DSP First, 2/e

MODIFIED TLH

Lecture 12
Linearity & Time-Invariance
Convolution

OVERVIEW

- IMPULSE RESPONSE, $h[n]$
  - FIR case: same as $\{b_k\}$
- CONVOLUTION
  - GENERAL: $y[n] = h[n] * x[n]$
  - GENERAL CLASS of SYSTEMS
  - LINEAR and TIME-INVARIANT
- ALL **LTI** systems have $h[n]$ & use convolution

DIGITAL FILTERING

- CONCENTRATE on the FILTER (DSP)
- DISCRETE-TIME SIGNALS
  - FUNCTIONS of n, the “time index”
  - INPUT $x[n]$
  - OUTPUT $y[n]$

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

  - For example, $\boxed{b_k = \{3, -1, 2, 1\}}$

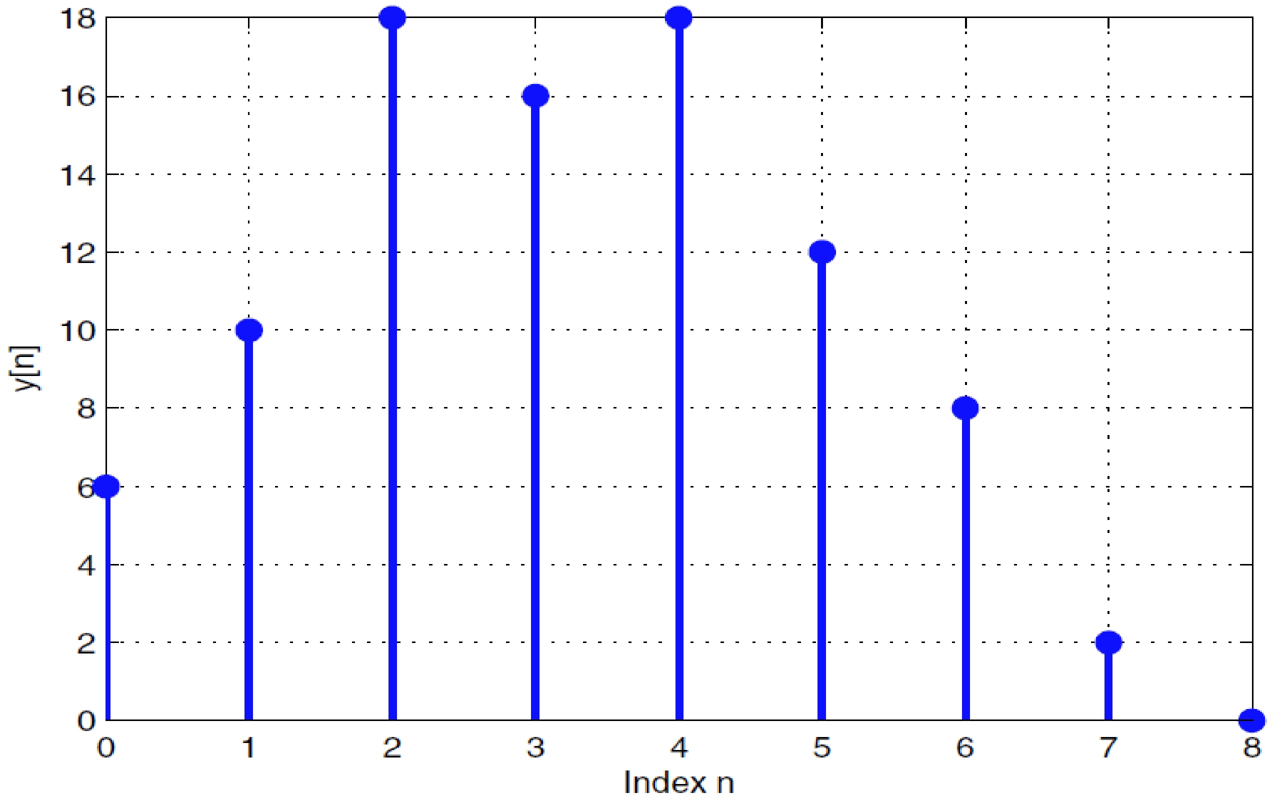
$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

- We can check the answers using MATLAB's filter function

```
>> n = 0:8;
>> x = [2 4 6 4 2 0 0 0 0];
>> h = [3 -1 2 1];
>> y = filter(h,1,x);
>> y

y = 6    10    18    16    18    12    8    2    0
```



