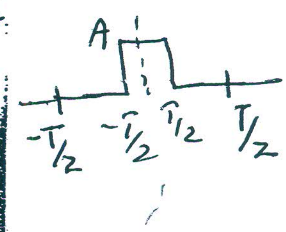
RELATIONSHIP BETWEEN F.S. AND F.T!

```
% Pulse plot and Pulse Train  Equal amplitudes and pulse widths
% The Pulse train frequencies samples the pulse specturm
% Harman pages 389, 401-403; K&H pages 112 and 119-120
clear, clf
A=1, tau=1, T0=2; %(Expect X=0 when n= 1,2,... w*tau=n*2*pi)
w=[0-5*pi:.005:5*pi];   % Plot from w=0 to 5*pi rad/sec
w=w+eps
Wpulseft= (A*tau)*sin(w*tau/2)./(w*tau/2);
figure(1)
wzero=zeros(size(w));   % Put in a zero line and plot f Hz.
plot(w,Wpulseft,w,wzero)
xlabel('\omega radians/sec'),ylabel('F(\omega)'),grid
title('Fourier Transform of pulse A=1, \tau=1')
% Divide the Fourier Transform of the pulse by T0
Wpulse= (1/T0)*(A*tau)*sin(w*tau/2)./(w*tau/2); %
wzero=zeros(size(w));   % Put in a zero line and plot f Hz.
% Add the pulse train spectrum T=2
% Form n*w0 for the pulse train w0=2*pi/T0=pi
nw0=pi*[-5:1:5];
nw0=nw0+eps
Ftrain= (A*tau/T0)*sin(nw0*tau/2)./(nw0*tau/2); %A/(n*pi*T0)
figure(2)
plot(w,Wpulse,w,wzero), hold
stem(nw0,Ftrain), xlabel('\omega radians/sec')
ylabel('Pulse and pulse train')
title('The pulse train spectrum samples the pulse spectrum/T0')
gtext(' A=1, tau =1, T0=2 for the pulse train'),gtext('\pi'),grid
hold off
```

RELATIONSHIP TO FOURIER SERIES

Comparing the coefficients of the Fourier series of Example 8.7 for a periodic pulse train of rectangular pulses and the Fourier transform of Example 8.11 for a single pulse shows that the series coefficients are

$$\alpha_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-in\omega_0 t}\,dt = \frac{A\tau}{T}\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}$$



