Problem Session 2

Pg 28

$z = x + jy$

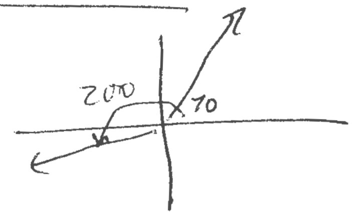
$z + z^* = 2x$

$z - z^* = x + jy - (x - jy) = 2jy$

$Im(z) = \frac{\frac{1}{2}(z - z^*)}{j}$

Pg 3 *

$x_1(t) = 1.7 \cos(20\pi t + 70\pi/180)$



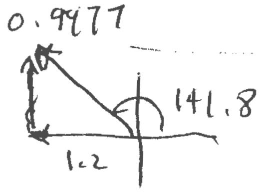
$x_2(t) = 1.9 \cos(20\pi t + 200\pi/180)$

$1.7\{\cos 20\pi t \cos \frac{70\pi}{180} - j \sin 20\pi t \sin 70\pi/180\}$

$1.9\{\cos 20\pi t \cos \frac{200\pi}{180} - j \sin 20\pi t \sin 200\pi/180\}$

$1.7 \cos \frac{70\pi}{180} \cos 20\pi t - j 1.7 \sin \frac{70\pi}{180} \sin 20\pi t$

$1.9 \cos \frac{200\pi}{180} \cos 20\pi t - j 1.9 \sin \frac{200\pi}{180} \sin 20\pi t$

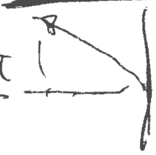


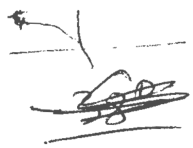
$a \cos 20\pi t - j b \sin 20\pi t$

So $\sqrt{a^2 + b^2}$; $\pi - \tan^{-1} \frac{Im}{Re}\left(\frac{+b}{a}\right) = \tan^{-1} \cancel{\frac{+0.9477}{-1.204}}$

$= 0.9$ $\quad \cancel{3.8207}$

$141.79^\circ = \frac{141.79\pi}{180}$





$\frac{\phi_{rad}}{2\pi} = \frac{\theta^\circ}{360} \qquad \frac{\phi_{rad}}{\pi} = \frac{\theta^\circ}{180} = \frac{141.79}{180} = \frac{\theta}{180}$

$= 0.7877\pi$ Rad

$= 2.4747$ Rad $= 141.8^\circ$

Pg 31

Pg 31-32

So $Re\left\{(-1.204 + j\,0.9477)\,e^{j\omega_0 t}\right\}$ (2)

$$Ae^{j\phi}e^{j\omega_0}$$

$$A = \left\{(-1.204)^2 + 0.9477^2\right\}^{1/2}$$

$$= 1.53223$$

$$\pi - \tan^{-1}\left(\frac{0.9477}{1.204}\right) = 141.79^\circ$$

Shift

$x_1(t)$ $\frac{70\pi}{180}$ to left so

$$\frac{t_{shift}}{T_0} = \frac{70\pi/180}{2\pi} = \frac{35}{180}\,T_0 = 0.194 \times 0.1$$

$$\stackrel{\circ}{=} 0.0194$$

$$= 0.02\ sec$$

$$X_2:\ \frac{200\pi/180}{2\pi} \times .1 = \frac{100}{180} \times .1 = .0555\ldots$$

EXAMPLES II

2-6.3 Pg 31

Complex Exponentials

January 22, 2019

For the complex number, $x + jy$,

$$\begin{aligned} r &= \sqrt{x^2+y^2} \\ \tan\theta &= \frac{y}{x} \end{aligned} \tag{1}$$

for $x \neq 0$. If $x = 0$, $\theta = \pi/2$ when $y > 0$ and $\theta = -\pi/2$ when $y < 0$.

It is important to go between the sinusoidal form and the phasor form. Assume the frequencies of the sinusoids are the same. We know the sum of such sinusoids will be a sinusoid of the same frequency.

Take the 10 Hz sinusoids (DSP First Page 31)

$$\begin{aligned} x_1(t) &= 1.7\cos(20\pi t + 70\pi/180) \\ x_2(t) &= 1.9\cos(20\pi t + 200\pi/180) \end{aligned} \tag{2}$$

The phasors involved are

$$\begin{aligned} X_1 &= A_1 e^{j\phi_1} = 1.7\, e^{j70\pi/180} \\ X_2 &= A_2 e^{j\phi_2} = 1.9\, e^{j200\pi/180} \end{aligned} \tag{3}$$

Then the steps to form $x_3(t) = x_1(t) + x_2(t)$ is as follows:

1. Convert both phasors to Rectangular form
2. Add the real and imaginary parts
3. Convert back to polar for the phasor X_3
4. Convert to the cosine form.