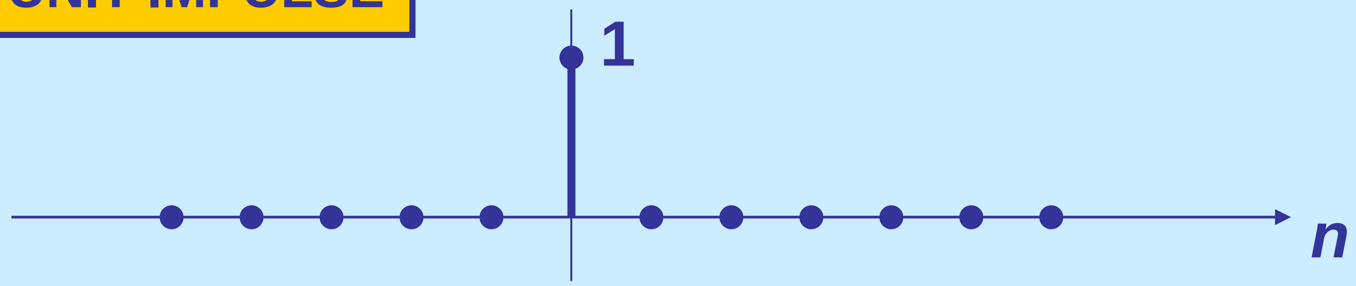
SPECIAL INPUT SIGNALS

- x[n] = SINUSOID
- x[n] has only one NON-ZERO VALUE

Later, sinusoid leads to the FREQUENCY RESPONSE

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



UNIT IMPULSE RESPONSE

- FIR filter <u>DIFFERENCE EQUATION</u> is specified by the filter coefficients b_k

$$\boxed{y[n] = \sum_{k=0}^{M} b_k x[n-k]}$$

- <u>EQUIVALENCE</u>: can we describe the filter using a SIGNAL instead?

FIR IMPULSE RESPONSE

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$h[n] = \sum_{k=0}^{M} b_k \delta[n-k]$$

n	$n < 0$	0	1	2	3	$\ldots$	M	$M+1$	$n > M+1$
$x[n] = \delta[n]$	0	1	0	0	0	0	0	0	0
$y[n] = h[n]$	0	b_0	b_1	b_2	b_3	$\ldots$	b_M	0	0

- Impulse response $h[k]=b_k$ is, in fact, a SIGNAL description of filter coefficients
- Allows us to write CONVOLUTION sum

LTI: Convolution Sum

- **Output = Convolution of x[n] & h[n]**
  - NOTATION: $y[n] = h[n] * x[n]$
  - FIR case:

FINITE LIMITS

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

Same as **b_k**

FINITE LIMITS

$$y[n] = h[n] * x[n]$$

LTI: Convolution Sum

- Delay the signal x[n] & then multiply by filter coefficients that come from h[n]

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

$$= h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + \ldots$$

FIR STRUCTURE

- Direct Form

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

SIGNAL FLOW GRAPH

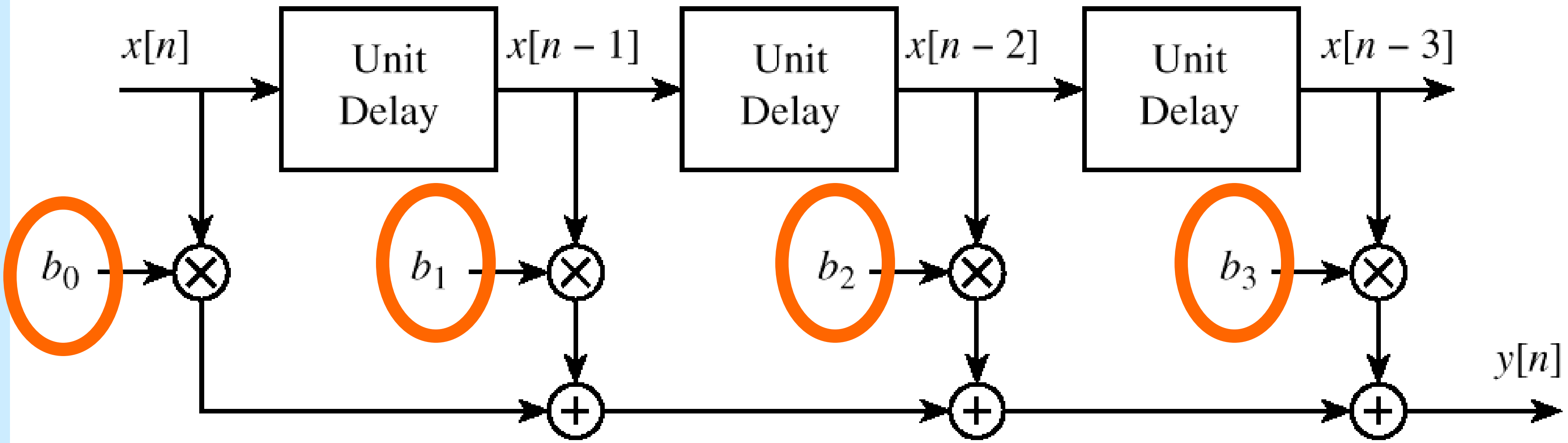


Figure 5.13 Block-diagram structure for the Mth order FIR filter.