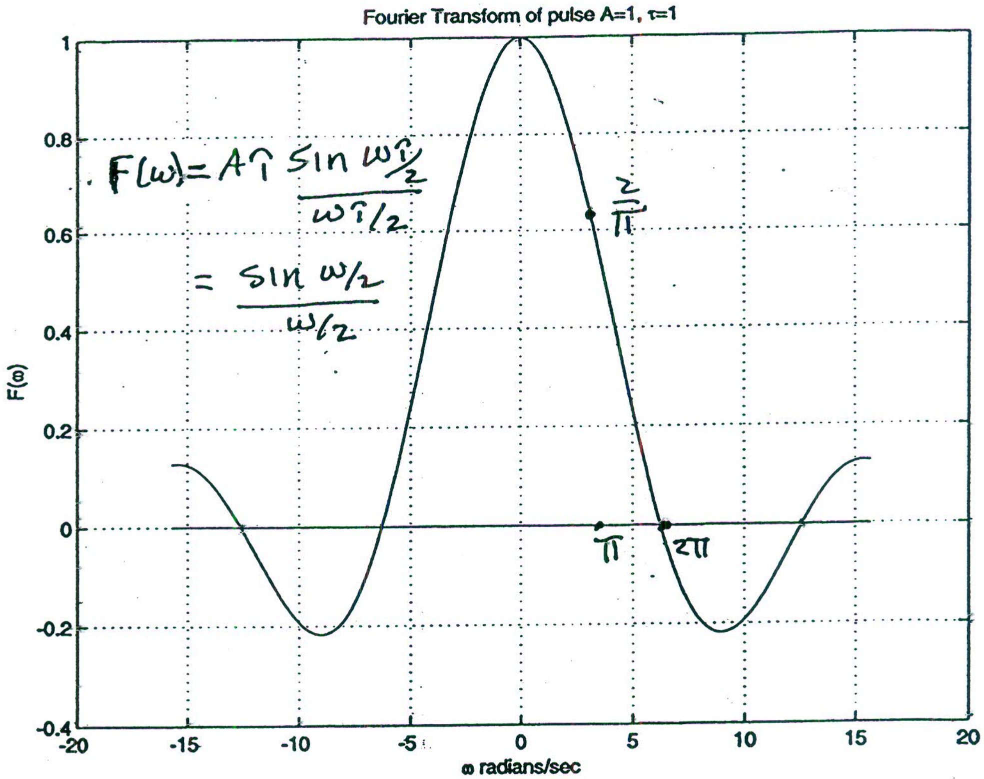
and the transform is

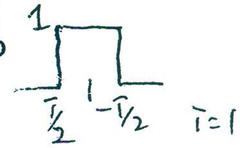
$$\mathcal{F}[f(t)] = F(i\omega) = \int_{-\tau/2}^{\tau/2} f(t)e^{-i\omega t}\,dt = A\tau\frac{\sin(\omega\tau/2)}{\omega\tau/2}.$$

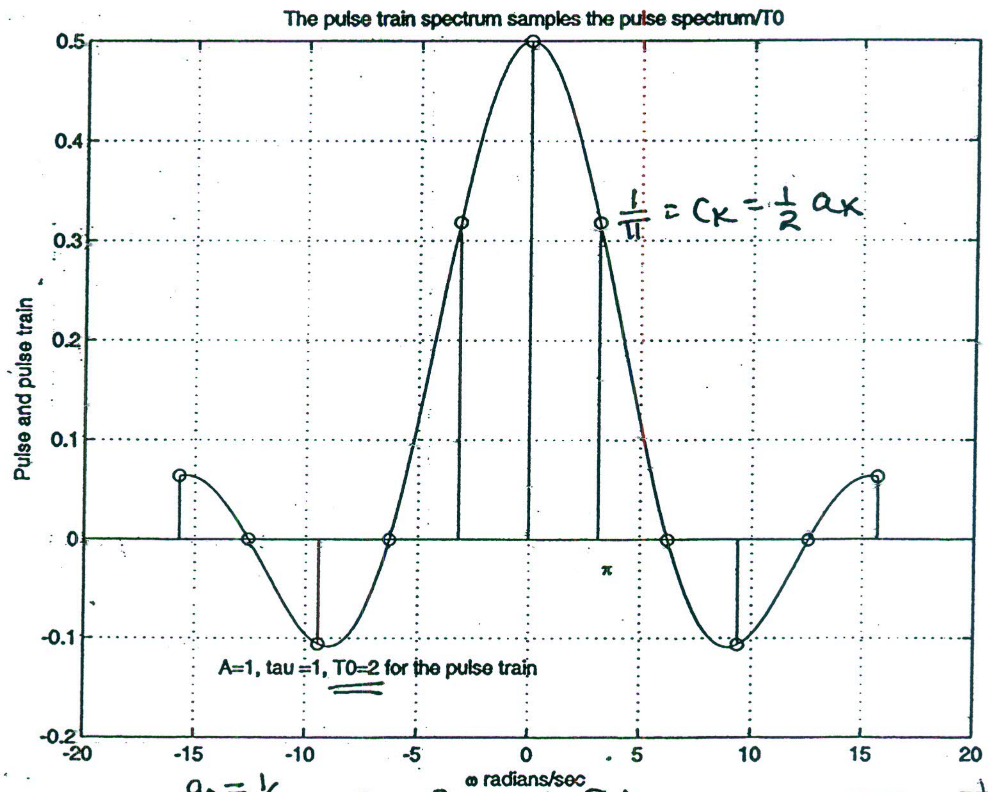
By comparing the two results, it is clear that designating the transform $F(i\omega) = \mathcal{F}[f(t)]$,

$$\frac{F(n\omega_0)}{T} = \frac{A\tau}{T}\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}.$$

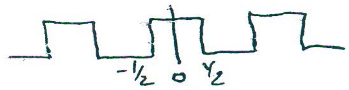
Thus, we conclude that the Fourier series coefficients are obtained by *sampling* the Fourier transform at the points $n\omega_0$ and dividing by the period T. However, the Fourier series itself is a continuous function of time, but the Fourier transform is a function of ω in the frequency domain.







