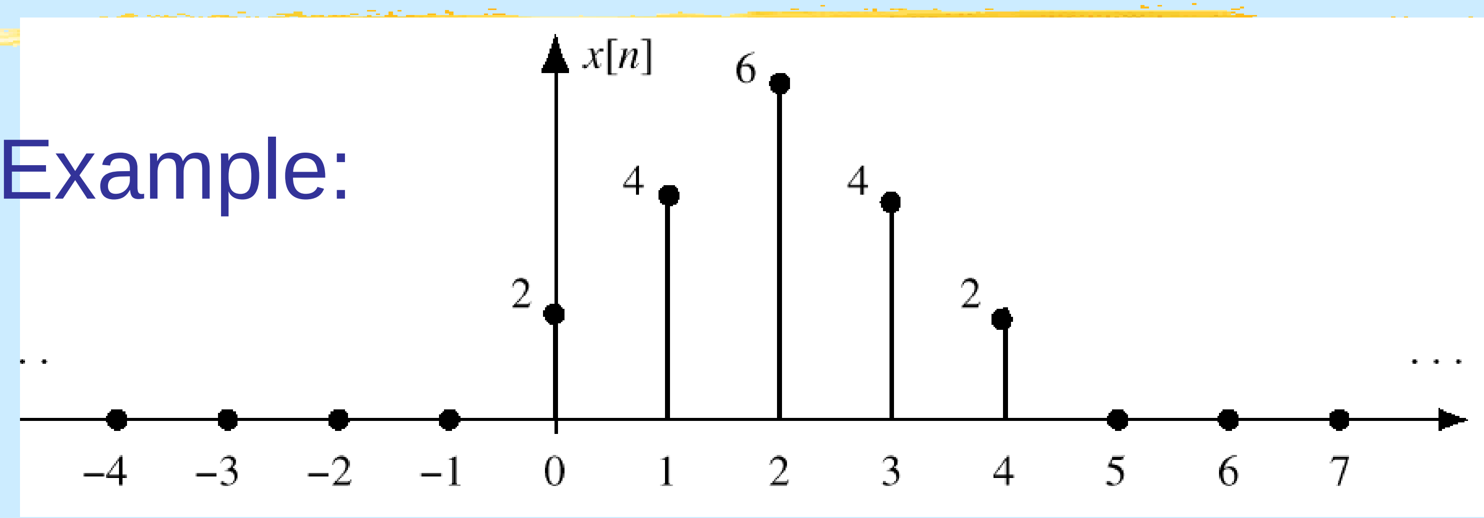
$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



Sequence Representation

Example:



$$x[n=0]=x[0]=2 \qquad x[n=1]=x[1]=4$$

$$x[n=2]=x[2]=6 \qquad x[n=3]=x[3]=4$$

$$x[n]=\cdots+\ \mathbf{0}\ \delta[n+1]+\ \mathbf{2}\ \delta[n]+\ \mathbf{4}\ \delta[n-1]$$

$$+\ \mathbf{6}\ \delta[n-2]+\mathbf{4}\ \delta[n-3]+\cdots$$

UNIT IMPULSE RESPONSE

- FIR filter description usually given in terms of coefficients b_k

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- Can we describe the filter using a **SIGNAL** instead?
- What happens if input is a unit impulse?

Example: 4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES

$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

- INPUT = UNIT IMPULSE SIGNAL = $\delta[n]$

$$x[n] = \delta[n]$$

$$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$

- OUTPUT is called **"IMPULSE RESPONSE"**
  - Denoted $h[n]=y[n]$ when $x[n]=\delta[n]$

FIR IMPULSE RESPONSE

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$h[n] = \sum_{k=0}^{M} b_k \delta[n-k]$$

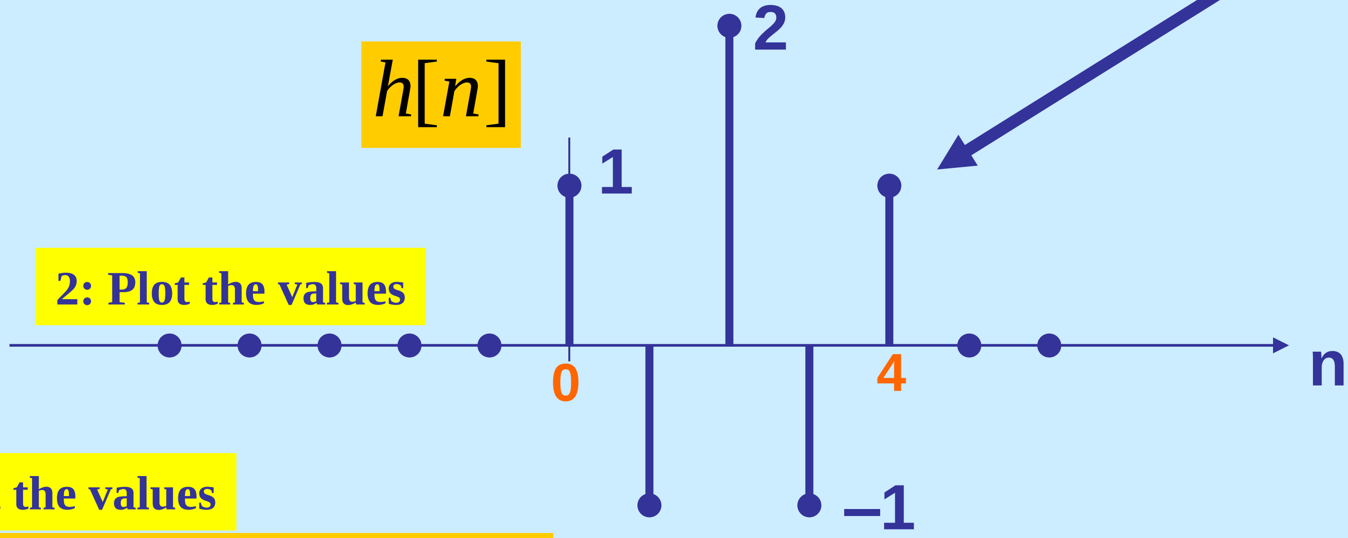
n	$n < 0$	0	1	2	3	$\ldots$	M	$M+1$	$n > M+1$
$x[n] = \delta[n]$	0	1	0	0	0	0	0	0	0
$y[n] = h[n]$	0	b_0	b_1	b_2	b_3	$\ldots$	b_M	0	0