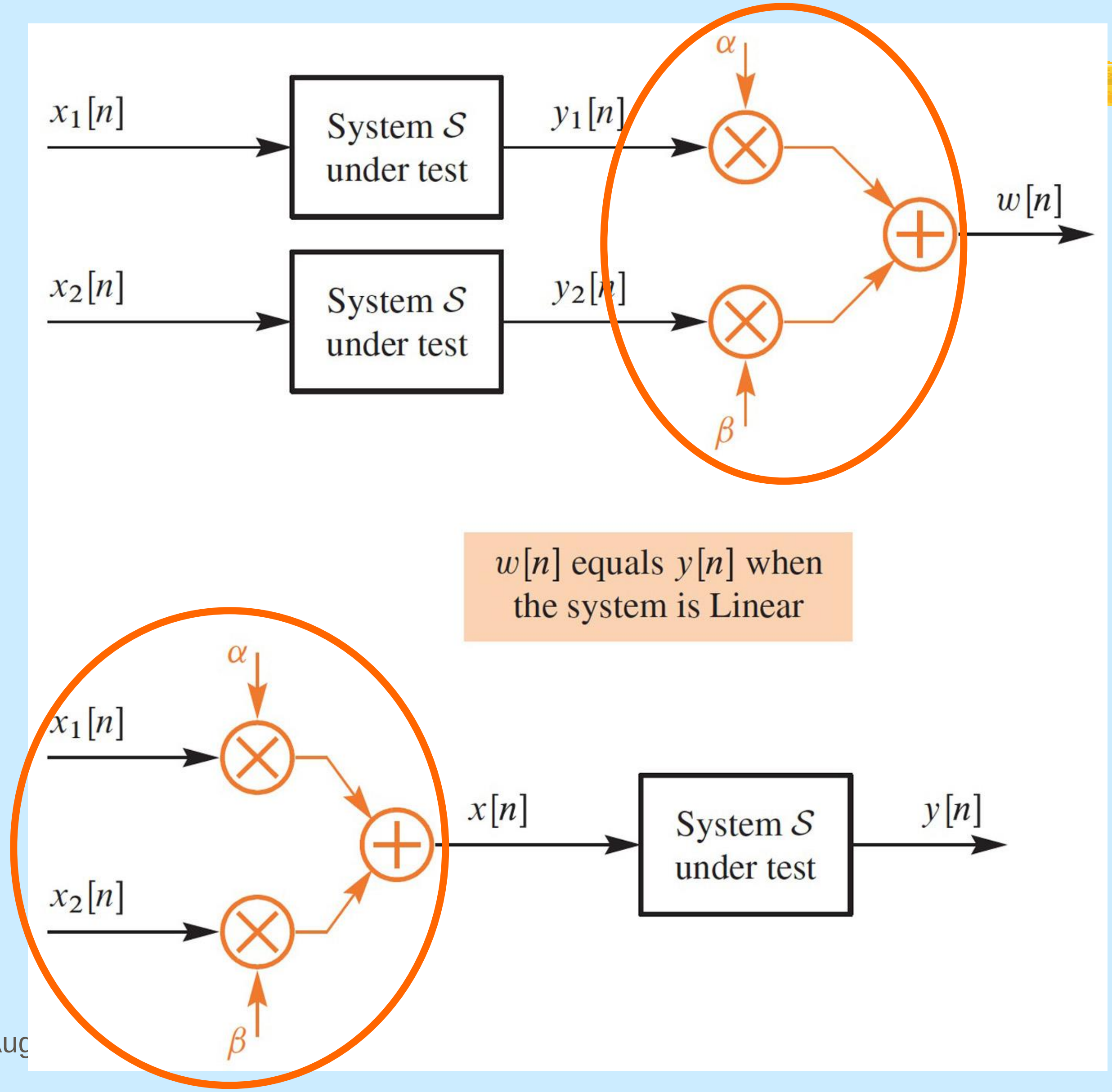
TIME-INVARIANCE

- IDEA:
  - "Time-Shifting the input will cause the same time-shift in the output"
- EQUIVALENTLY,
  - We can prove that
    - The time origin (n=0) is picked arbitrary

