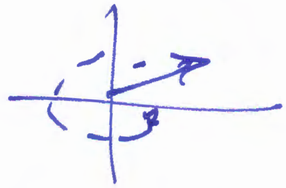
TLH EXAMPLES PROBLEMS CH2 CH2 (1)

DSP FIRST

COMBINE PGS 13 – 16

$\cos(\theta)$ or $\sin(\theta)$

PERIOD is 2π RADIANS



1. How Long to rotate 2π radians? T_0

e.g. $T_0 = 1$ sec $\Rightarrow$ rotation 2π rad/sec

$T_0 = \frac{1}{10}$ sec $\Rightarrow$ $10 \times 2\pi$ rad/sec

So rotation $= \frac{2\pi}{T_0}$ rad/sec

call This $\omega = \frac{2\pi}{T_0}$

2. Since

θ $\quad 0 \longrightarrow 2\pi \longrightarrow 4\pi \longrightarrow \cdots$

t $\quad 0 \longrightarrow T_0 \longrightarrow 2T_0 \longrightarrow \cdots$

number of cycles/sec $= \frac{1}{T_0}$ = frequency in Hertz

So $T_0 = 1$ s $\quad f = 1$ Hz

$T_0 = \frac{1}{10}$ s $\quad f = 10$ Hz

Eg 2.4 Pg 16

3. So we can write

$x(t) = A\cos(\omega_0 t)$ $\quad \omega_0 = 2\pi f_0$

with $\omega_0 t = \theta$ and

$2\pi f_0 t = \theta$

$2\pi \frac{t}{T_0} = \theta \Rightarrow \frac{t}{T_0} = \frac{\theta}{2\pi}$ Pg 18

```
% Table of frequency and Period
clc,clear all
format rat   % Fractions
frequency = [1 10 100 1000 10000 100000 ];
period = 1./[frequency];
% c = rdivide(a,b) and c = a./b perform right-array division
%by dividing each element of a by the corresponding element of b.
%If inputs a and b are not the same size,
%one of them must be a scalar value.
%
% One Way to do it - display and align
A=[frequency;period]'; % Make 2X6 Array
disp('  frequency-Hz  period-Seconds')
disp(A)
% Better way Make a table
varNames={'Frequency','Period'}
T=table(frequency',period','VariableNames',{'Frequency','Period'})
```

```
  frequency-Hz  period-Seconds
       1             1
      10             1/10
     100             1/100
    1000             1/1000
   10000             1/10000
  100000             1/100000


varNames =

  1×2 cell array

    {'Frequency'}    {'Period'}


T =

  6×2 table

    Frequency    Period
    _________    ______

          1          1
         10        0.1
        100       0.01
       1000      0.001
      10000     0.0001
      1e+05      1e-05
```

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2-4

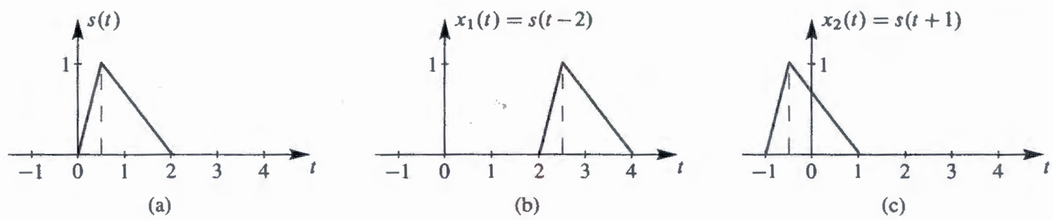


Figure 2-8 Illustration of time-shifting: (a) the triangular signal $s(t)$; (b) shifted to the right by 2 s, $x_1(t) = s(t-2)$; (c) shifted to the left by 1 s, $x_2(t) = s(t+1)$.

EXERCISE 2.2 Derive the equations for the shifted signal $x_2(t) = s(t+1)$.

