HW 2 5131 Fall 2014 Due Sept 8

September 2, 2014

Put your answers neatly on separate sheets. Include the MATLAB code and figures - with your name on the pages and the Course Number 5131.

Do the problems by hand and **check** the results whenever possible. However, you may wish to verify your results with MATLAB solutions to the problems when appropriate. You can use symbolic MATLAB to check results if you have access to it.

`Harman Chapters 1, 2, 6, 12 will be helpful.`

Problem 1 20 Points

Complex plane For the equation with z complex

$$\left|\frac{z}{z-1}\right| = 2,$$

what is the locus of points in the complex (xy) plane? Write the equation and describe the curve.

Problem 2 20 Points

1. Find the solutions to the equation $z^3 + 1 = 0$ and write the answers as both $a + ib$ and $|z|\angle\theta$.
2. Compute the solution by finding the cube root using MATLAB "roots" and multiply the roots together to check. Turn in the MATLAB program with comments.

Problem 3 20 Points

(a) Prove the relationship that the sum of the sinusoids can be written as a shifted cosine in the expression

$$a\cos\theta + b\sin\theta = c\ \cos(\theta - \alpha)$$

where

$$c = \sqrt{a^2 + b^2}, \quad \tan\alpha = \frac{b}{a}.$$

(b) If we associate the vector components a with $a\cos\omega t$ and $-jb$ with $b\sin\omega t$, show that the magnitude and phase of the equation

$$a\cos\theta + b\sin\theta \rightleftharpoons a - jb$$

yields the same result as in part (a) using the vector representation. What is the justification for this use of a vector to represent the trigonometric functions?

(c) Transform $4\cos\omega t + 3\sin\omega t$ to the shifted cosine form.

Problem 4
Easy Ones 20 points

1. Convert decimal to binary using MATLAB commands and convert back.
2. MATLAB has commands (int,etc) to create computer numbers. Convert 325.499 to a 16-bit integer. (Hint: Commands are like those data types in C.)
3. Convert 0.3891 to 8-bit binary by hand. (MATLAB does integers).
4. Does the *harmonic* series

$$\sum_{n=1}^{n=\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$$

converge or diverge?

Problem 5 20 Points
Euler's formula states that $e^{i\theta} = \cos\theta + i\sin\theta$. Do the following:

(a) Derive the Taylor series for $e^{i\theta}$, $\cos\theta$, and $\sin\theta$ and show Euler was right. (Review Sections 6.2 and 6.3 in text.)

(b) Writing Euler's formula for $e^{i\theta}$ and $e^{-i\theta}$, show that

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

and

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}.$$