3315
CH4 PROBLEM SESSION

PLOS

consider

$$x[n] = x(nT_s) = A\cos(\omega nT_s + \phi)$$
$$= A\cos(\hat{\omega} n + \phi)$$

I. $\hat{\omega}$ IS DISCRETE TIME FREQUENCY!
(sometimes Ω); $F = \Omega/2\pi$ radians/sample

ω is continuous rad/sec / $\omega = 2\pi f$

II Define f_s = sampling rate = samples/sec
(sometimes given as S H_3); $T_s = \frac{1}{f_s}$ sec

III SUMMARY

If $x(t)$ contains frequencies 0 to f_{max}
for no Aliasing

1. $f_s = S > 2f_{max}$ pg 111

2. $-\pi < \hat{\omega}_0 = \omega_0 T_s < \pi$ (4.9)
$= 2\pi f_0 T_s < \pi$

IV If Aliased,

The frequencies become

$f_a = f_0 - M f_s$ for $\cos(2\pi F_0 + \phi)$

look at 4.9

$-\pi < \hat{\omega}_0 = \omega_0 T_s < \pi$ $\hat{\omega}_0$ DISCRETE

OR $-\pi < \frac{2\pi F_0}{F_s} < \pi$

$-0.5 f_s < f < 0.5 f_s$ ANALOG

pg 113 $\hat{\omega} = \omega_0/f_s + 2\pi\ell \quad \ell = 0, \pm 1, \pm 2$

$\hat{\omega} = -\omega_0/f_s + 2\pi\ell$

we can look at it in analog

If aliased $f_a = f_0 - Mf_s$

So that the

digital falls between $-\pi, \pi$

analog falls between $-0.5f_s, 0.5f_s$

Example

$\cos(200\pi t + \theta) \qquad f = 100\ Hz$

a) Need $f_s = S > 200$ samples/sec

b) frequencies

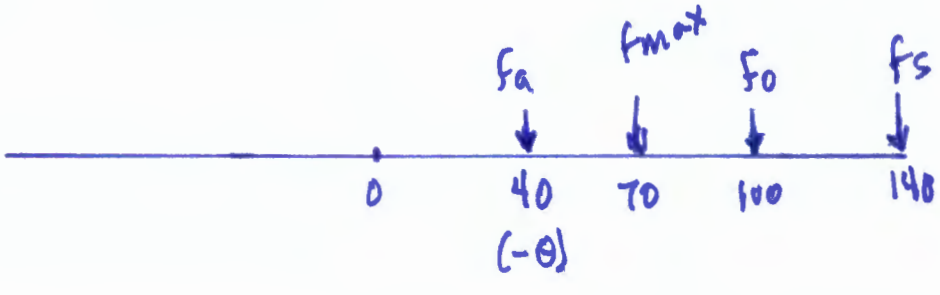
$f_s = 240\ Hz \quad f = 100\ Hz \quad OK$