Delay vs Advanced

We will have many occasions to consider time-shifted signals. Whenever a signal can be expressed in the form $x_1(t) = s(t - t_1)$, we say that $x_1(t)$ is a time-shifted version of $s(t)$. If t_1 is a positive number, then the shift is to the right, and we say that the signal $s(t)$ has been ***delayed*** in time. When t_1 is a negative number, then the shift is to the left, and we say that the signal $s(t)$ was ***advanced*** in time. In summary, time shifting is essentially a redefinition of the time origin of the signal. In general, any function of the form $s(t - t_1)$ has its time origin moved to the location $t = t_1$.

For a sinusoid, the time shift is defined with respect to a zero-phase cosine that has a positive peak at $t = 0$. Since a sinusoid has many positive peaks, we must pick one to define the time shift, so we pick the positive peak of the sinusoid that is closest to $t = 0$. In the plot of Fig. 2-6, the time where this positive peak occurs is $t_1 = 0.005$ s. Since the peak in this case occurs at a positive time (to the right of $t = 0$), we say that the time shift is a delay of the zero-phase cosine signal. Now we can relate the time delay to phase. Let $x_0(t) = A\cos(\omega_0 t)$ denote a cosine signal with zero phase. A delay of $x_0(t)$ can be converted to a phase φ by making the following comparison:

??

WATCH SIGN!

$$x_0(t - t_1) = A\cos(\omega_0(t - t_1)) = A\cos(\omega_0 t + \varphi)$$

$$\Rightarrow \cos(\omega_0 t - \omega_0 t_1) = \cos(\omega_0 t + \varphi)$$

Time and Phase

Since the second equation must hold for all t, we conclude that $\varphi = -\omega_0 t_1$. Notice that the phase is negative when the time shift t_1 is positive (a delay). We can also express the phase in terms of the period ($T_0 = 1/f_0$) where we get the more intuitive formula

!!

See YouTube Timothy Schulz Intro to the Sinusoid

$$\varphi = -\omega_0 t_1 = -2\pi\left(\frac{t_1}{T_0}\right) \tag{2.7a}$$

which states that the phase is 2π times the fraction of a period given by the ratio of the time shift to the period. If we need to get the time shift from the phase, we solve for t_1

DEMO
Sine Drill

$$t_1 = -\frac{\varphi}{\omega_0} = -\frac{\varphi}{2\pi f_0} = -\frac{\varphi T_0}{2\pi} \tag{2.7b}$$

Since the positive peak nearest to $t = 0$ must always lie within the interval $[-\frac{1}{2}T_0, \frac{1}{2}T_0]$, the phase calculated in (2.7a) will always satisfy $-\pi < \varphi \le \pi$. However, TCH Ex 2

YouTube DSP First Pg (2)

Timothy Scholz Intro to the sinusoidal signal Pg 14

$s(t) = A\cos(2\pi f t + \phi)$

4:15

$= A\cos\left[2\pi f\left(t - \frac{\phi}{2\pi f}\right)\right]$

Looks like 0 phase with delay $-\frac{\phi}{2\pi f}$

ϕ radians / delay seconds in seconds

see Page 13 sine/cosine

So 1 cycle $= 2\pi$ radians

$= T$ seconds $=$ Period

$= 1/f$ cycles/second $=$ frequency

So $\frac{\phi}{2\pi} = \frac{t_{delay}}{T}$ to the right i.e. $+$time

Example: 2π rad; $1/f$ period; shift in $+t$

Shift for $A\cos(2\pi t - \phi) = A\cos[\omega(t - t_d)]$

Here $f = 1$ Hz so $T = 1$ sec

ϕ shift	$t_{delay} = \phi/2\pi \times T$
$\pi/4$	$\frac{\pi/4}{2\pi} \cdot 1 = 1/8$ sec check $\frac{1}{8} \cdot 2\pi = \pi/4$ ✓
$\pi/2$	$\frac{\pi/2}{2\pi} \cdot 1 = 1/4$ sec $\Rightarrow A\cos[2\pi(t - 1/4)]$
π	$1/2$
⋮	

