

3/25/2019

# Lecture

①

## Review

Ch 4 pg 105

$$x[n] = x(nT_s)$$

$$-\pi \leq \hat{\omega} \leq \pi$$

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

$$= 2\pi \frac{f}{f_s}$$

Prob  $x(t) = \cos(200\pi t) = \cos(2\pi \cdot 100t)$

$f_0 = 100 \text{ Hz}$  let  $f_s = 500 \text{ Hz}$

Figure 4-3

$$\hat{\omega} = \frac{2\pi \cdot 100}{500} = 0.4\pi \quad 72^\circ$$



Alias Note

$$\checkmark \cos(2.4\pi n) = \cos(0.4\pi n + 2\pi n)$$

Proof  $\cos(0.4\pi n + 2\pi n) = \underbrace{\cos(0.4\pi n)}_{\rightarrow 0} \underbrace{\cos(2\pi n)}_{\rightarrow 1} - \sin(0.4\pi n) \sin(2\pi n)$

$$\text{So } \hat{\omega} = \frac{2\pi f}{500} = 2.4\pi \quad f = \frac{2.4 \times 500}{2} = 1.2 \times 500 = 600 \text{ Hz}$$

$$f_{\text{alias}} = 600 \text{ Hz} - 500 \text{ Hz} = 100 \text{ Hz}$$

Prob 50

$$-\pi < \hat{\omega}_0 = \omega_0 T_s < \pi$$

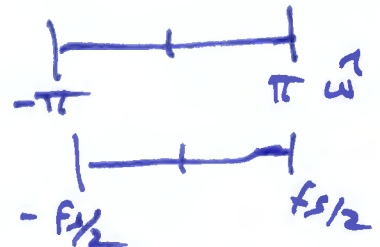
$$\omega_0 = 2\pi f_0$$

$$T_s = \frac{1}{f_s}$$

$$-\pi < 2\pi \frac{f_0}{f_s} < \pi$$

$$\text{So } -\frac{1}{2} < \frac{f_0}{f_s} < \frac{1}{2}$$

If result is < 0; negate phase



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Pg 113

$$x[n] = x[\sum nT_s] = x[n/f_s] =$$

$$A \cos[(\omega_0/f_s)n + \phi] = A_+ e^{j\omega_0/f_s n} + A_- e^{-j\omega_0/f_s n}$$

$$A_+ = \frac{1}{2} A e^{j\phi} \quad A_- = \frac{1}{2} A e^{-j\phi}$$

Frequencies eg 4.13

$$\hat{\omega} = \omega_0/f_s + 2\pi l \quad l = 0, \pm 1, \dots$$

$$\hat{\omega} = -\omega_0/f_s + 2\pi l$$

Alias  $f_s = 125 \text{ s/s (Hz)}$   $f_0 = 100 \text{ Hz}$

pg 117  $\hat{\omega} = \pm 2\pi (100/125) = \pm 1.6\pi$

So shift to  $\pm\pi$  for Aliased frequency

$$\hat{\omega} = 1.6\pi + 2\pi l \geq \pi$$

$$\hat{\omega} = \underline{-1.6\pi + 2\pi \cdot 1} = \underline{+0.4\pi}$$

$$\text{So } f = 0.4\pi f_s / 2\pi = \frac{0.4}{2} \times 125 = \boxed{25 \text{ Hz}}$$

Phase change  $+\phi$  to  $-\phi$

OR  $f_{\text{alias}} = 100 \text{ Hz} - 125 \text{ Hz} = -25 \text{ Hz}$

so  $\cos(\omega_0 t + \phi) \Rightarrow \cos(\omega_a t - \phi)$   
 $\omega_0 = 2\pi \cdot 100$   $\omega_a = 2\pi \cdot 25$

P 169 OUTPUT OF FIR FILTER

CONVOLVE  $y[n] = \sum_{k=0}^M b_k x[n-k]$  Eq 5.24

ASSUMING  $h[m] = 0$  for  $m < 0$

$y[n] = \sum_{k=0}^M h[k] x[n-k]$  Eq 5.13 P 161  
↑ Impulse Response

but  $h[n] = \sum_{k=0}^M b_k \delta[n-k] = \begin{cases} b_n & n=0,1,\dots,M \\ 0 & \text{otherwise} \end{cases}$   
Eq Pg 160