$f_s = 140\ Hz$ - Aliased

$f_a = 100 - 140 = -40\ Hz$

$x_a(t) = \cos(-2\pi(40)t + \theta) = \cos(80\pi t - \theta)$

Phase Reversal



Let's convert

$x(t) = \cos(2\pi f_0 t)$ sampled every T_s seconds; so $f_s = \frac{1}{T_s}$

$x[n] = \cos(2\pi f_n T_s + \theta) =$

$\cos(2\pi n \frac{f}{f_s} + \theta) =$

$\cos(2\pi n F + \theta) =$

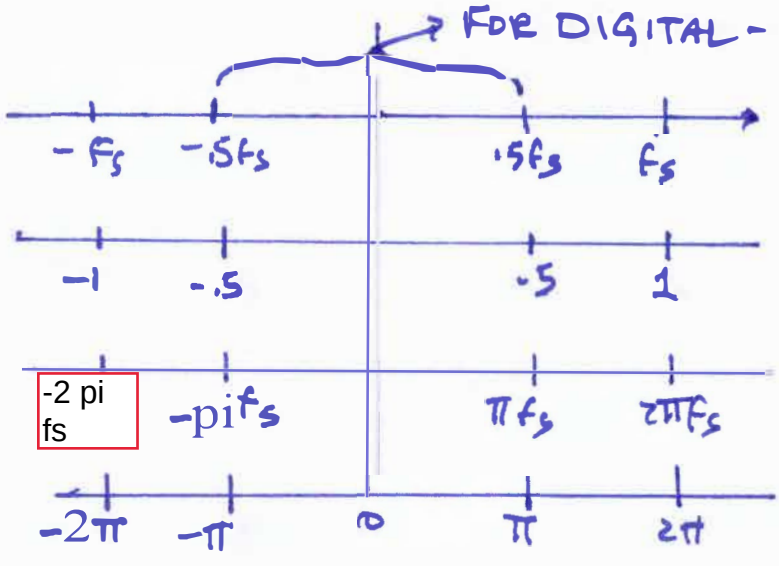
$\cos(\hat{\omega} n + \theta)$

OR

$\tilde{x}[n] = e^{j(2\pi n F + \theta)}$

$= \cos(2\pi n F + \theta) + j \sin(2\pi n F + \theta)$

Here $F = f/f_s$ $\hat{\omega} = 2\pi F$



$-.5 < F < .5$

$-\pi < \hat{\omega} < \pi$

$\hat{\omega} = \Omega = 2\pi F$

$= 2\pi f/f_s$

Digital

DSP FIRST

CH4

P106 $x(t) = \cos(2\pi(100)t)$

$T_s = 0.5 \times 10^{-3}$ sec

$f_s = \frac{1}{T_s} = 2000$ samples/sec

$\hat{\omega} = \frac{\omega}{f_s} = \frac{2\pi(100)}{2000} = 0.1\pi$ rad

$\approx 18°$

Note: $f_{s_1} = 200$ samp/sec

$\hat{\omega}_1 = \frac{2\pi(100)}{200} = \pi$ rad

see figure 4.3 Pg 106

(**) YouTube

Why car wheels rotate backwards

Physics Made fun 4:26

CAMERA - SAMPLES

Wheel 157 cm

speed 0 to 21 rotations/sec

0, 72, 144, 216° deg/sec steady

(**) Audio Sampling Rate Demo

3 examples Dan Hoskem

(**) SPECTRUM from θ-scope

P-4.13 $7/\pi \cos(1.8\pi nn + 2.03)$

(a) f = 1600 Hz. Phase will be −2.03 rad.

(b) ~~fsamp~~ = ~~9600~~ Hz.

(c) ~~f~~ = ~~2200~~ Hz. Duration = ~~12.8~~ s.

$\hat{w} = 1.8\pi = w/f_s$

Reduce to

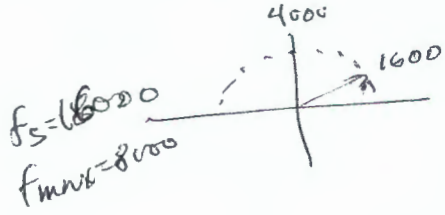
a) $\hat{\omega} = 1.8\pi$ X so $\hat{\omega} = 1.8\pi - 2\pi = -0.2\pi$

Then $0.2\pi = \frac{\omega}{16000} = \frac{2\pi f}{16000\ Hz}$

$f = \frac{16000 \times 0.2\pi\ Hz}{2\pi} = 8000 \times 0.2 = 1600\ Hz$

But Negative

$x[nn] = \frac{7}{\pi} \cos(\underbrace{0.2\pi nn}_{\hat{\omega}} - 2.03\ rad)$!



$\pi/5 = 36^\circ$

See MATLAB

DSPF_Prob4-13.M

C:\Users\Harman\Documents\Data\CENGc...\DSPF_Prob4_13.m Page 1

```
% DSP First Problem 4-13
% Use soundsc
% Compute the frequency from w_hat = 1.8*pi
fs = 16000    % Sample rate/sec
w_hat1 = 1.8*pi-2*pi  % Bring it to -pi to pi
f= abs(fs*w_hat1/(2*pi))  % 1600 Hz
T0 = 1/f    % Seconds per Cycle   6.2500e-04 sec
% Play 5 seconds
nnmax= 5*fs        % 80000
nn=0:nnmax;
xx=(7/pi)*cos(1.8*pi*nn+2.03);
To = 1/1600   ; %sec
soundsc(xx,fs)
```