LINEAR SYSTEM

- LINEARITY = Two Properties
- SCALING
  - “Doubling x[n] will double y[n]”
- SUPERPOSITION:
  - “Adding two inputs gives an output that is the sum of the individual outputs”

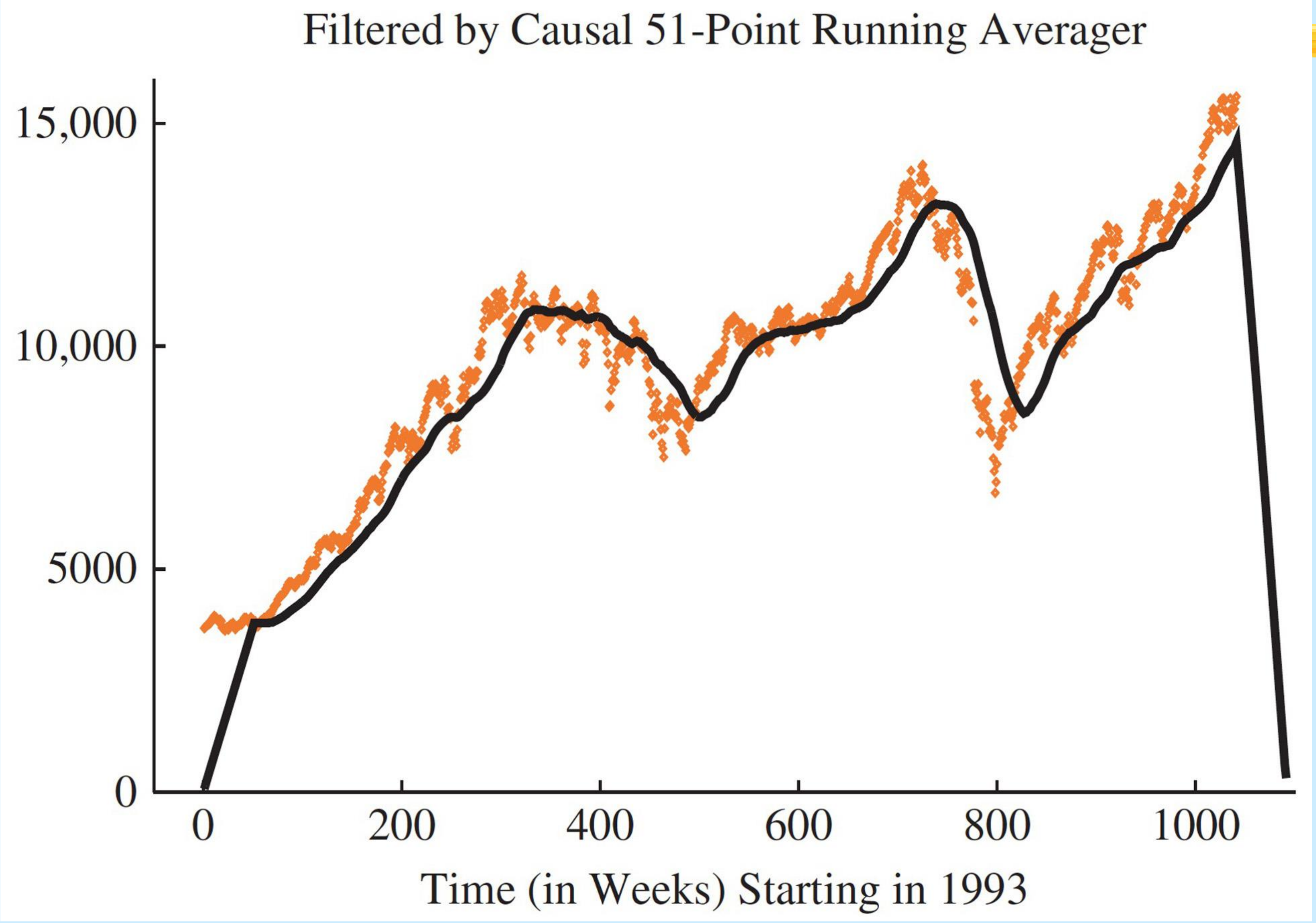
TESTING LINEARITY



LTI SYSTEMS $y[n] = h[n] * x[n]$

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - **IMPULSE RESPONSE** h[n]
  - **CONVOLUTION**:
    - The “rule” defining the system can ALWAYS be re-written as convolution
- FIR Example: h[n] is same as b_k

FILTER STOCK PRICES



STOCK PRICES FILTERED (2)