HW 7 Fourier Series and Transform 5131 Due Oct 27 2014

Do the problems by hand and **check** the results whenever possible. However, you may wish to verify your results with MATLAB solutions to the problems when appropriate. You can use symbolic MATLAB to check results if you have access to it.

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Harman Chapter 5, 6 and 8 will be helpful.
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Problem 1 20 Points
Consider the even, periodic pulse train in Figure 8.6 in my book or on the WEB. This is an important test signal in electronics and the signal is also of interest because of the characteristics of its Fourier components. The period is T, the amplitude is A, and the pulse has duration τ in each period. The function is an even function with average value over a period of $A\tau/T$.

The Fourier series for the pulse train is

$$f(t) = \frac{A\tau}{T} + 2\frac{A\tau}{T} \sum_{n=1}^{\infty} \text{sinc}(n\omega_0\tau/2)\cos(n\omega_0 t).$$

The sinc function here
$$\text{sinc}\, x = \frac{\sin x}{x}.$$

This is not the convention used in the MATLAB *Signal Processing Toolbox* for the function **sinc**. There the x is divided by π on the right-hand side.

Problem: Write the Fourier Series for a similar pulse train shifted to the right by t_0 seconds.

Problem 2 20 Points

For the time function

$$x(t) = \alpha + \beta \cos(2\pi f_0 t) + \gamma \sin(2\pi f_0 t)$$

use the equations and determine the Fourier Series.

Problem 3
10 Points

(a) Show that if $x(t)$ has the Fourier transform $X(\omega)$

$$tx(t) \text{ has transform } j\frac{dX(\omega)}{d\omega}$$

by differentiating both sides of the Fourier Transform definition with respect to ω and multiplying by j.

(b) Using this property find the Fourier transform of the function

$$x(t) = 2te^{-2t}u(t)$$

Problem 4 20 Points
Fourier series of clock signal Consider the computer clock signal shown in the Figure, with a pulse rate of 8 million pulses per second ($f_c = 8$ Megahertz) and amplitude of 4 volts and a pulse width of 0.05 microseconds. NOTE: The figure does not show the signal to scale.

1. Find the Fourier series by hand calculation using the basic definitions of the coefficients.

2. Using MATLAB plot the Fourier amplitude spectrum in Hertz of the clock signal as discrete values up to 80 Megahertz.

3. Using MATLAB Plot $f(t)$ in seconds from the Fourier Series you calculated for 6 periods of the pulses.
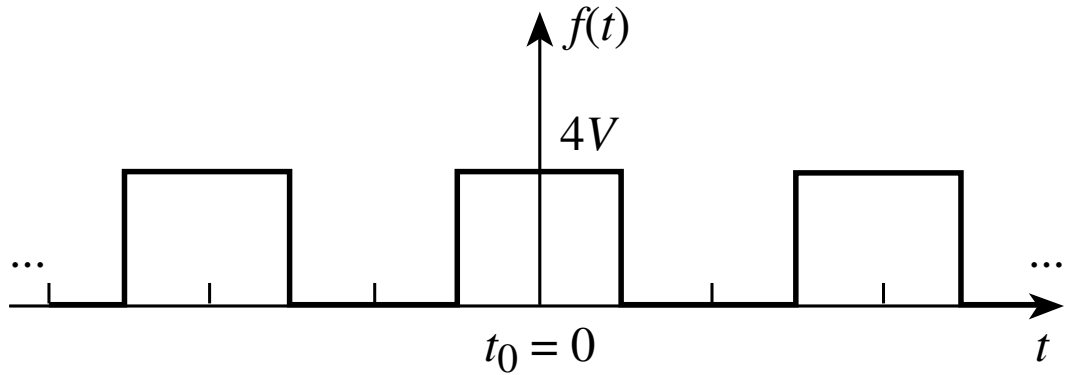

Figure 1: Clock Signal

You can write the series for many pulse trains by letting the period be T, the pulse width be τ and the amplitude be A and then computing the Fourier Series. Then, just plug in the numbers.

Problem 5
30 Points
Consider the system described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = P(t),\tag{1}$$

where $P(t)$ is the pulse

$$P(t) = \begin{cases} A, & 0 \le t \le \tau, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Write the Fourier Transform of the Pulse.

(b) Determine the transfer function $H(j\omega)$ of the system described by the differential equation 1.

(c) Determine the result $|Y(j\omega)|$ if the input is the pulse with $A = 1$ and $\tau = 1$. What is $|Y(0)|$?