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Two steps (shown in color) are important in this proof. In the third line, the complex exponential $e^{j\omega_0 t}$ is factored out of the summation because all the sinusoids have the same frequency. In going from the third line to the fourth, the crucial step is replacing the summation term in parentheses with a single complex number, $Ae^{j\varphi}$, as defined in (2.23), because we are adding N complex constants.

2-6.3 Phasor Addition Rule: Example

same frequency!

We now consider an example of adding two sinusoids, where

Shift $\frac{70\pi/180}{2\pi} T_0 \doteq .0125$

$$x_1(t) = 1.7\cos(20\pi t + 70\pi/180)$$

$e^{j(2\pi 10 t)}$

$$x_2(t) = 1.9\cos(20\pi t + 200\pi/180)$$

$\frac{200\pi/180}{2\pi} T_0 = 0.055\ldots$

The frequency of both sinusoids is 10 Hz, so the period is $T_0 = 0.1$ s. The sum $x_1(t)+x_2(t)$ is done via phasor addition (2.23) of the complex amplitudes which requires four steps:

$\frac{\theta_{rad}}{\pi} = \frac{\theta_{deg}}{180}$

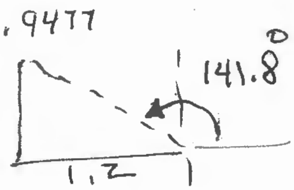
so $\theta_{rad} = \theta_{deg} \cdot \pi/180$

1. Represent $x_1(t)$ and $x_2(t)$ by the phasors:

$$X_1 = A_1 e^{j\varphi_1} = 1.7e^{j70\pi/180}$$
$$X_2 = A_2 e^{j\varphi_2} = 1.9e^{j200\pi/180}$$

2. Convert both phasors to rectangular form:

$$X_1 = 0.5814 + j1.5975$$
$$X_2 = -1.7854 - j0.6498$$



3. Add the real parts and the imaginary parts:

$\{(-1.204)^2 + 0.9477^2\}^{1/2} = 1.5322\ldots$

$$\begin{aligned} X_3 &= X_1 + X_2 \\ &= (0.5814 + j1.5975) + (-1.7854 - j0.6498) \\ &= -1.204 + j0.9477 \end{aligned}$$

$\pi - \tan^{-1}\left(\frac{0.9477}{1.204}\right) = 141.79\ldots$

4. Convert back to polar form, obtaining[6]

$141.79°$ ✓

$$X_3 = 1.5322e^{j141.79\pi/180}$$

[6]With a modern scientific calculator, step 1 is data entry, and then a single button would be pushed to do steps 2–4.

The resultant phasor X_3 is converted back to a 10-Hz sinusoid, so the final formula for $x_3(t)$ is

$\angle \theta^\circ$ 2.474 Radians

$\frac{141.79\pi/180}{2\pi} T_0 = 0.3938 \cdots \times .1 \doteq 0.039$ sec

$$x_3(t) = 1.5322\cos(20\pi t + 141.79\pi/180)$$

or

$$x_3(t) = 1.5322\cos(20\pi(t + 0.0394))$$

The waveforms of the three signals are shown Fig. 2-15(b) and the phasors used to solve the problem are shown on the left in Fig. 2-15(a). Notice that the times where the maximum of each cosine signal occurs can be derived from the phase through the formula

$$t_m = -\frac{\varphi T_0}{2\pi}$$

If $\phi < 0$, shift is to Right

which gives

$$t_{m1} = -0.0194, \quad t_{m2} = -0.0556, \quad t_{m3} = -0.0394 \text{ s}$$

These times are marked with vertical dashed lines in the corresponding waveform plots in Fig. 2-15(b).

