Laplace and Z Homework 7 CENG 4331 Due Nov 30

Problem 1
10 Points
Compute the Laplace transform of the following functions by direct integration:

(a) $f(t) = 2t$

(b) $f(t) = t - 3$

(c) $f(t) = 2 \sin t$

Problem 2
10 Points
Now compute the Laplace transform of the following functions by using the theorems, i.e. the shifting properties and the linearity property and the previous results in Problem 1:

(a) $f(t) = 3te^{3t}$

(b) $f(t) = e^{-2t} \cos 4t$

Problem 3
10 Points
By partial fraction expansion, after checking that the expansion is correct by multiplying the factors together, find the function $f(t)$ corresponding to the Laplace transform:

$$F(s) = \frac{120s}{(s-1)(s+2)(s^2-2s-3)}.$$

Problem 4
10 Points
Determine the solution of the following initial value problems using Laplace transforms and check the answer:

(a) $\dfrac{d^2y}{dt^2} + 4y = 0, \quad y(0) = 0,\ y'(0) = 10.$

(b) $\dfrac{d^2y}{dt^2} + y = 2u(t) \quad y(0) = 0,\ y'(0) = 2.$

Problem 5
15 Points

Consider Figure 6.20 on K&H Page 340 and let

$$H_1(s) = \frac{s+3}{s-2}$$

and

$$H_2(s) = K = 4,$$

a constant for the feedback element. Using Laplace transforms and the inverse transform, compute the following

(a) The open-loop step response $y(t)$ for $H_1(s)$.

(b) The closed-loop step response of $H_1(s)$ in the loop with $H_2(s)$ as the feedback element. (Equation 6.136)

(c) Compare the time responses in the problems a and b above and discuss the effect feedback has had on the response in part b.

Z Homework CENG 4331

Review parts of Chapter 7 in the textbook.

A *geometric series* is a series with each term after the first being a fixed multiple of the preceding term. The multiplier is a real number r, called the *ratio*, so that $a_{n+1} = ra_n$. If the sum is taken from $n = 0$, the geometric series is represented as

$$\sum_{n=0}^{\infty} ar^n = a + ar + \cdots \quad (a \neq 0). \tag{1}$$

The series converges to the sum

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \tag{2}$$

if $-1 < r < 1$ but diverges if $|r| > 1$.

The nth partial sum for the geometric series is found by subtracting the terms

$$\begin{aligned} S_n - rS_n &= a + ar + \cdots + ar^n - \left(ar + ar^2 + \cdots + ar^{n+1}\right) \\ &= a - ar^{n+1}, \end{aligned}$$

so that $S_n - rS_n = a(1 - r^{n+1})$. Thus, solving for S_n leads to the result

$$S_n = \frac{a(1-r^{n+1})}{1-r}$$

for the sum of the first $n+1$ terms. Taking the limit as n goes to infinity with $|r| < 1$ shows that the sum of the series is $a/(1-r)$, as shown in Equation 2.

Power series are used extensively for computing or approximating values of functions. Suppose that a power series converges to the value of $f(x)$, so that

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + \cdots + a_n x^n + \cdots. \tag{3}$$

Such a series is called a *power series representation*, or a *power series expansion*, of $f(x)$. Alternatively, the function is said to be *represented*, or *expressed*, by the series. Although many functions of interest can be represented by a power series, there are many functions that cannot. If $f(x)$ can be represented by a power series in an interval, the function is termed *analytic* in the interval. The Taylor series is an important example of a power series.

Mathematically, the Z-transform is a rule by which a sequence of numbers is transformed into a function of the complex variable $z = x + iy$, which can be written

$$z = \rho \exp(i2\pi F) = \rho \exp(i\Omega).$$

If $\{f(n)\}$ is a sequence, then we write

$$\mathcal{Z}[f(n)] = F(z) \quad \text{and} \quad f(n) = \mathcal{Z}^{-1}[F(z)]. \tag{4}$$

The Z-transform is defined by the series

$$\begin{aligned}
\mathcal{Z}[f(n)] &= \sum_{n=0}^{\infty} f(n) z^{-n} \\
&= f(0) + \frac{f(1)}{z} + \frac{f(2)}{z^2} + \cdots \qquad (5) \\
&= f(0) + f(1)z^{-1} + f(2)z^{-2} + \cdots . \qquad (6)
\end{aligned}$$

This transform is typically used to analyze sequences for which $f(n) = 0$ for $n < 0$ so this transform is called the *unilateral*, or *single-sided* Z transform since the sum begins at zero.

Do the problems by hand unless otherwise indicated and **check** the results whenever possible. However, you may wish to verify your results with MATLAB solutions to the problems when appropriate. You can use symbolic MATLAB to check results if you have access to it.

Problem 6
10 Points
Sum the following series

(a) $1 + \frac{1}{2} + \frac{1}{4} + \cdots$

(b) $\pi + \frac{\pi}{\sqrt{2}} + \cdots + \frac{\pi}{\sqrt{2^n}} + \cdots$

Problem 7
10 Points
Write the power series for the following fractions

(a)

$$\frac{1}{1+x}$$

by direct polynomial division of 1 by $1 + x$

(b) Using the result of Part a write the power series for

$$\frac{1}{1-x}$$

Problem 8
10 Points
Find the first 4 values of $e(k)$ from the inverse z-transform by long division.

$$E(z) = \frac{z}{z^2 - 3z + 2}.$$

See K&H Example 7.15 Page 369.

Problem 9
15 Points
Solve the following difference equation using z-transforms:

$$y[n] - 0.5y[n-1] = 4u[n]; \qquad y[-1] = 0;$$

Use the z-transform method to determine $y[n]$. That is, take the z-transform of the equation and invert it to get $y[n]$. K&H Page 378, 379.