HW 4 5131 Fall 2014 Due Sept 29

September 22, 2014 Do the problems by hand and **check** the results whenever

possible by substituting the solution into the differential equation and also verifying the initial conditions. You may wish to verify your results with MATLAB solutions to the problems when appropriate. You can use symbolic MATLAB to check results.

`Harman Chapter 5 will be helpful.`

Problem 1 10 Points if checked
Solve the equation

$$\dot{y}(t) + 5y(t) = 50 \quad \text{with} \quad y(0) = y_0$$

by the following two methods:

1. Integrating factor method;
2. Method of undetermined coefficients.

Problem 2 30 Points

(a) Solve the equation by hand

$$\ddot{\theta}(t) + 3\dot{\theta}(t) + 2\theta(t) = f(t), \qquad t \geq 0. \tag{1}$$

with initial conditions $\theta(0) = 1, \dot{\theta}(0) = 0$ and $f(t) = 0$. Plot the solution with MATLAB for $[0, 5]$ seconds. Be sure to add comments to the program and label the plot.

(b) Let $f(t) = U(t)$ with the same initial conditions and plot the step response.

Problem 3 20 Points if checked
Write the solutions of the 2nd order differential equation

$$y''(x) + y' - 2y(x) = xe^x.$$

in terms of arbitrary constants c_1 and c_2.

Problem 4 20 Points if checked and plotted
Let the physical components have the following properties

$$\begin{aligned} m &= 1 \text{ kilogram}, \\ \beta &= 4 \text{ newton-seconds/meter}, \\ k &= 40 \text{ newtons/meter}, \end{aligned}$$

where the International System of Units (SI) is used to define the mass m in kilograms, the viscous friction β in newton-seconds/meter, and the spring constant k in newtons/meter. The forcing function is $f(t) = mg$, where g is the force of gravity (9.81 meters per second^2). The solution will be given as displacement measured in meters.

Dividing the equation by the value of m yields

$$\ddot{y}(t) + 4\dot{y}(t) + 40y(t) = 9.81, \qquad t \geq 0. \tag{2}$$

Solve this equation using Symbolic MATLAB and simplify the answer to show that it agrees with Harman Page 257 Equation 5.88.

Problem 5 20 Points if checked and plotted
Given the differential equation representing a *critical damping* situation

$$\ddot{x}(t) + 16\dot{x}(t) + 64x(t) = 0$$

a. Solve it by hand with $x(0) = 1/3$ and $\dot{x}(0) = 0$.

b. $x(0) = 0$ and $\dot{x}(0) = 5$

c. Plot both cases and discuss the resulting motion if this is a mass, spring, damper problem.