$a_0 = 1/2 \qquad a_k = \frac{2}{\pi} \sin\left(\frac{n\pi}{2}\right)$



$$\text{FS} \quad f(t) = \frac{A\tau}{T_0} + \frac{2A\tau}{T_0} \sum \frac{\sin n\omega_0 \tau/2}{n\omega_0 \tau/2} \cos(n\omega_0 t) \qquad \omega_0 = \frac{2\pi}{T_0} = \pi$$

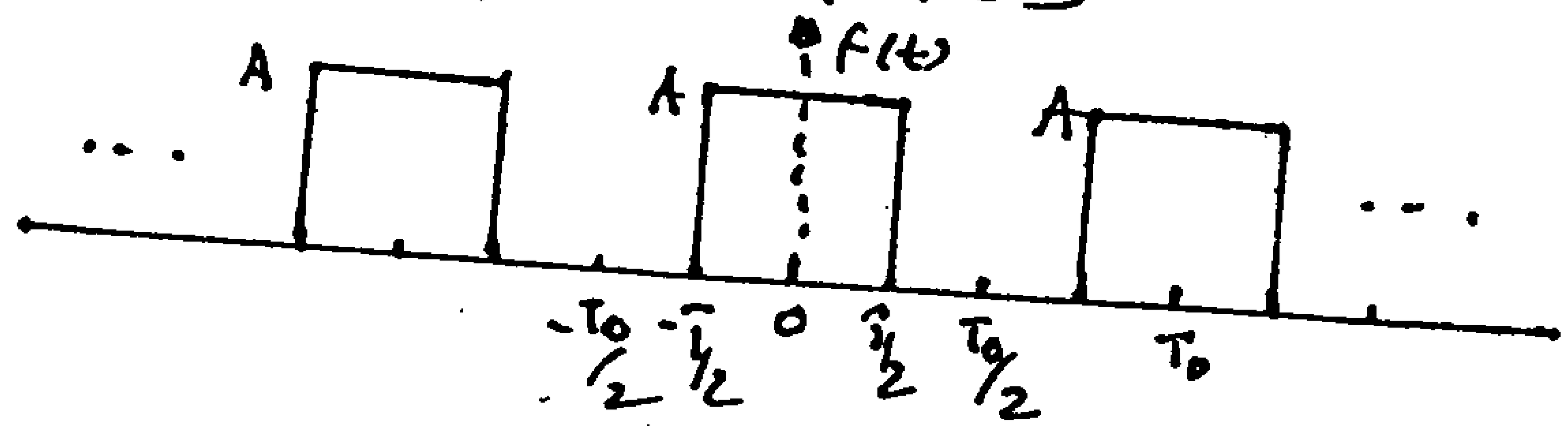
$A = 1, \tau = 1$

$$= \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \qquad c_0 = a_0 \qquad c_K = \frac{1}{2} a_K$$

FOURIER PROPERTIES

Roberts p231
HARMAN p389
K+H p101

I TIME SHIFT IN SERIES



EVEN PULSE TRAIN

SERIES

$$f(t) = \frac{A\tau}{T_0} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi}{T_0} t\right) \qquad \omega_0 = \frac{2\pi}{T_0}$$

