HW 4 Fourier Transform 4331 Fall Due October 5

September 24, 2015

`K&H Chapter 3, Section 3.4-3.6, Section 3.3.3`
`See K&H WEB site Worked Problems Chapter 3 Fourier Transforms for some examples.`

Problem 1
15 Points
A waveform has the Fourier Series

$$f(t) = \frac{2}{\pi}(\cos t + \frac{\cos 3t}{3} + \frac{\cos 5t}{5} + \cdots).$$

(a) What is the fundamental frequency in radians/sec and Hertz and what is the fundamental period?

(b) What is the power in the third harmonic $\cos 3t$ term in watts if the signal is a voltage signal across a 1 Ohm resistor? Do the calculation by Parseval's Theorem.

($\pi^2 \approx 9.87$)

Problem 2 15 Points
Compare the Fourier transform for pulses of various widths τ centered at the origin. On the same plot, compare the transforms for $\tau = 4, 8, 16$ seconds. See the Example on Page 119 and Figure 3.14. Also, study Time Scaling on page 128-130.

Plot the sinc function result from -0.5 to 0.5 Hertz. To better see the frequency spectrum, divide the Fourier Transform by the width of the pulse τ for each width so the plot amplitudes are normalized to 1. Apply a grid and a legend to the plot to have a nice result.

The MATLAB function sinc will create the function in the form

$$sinc(x) = sin(\pi x)/\pi x$$

or just plot using sine form.

As a partial check the results, we expect zeros at frequencies that cause the sine function to be zero. For each τ, determine the zero locations on the frequency axis in Hertz. Then, check the plot to see if it is correct.

Problem 3 30 Points In Example 3.12 K&H show the Fourier Transform for the function

$$x(t) = e^{-at}\sin(b\pi t)u(t)$$

with $a = 2$ and $b = 2$ as

$$X(\omega) = \frac{2\pi}{4 - \omega^2 + 4\pi^2 + 4j\omega}.$$

(a) Take the Fourier Transform of $x(t)$ and prove their result is correct. Use the multiplication by $\sin(\omega t)$ property to help you.

(b) Run the symbolic program and plot $x(t)$ and $X(\omega)$ as shown in Figure 3.17.

Problem 4 20 Points
Taking the standard ac wave with $f = 60$ Hz written as

$$v(t) = 170\sin\omega t \quad \text{volts}$$

(a) Determine the period T in milliseconds and the radian frequency ω in rad/sec.

(b) Determine the peak-to-peak voltage of the wave.

(c) Determine the rms (root-mean-square) voltage of the wave. This is the square root of the power as defined by Equation 3.28, page 113. It is the equivalent direct-current voltage that would produce the same heating (Power dissipation) in a resistor as $P = V_{rms}^2/R$.

(d) Compute the average power of the wave over a period.

Problem 5 20 Points
Do K&H Problem 3.16(a). To do this, use superposition of two pulses as follows: $p(1)$ - height 2 and $p(2)$ height 1 but shifted in time from its even position by 1.5 seconds.

(a) Calculate the Fourier transform using the sinc results as shown in the text with the appropriate arguments and scaling for this problem.

(b) Write the complex exponential Fourier Series given the result in Part a. Compare the equations on Page 109 and the equations on Page 114, Eq. 3.30 to help understand the relationship.

Study Examples 3.13 and 3.14 to help.