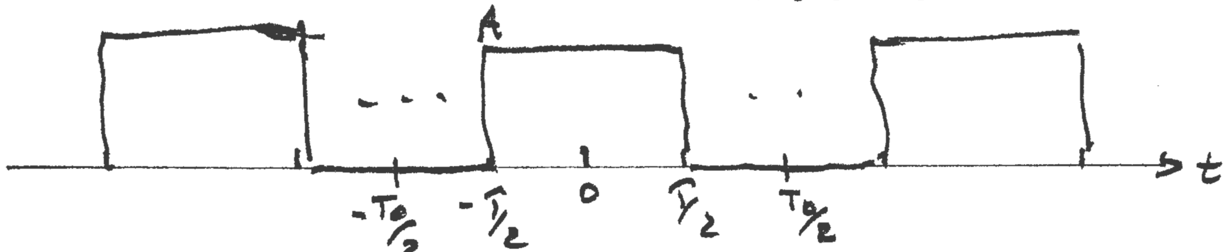
PULSE WAVE

Lecture 3~~5~~2 slide 15 (7C)

$x(t) = x(t+T)$



SLIDE 5

Find the series $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k F_0 t}$

SLIDE 4

I. $a_k = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)\, e^{-j(2\pi/T_0)kt}\, dt$

If we do the general case - all the coefficients will be found

$$a_k = \frac{1}{T_0} \int_{-T/2}^{T/2} A \cdot e^{-j\overbrace{(2\pi/T_0)}^{=\omega_0} k t}\, dt$$

integrate

$$\frac{A}{T_0} \left. \frac{e^{-j\omega_0 k t}}{-j\omega_0 k} \right|_{-T/2}^{T/2}$$

WATCH SIGNS !

$$\frac{A}{T_0} \; \frac{e^{-j\omega_0 k T/2} - e^{-j\omega_0 k(-T/2)}}{-j\, 2\pi/T_0\, k}$$

so write as ratio T/T_0 in Exponent

PULSE 2

$$\underline{\text{SO}}\; A \cdot \frac{e^{j\pi(\tau/T_0)k} - e^{-j\pi(\tau/T_0)k}}{+j\,2\pi k} = a_k$$

This is the sine

$$a_k = \frac{A \sin(k\pi\ \tau/T_0)}{\pi k} \qquad k = 0, \pm 1, \pm 2, \ldots$$

These are the coefficients - <u>NOT</u> the series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{j 2\pi k f_0 t}$$

For a_k $\quad \sin[k \cdot \pi(\tau/T_0)]$

SPECIAL CASE suppose $\tau/T_0 = 1/2$ so Duty cycle = 50%

$\sin(k\pi/2) \rightarrow$

$k = \pm 1$	± 2	± 3	...
± 1	0	± 1	

SO HERE, EVEN HARMONICS ARE ZERO (SYMMETRICAL ABOUT ZERO = $x(t)$)

WE ARE STILL NOT DONE IF WE WANT REAL FORM,

BUT FOR SPECTRUM

SLIDE 1B

Suppose $\tau/T_0 = 1/2$, Then

$$a_k = \frac{A}{\pi k} \sin(k\pi/2)$$

Plot $x(t) = A(1/2) + \frac{A}{\pi} \sum_{k=-\infty}^{\infty} \frac{1}{k} \sin(k\frac{\pi}{2}) e^{jk}$

take + and - terms say $k = \pm 1$

$$\frac{A}{\pi} \frac{1}{(-k)} \underbrace{\sin(-\pi/2)}_{=-1} e^{-jk\pi t}$$

$$+ \frac{A}{\pi} \frac{1}{k} \underbrace{\sin(\pi/2)}_{=+1} e^{+jk\pi t}$$

$$= \frac{A}{\pi}\left(\frac{1}{k}\right) 2\cos(k\pi t) \quad ..$$

So

$$x(t) = \frac{A}{2} + \frac{2A}{\pi} \sum_{k=1}^{N} \frac{1}{k} \cos(k\omega_0 t)$$

because a_k is real

SEE MY MATLAB EXAMPLES - MATLAB_FOURIER ON WEB

Lets Plot

See FourierChB_TCH pg 392

Define A, T_0, ω_0, τ/T_0 and
number of components.
Plot discrete values OR use STEM

Example for $T_0 = 2$ sec, $\tau = 1$ sec

$f_0 = .5$ Hz $\quad \omega_0 = 2\pi/2 = \pi$

MATLAB Pulsetrainspec.m
Expect a Positive spectrum

$a_0 = \frac{1 \cdot 1}{2} = 0.5$ ✓

$a_1 = \frac{2A}{\pi} = \frac{2}{\pi} = 0.6366$

WATCH IT PLOTTING $-f$ to f
VALUES ARE $\frac{1}{2}$

Now
$a_3 = \frac{2}{3\pi} = 0.2122$ ✓ but $\angle 0$