$\boxed{\text{Page 15 figure 2.6}}$ PROBLEM 2

$$\frac{t}{25ms} = \frac{-0.4\pi}{2\pi} = -0.2 \quad \text{Shift Right}$$

$$t = -0.2(25ms) = \underline{5.0\,ms}$$

$$\underline{f(t) = \cos(2\pi 40t - 0.4\pi)}$$

Note peak at zero

$$2\pi 40\, t - 0.4\pi = 0$$

$$t = \frac{0.4\pi}{2\pi 40} = \frac{0.4}{2\times 40} = \frac{0.2}{40} = 0.2(25ms)$$

Positive shift

$$= \underline{5.0\,ms}$$

Fig 2.1 Pg 10

$$10\cos(2\pi\, 440t - 0.4\pi)$$

$$\text{So} \quad t = \frac{0.4\pi}{2\pi\, 440} = \frac{0.2}{440} = \underline{0.2(2.27ms)}$$

$$= \underline{0454\,ms}$$

For phase learn cos/sin

	cos	sin
0°	1	0
45	$1/\sqrt{2}$	$1/\sqrt{2}$
60	1/2	$\sqrt{3}/2$

so we define the phase to be $\varphi = \varphi' - \pi/2$ in (2.2). For simplicity and to prevent confusion, we often avoid using the sine function.

(c) ω_0 is called the ***radian frequency***. Since the argument of the cosine function must be in radians, which is dimensionless, the quantity $\omega_0 t$ must likewise be dimensionless. Thus, ω_0 must have units of rad/s if t has units of seconds. Similarly, $f_0 = \omega_0/2\pi$ is called the ***cyclic frequency***, and f_0 must have units of s^{-1}, or hertz.

EXAMPLE 2-1 Plotting Sinusoids

Figure 2-6 shows a plot of the signal

$$x(t) = 20\cos(2\pi(40)t - 0.4\pi) \tag{2.3}$$

$T = \frac{1}{40}$ sec $= 25$ms

In terms of our definitions, the signal parameters are $A = 20$, $\omega_0 = 2\pi(40)$ rad/s, $f_0 = 40$ Hz, and $\varphi = -0.4\pi$ rad. The signal size depends on the amplitude parameter A; its maximum and minimum values are $+20$ and -20, respectively. In Fig. 2-6 the maxima occur at

$$t = \ldots, -0.02, 0.005, 0.03, \ldots$$

and the minima at

$$\ldots, -0.0325, -0.0075, 0.0175, \ldots$$

DEMO
Sinusoids

The time interval between successive maxima in Fig. 2-6 is 0.025 s, which is equal to $1/f_0$. To understand why the signal has these properties, we will need to do more analysis.

2-3.1 Relation of Frequency to Period

Repeats 2π radians

PERIOD

The sinusoid plotted in Fig. 2-6 is a periodic signal. The ***period*** of the sinusoid, denoted by T_0, is the time duration of one cycle of the sinusoid. In general, the frequency of the sinusoid determines its period, and the relationship can be found by applying the definition of periodicity $x(t + T_0) = x(t)$ as follows:

Van Veen video Periodic

$$A\cos(\omega_0(t + T_0) + \varphi) = A\cos(\omega_0 t + \varphi)$$

$$\cos(\omega_0 t + \underbrace{\omega_0 T_0}_{=2\pi?} + \varphi) = \cos(\omega_0 t + \varphi)$$

✓

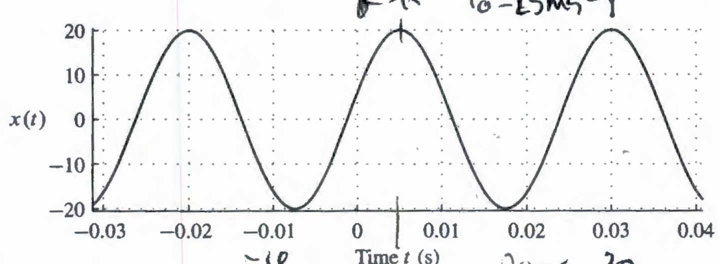
$\frac{\omega_0}{2\pi} = \frac{40\,r/s}{2\pi}$; $\frac{2\pi}{2\pi T_0} = \frac{1}{T_0}$ = 25ms +5.0ms

