

HW 3 Fourier Series 4331 Fall Due September 23

September 16, 2015

Do the problems by hand and **check** the results whenever possible. However, you may wish to verify your results with MATLAB solutions to the problems when appropriate. You can use symbolic MATLAB to check results if you wish (See Example on Page 110).

KandH Chapter 3, Appendix A

Problem 1 10 Points

Euler's formula states that $e^{i\theta} = \cos \theta + i \sin \theta$. Do the following:

Writing Euler's formula for $e^{i\theta}$ and $e^{-i\theta}$, show that

$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

and

$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}.$$

Problem 2 10 Points - Study Appendix A carefully and particularly Euler's Formula which yields one of the most interesting equations in the world

$$e^{i\pi} = -1.$$

1. Prove Equation 3.20 Page 108 that relates the Fourier Series trig coefficients to the complex exponential series.
2. Show that Equation for $x(t)$ on page 112 is the same as the Fourier Series of Equation 3.11 on Page 103.

Problem 3 30 Points

- (a) Prove the relationship that the sum of the sinusoids can be written as a shifted cosine in the expression

$$a \cos \theta + b \sin \theta = c \cos(\theta + \alpha)$$

where

$$c = \sqrt{a^2 + b^2}, \quad \tan \alpha = -\frac{b}{a} \quad a \geq 0.$$

- (b) Explain the angle

$$\alpha = \pi + \arctan\left(-\frac{b}{a}\right) \quad a < 0.$$

- (c) If we associate the vector components a with $a \cos \omega t$ and $-jb$ with $b \sin \omega t$, show that the magnitude and phase of the equation

$$a \cos \theta + b \sin \theta \Leftrightarrow a - jb$$

yields the same result as in part (a) using the vector representation. What is the justification for this use of a vector to represent the trigonometric functions?

- (d) Transform $4 \cos \omega t + 3 \sin \omega t$ to the shifted cosine form.

This should prove Equations 3.8, 3.9, and 3.10 (Page 102).

Problem 4 20 Points

Integrate the following by parts:

(a)

$$\int e^x \cos x \, dx$$

(b)

$$\int e^x \sin x \, dx$$

(c)

$$\int e^x e^{ix} \, dx$$

What is the relationship between the integrals in Parts a and b and the integral of Part c? How do we get the integrals of Parts a and b from the result in Part c?

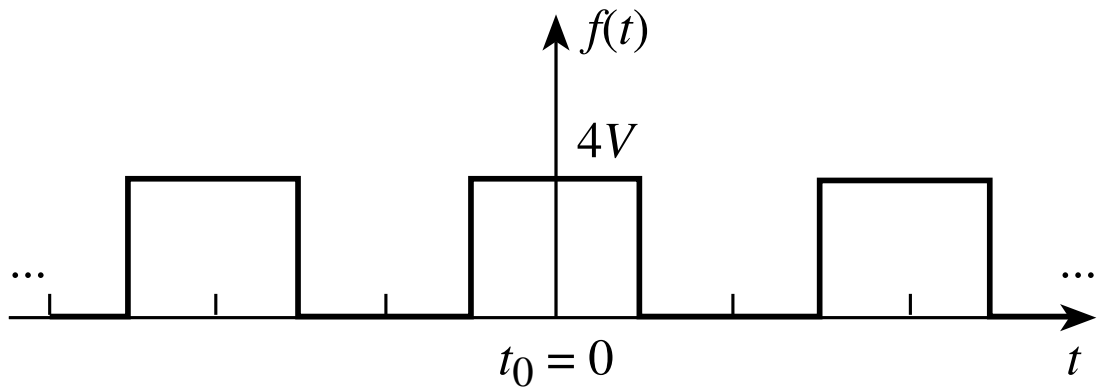


Figure 1: Caption for F08-13.new

Problem 5 30 Points

Fourier series of clock signal Consider the computer clock signal shown in the Figure, with a pulse rate of 8 million pulses per second ($f_c = 8$ Megahertz) and amplitude of 4 volts and a pulse width of 0.05 microseconds. NOTE: The figure does not show the signal to scale.

1. Find the Fourier series by hand calculation using the basic definitions of the coefficients.
2. Using MATLAB plot the Fourier amplitude spectrum in Hertz of the clock signal as discrete values up to 80 Megahertz.
3. Using MATLAB Plot $f(t)$ in seconds from the Fourier Series you calculated for 6 periods of the pulses.

Hint: You can write the series for many pulse trains by letting the period be T , the pulse width be τ and the amplitude be A and then computing the Fourier Series. Then, just plug in the numbers.