MATLAB CONV(hn, xn)  
OR USE FILTER

VIDEOS SPEAKING IN "CHURCH"

FIR NOTCH  
SHOW MATLAB



HW6 GO OVER

SUGGEST REVIEW

CH2 REFERENCES

TEST REVIEW 1 + Summary for FINAL

FOUR-convolve Review

# CH 6

④

PIQS SPECIAL INPUT

$$A e^{j[\hat{\omega} n + \phi]}$$

Complex sinusoid

$$\text{so } y[n] = H(\hat{\omega}) A e^{j\phi} e^{j\hat{\omega} n} \quad \text{eg 6.2}$$

$$\text{To get } H(\hat{\omega}) \equiv H(e^{j\hat{\omega}}) = \quad \text{eg 6.4}$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega} k} = \sum_{k=0}^M h[k] e^{-j\hat{\omega} k}$$

## FREQUENCY RESPONSE

EXAMPLE 6.1  $b_k = \{1, 2, 1\}$  so

$$H(e^{j\hat{\omega}}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

write as  $= e^{-j\hat{\omega}} (2 + 2\cos \hat{\omega})$  to plot

$$-\pi < \hat{\omega} < \pi$$

This is  $|H(e^{j\hat{\omega}})|$   $1/\hat{\omega}$

Ex 6.2 what does  $H(e^{j\hat{\omega}})$  do?

CHANGES FREQUENCY OF OUTPUT

JUST MULTIPLY AT FREQUENCIES

$$y[n] = H(e^{j\pi/3}) \cdot (2e^{j\pi/4} e^{j\pi n/3})$$

$$= \left[ e^{-j\pi/3} \cdot (2 + 2\cos^{1/2} \pi/3) \right] x[n]$$

$$= 3e^{-j\pi/3} x[n] =$$

$$\underline{6e^{-j\pi/2} e^{j\pi n/3}}$$

pg 197

CH6

$$2 e^{-j\pi/4} e^{j(\pi/3)n} = x[n]$$

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$$y[n] = 6 e^{-j\pi/2} e^{j\pi n/3}$$

Rewrite as multiple of  $\delta$

$$e^{-j\pi/2} = e^{j\pi/4} e^{-j\pi/3}$$

$$\text{So } y[n] = 6 e^{j\pi/4} e^{j\pi n/3} \cdot e^{-j\pi/3}$$

$$= 6 e^{j\pi/4} e^{j\pi(n-1)/3}$$

↑ delay 1

PI99 cosine input CH6

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$$x[n] = 3 \cos\left(\frac{\pi}{3}n - \frac{\pi}{2}\right)$$

$$\begin{aligned} H(e^{j\pi/3}) &= e^{-j\pi/3} (2 + 2\cos\pi/3) \\ &= 3 e^{-j\pi/3} \\ &= 3 \angle -\pi/3 \end{aligned}$$

$$\begin{aligned} H(e^{j\pi/3}) \cdot x[n] &= (3) \cdot 3 \left( \cos\left[\frac{\pi}{3}n - \frac{\pi}{3} - \frac{\pi}{2}\right] \right) \\ &= 9 \cos\left(\frac{\pi}{3}(n-1) - \frac{\pi}{2}\right) \\ &\quad \uparrow \text{delay } 1 \end{aligned}$$

Look at

Equation Page 203  
 Time  $\longleftrightarrow$  Freq

SH

$$h[n] = \sum_0^m h[k] \delta[n-k] \longleftrightarrow$$

$$H(e^{j\hat{\omega}}) = \sum_0^m h[k] e^{-j\hat{\omega}k}$$

meaning of shift in  $n$  - page 206

$$y[n] = x[n - n_0]$$

$$H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}n_0}$$

$$|H(e^{j\hat{\omega}})| = |e^{-j\hat{\omega}n_0}| = 1$$

$$\angle e^{-j\hat{\omega}n_0}$$

$$= -n_0 \hat{\omega}$$

slope is  $-n_0$

see figure 6-2