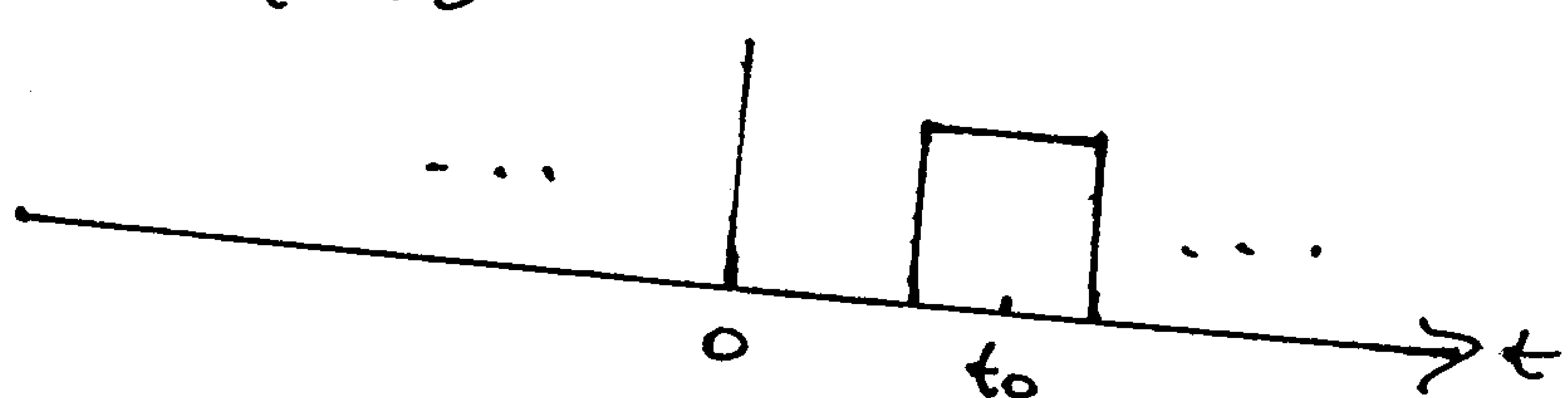
$$a_n = \frac{2}{T_0}\int_{-\tau/2}^{\tau/2} A\cos\left(\frac{2n\pi}{T} t\right) = \frac{2A}{n\pi}\sin\left(\frac{2n\pi\tau}{2T_0}\right)$$

$$= \frac{2A\tau}{T_0}\left[\frac{\sin\left(\frac{2n\pi\tau}{2T_0}\right)}{\left(\frac{2n\pi\tau}{2T_0}\right)}\right]$$

Thus

$$f(t) = \frac{A\tau}{T_0} + \frac{2A\tau}{T_0}\sum_{n=1}^{\infty}\frac{\sin(n\pi\tau/T_0)}{\left(\frac{n\pi\tau}{T_0}\right)}\cos\left(\frac{2n\pi t}{T_0}\right) \qquad (1)$$

SHIFT BY t_0



$$f(t) = \frac{A\tau}{T_0} + \frac{2A\tau}{T_0}\sum\frac{\sin(n\pi\tau/T_0)}{\left(\frac{n\pi\tau}{T_0}\right)}\cos\left[\frac{2n\pi}{T_0}(t-t_0)\right]$$

EVEN WRT $(t-t_0)$ (2)

I CONT. CONSIDER THE TIME SHIFT THEOREM FOR SERIES

$$x(t-t_0) \overset{FS}{\longleftrightarrow} e^{-j2\pi k t_0/T_0}\left[C_k\right]$$

Where C_k are the complex EXPONENTIAL COEFFICIENTS

$$C_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-jk\left(\frac{2\pi t}{T_0}\right)} dt \quad k=0, \pm 1, \dots$$

Note $C_k = \frac{1}{2}(a_k - jb_k)$; $C_{-k} = \frac{1}{2}(a_k + jb_k)$ $k=1,2,\dots$

So $$C_k = \frac{A\tau}{T_0}\,\frac{\sin\left(n\frac{2\pi\tau}{2T_0}\right)}{n\frac{2\pi}{T_0}\cdot\tau/2} \times e^{-jk\frac{2\pi t_0}{T_0}}$$

USING (1), THE CHANGE IS

$$\cos\left(\frac{2k\pi t}{T_0}\right) \times e^{-jk\frac{2\pi t_0}{T_0}} = \left[\frac{e^{jk\omega_0 t} + e^{-jk\omega_0 t}}{2}\right] e^{-jk\omega_0 t_0}$$

This is

$$\cos\left[\frac{2k\pi}{T_0}(t-t_0)\right]$$

and letting $k=n$

we have (2)