3 Ways to Represent the FIR filter

1 Use SHIFTED IMPULSES to write $h[n]$

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$

2: Plot the values



3: List the values

$$b_k = \{\,1, -1, 2, -1, 1\,\}$$

True for any signal, x[n]

FILTERING EXAMPLE

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

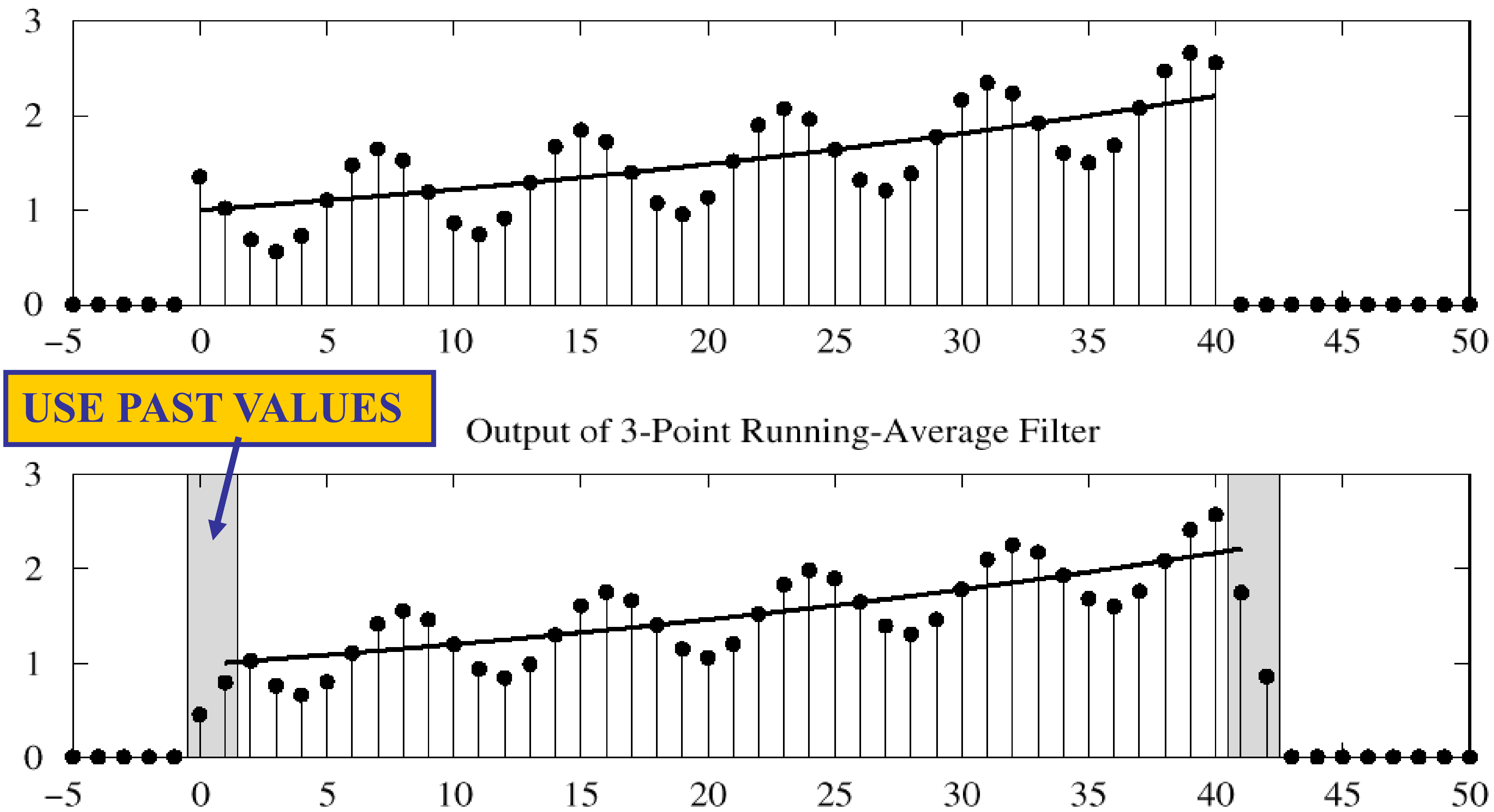
$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER
  - Changes A slightly

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

3-pt AVG EXAMPLE

$$\text{Input} : x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$



7-pt FIR EXAMPLE (AVG)

$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$