$T_0 = 25$ms

$\frac{t}{25\text{ms}} = \frac{-0.4\pi}{2\pi}$ $t = 5$ ms To Right

$2\pi f_0 T_0 = 2\pi$ $T_0 = 1/f_0$

See Fig 2.1



−10 ms 5ms 20ms 30

Figure 2-6 Sinusoidal signal with parameters $A = 20$, $\omega_0 = 2\pi(40)$, $f_0 = 40$ Hz, and $\varphi = -0.4\pi$ rad.

$T_0 = \frac{1}{40\,c/s} = 25$

See Page 24 F 2.11 For $e^{j(\omega_0 t + \varphi)}$

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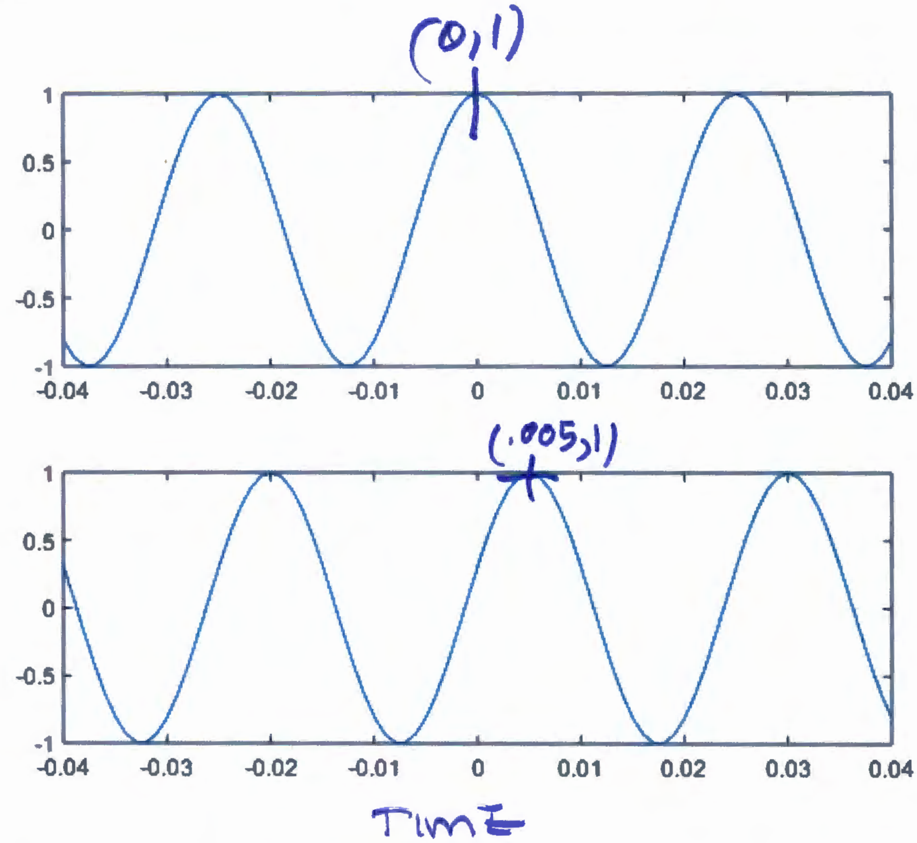
```
% Plot of cosine and shifted cosine
% 20*cosine(2*pi*40 -0.4*pi) vs 20*cosine(2*pi*40)
% T0= 1/40 = 25ms. Consider a time axis from -.04 to +.04 seconds
taxis = -.04:.001: .04;
x1=cos(2*pi*40*taxis) ;
x2=cos(2*pi*40*taxis -0.4*pi);
subplot(2,1,1); plot(taxis,x1)
subplot(2,1,2); plot(taxis,x2)
grid on
```

FIG 2-0



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