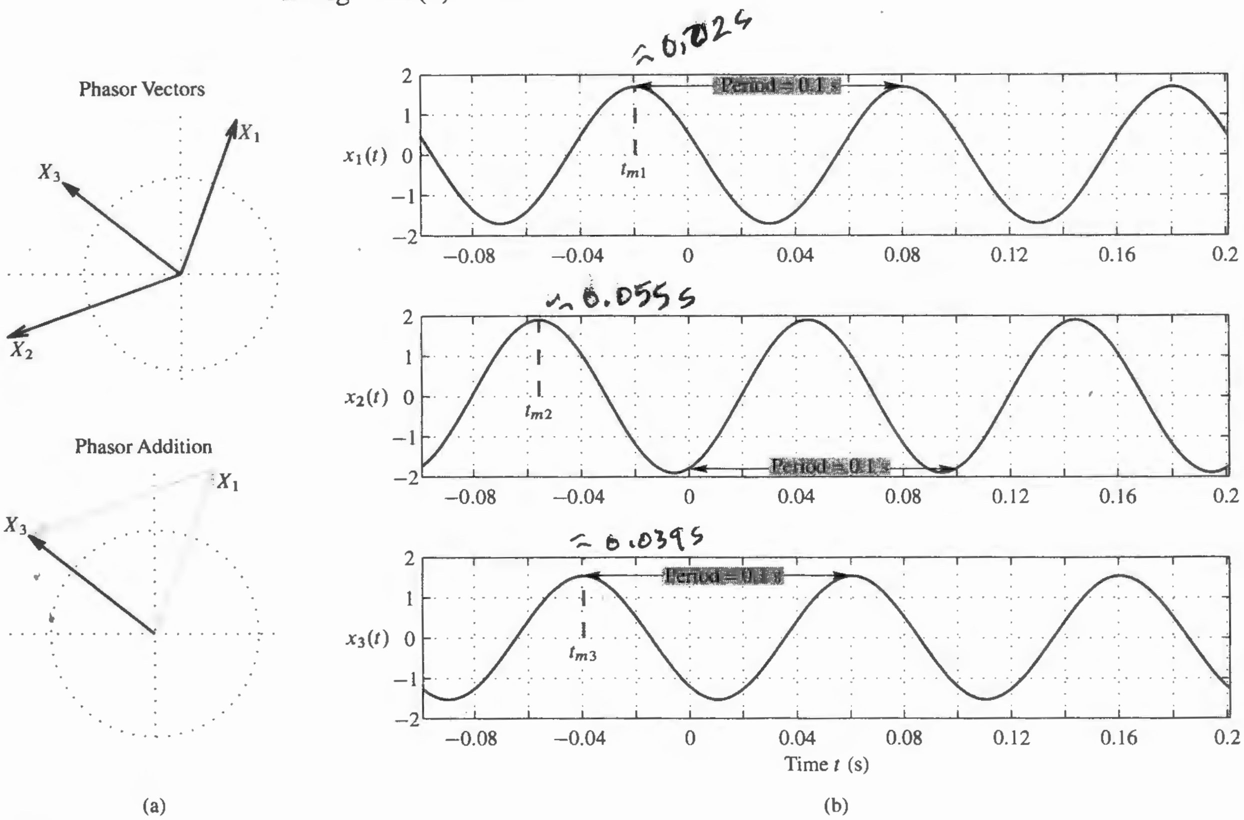


Figure 2-15 (a) Adding sinusoids by doing a phasor addition, which is actually a graphical vector sum. (b) The time of the signal maximum is marked on each $x_i(t)$ plot.

```
% Example 2-6.3 Add two cosines
% x1(t) = 1.7 cos(20*pi*t +70*pi/180) + 1.9 cos(20*pi*t +200*pi/180)
format short
%  Convert Phasor to rectangular

x1=1.7*exp(j*70*pi/180)    % x1 =  0.5814 + 1.5975i

x2=1.9*exp(j*200*pi/180)   % x2 = -1.7854 - 0.6498i
x3=x1+x2                   % x3 = -1.2040 + 0.9476i
% Convert x3 to polar
magx3=abs(x3)              % magx3   = 1.5322
x3theta=angle(x3)          % x3theta = 2.4748  theta in rad
%
thetadeg=x3theta*180/pi    % thetadeg =   141.7942 degrees
% Consider x(t-t0); the shift in time, omega = 2*pi/T0 = 20*pi
omega = 20 *pi
To = 2*pi/omega % 0.1 Seconds
td= - x3theta*(To/(2*pi)) % td= -0.0394
```

$$x_3(t) = 1.5322 \cos(20\pi t + 141.79\pi/180)$$

$$x_3(t) = 1.5322 \cos(20\pi t + 2.475)$$

IN TIME: $\cos[\omega(t \pm t_d)]$

Here $\theta > 0$, so shift is to <u>LEFT</u>, t_d

so $$\frac{-141.79\pi/180}{2\pi} = \frac{t_d}{T_0}$$ or $t_d = -.0394$

Another way
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$$\frac{\theta_{rad}}{2\pi} = -\frac{t_d}{T_0} \quad \text{for } \cos\omega(t - t_d)$$