Harman Outline 1B CENG 5131 PDF

D. Polynomials (Harman P 160)
Review Harman Pages 160-161.

Let $a_0, a_1, \ldots, a_n$ be $n+1$ arbitrary numbers with $a_n \neq 0$. Then, the function

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0 \tag{1}$$

is a *polynomial* of degree n. The $n+1$ constants $a_0, a_1, \ldots, a_n$ are the *coefficients* of the polynomial. A polynomial is a *real polynomial* if all its coefficients are real numbers. This text considers only polynomials with real coefficients unless otherwise stated, because these are associated with mathematical models of physical systems.

The numbers z that are solutions to the equation

$$P(z) = 0 \tag{2}$$

are called the *roots* or sometimes the *zeros* of the polynomial. The values of the roots are not necessarily real numbers. Thus, a root z may have the form $z = x + iy$, where i is the imaginary number $\sqrt{-1}$. In electrical engineering problems, this is often written j so that no confusion would result with the current if it is designated by i. As described in Chapter 2, the number $\bar{z} = x - iy$ is the complex *conjugate* of z. The notation z^* is also used to designate the complex conjugate of z.

Important properties of *real* polynomials and their roots are as follows:

1. A polynomial of degree $n \geq 1$ has n roots.
2. A polynomial of odd degree has at least one real root.
3. If z is a complex root of a real polynomial, then the complex conjugate $\bar{z}$ is a root also.

The polynomial $P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0$ can always be written in the form

$$P(z) = (z - z_1)(z - z_2) \cdots (z - z_n) a_n \tag{3}$$

as the product of linear factors using the roots $z_i, i = 1, 2, \ldots, n$ of

$$P(z) = 0.$$

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MATLAB roots and poly

>> %p= x^3-7x^2+40x-34

>> r=roots([1 -7 40 -34])

r =

   3.0000 + 5.0000i
   3.0000 - 5.0000i
   1.0000

>> p=poly(r)

p =

    1.0000   -7.0000   40.0000  -34.0000
```

See also **conv** and **deconv** Harman p431 and **polyval** Harman p350.

Polynomial fit (7.1, P351) Go over Example 7.1 Harman P353.

Handout 4. Functions, Sequences, Series (Harman P 276)
Review Harman Pages 276-277,282-296.

1. Continuous functions definition P276
2. Sequences Section 6.2 P 282
3. Infinite Series P 284-287
4. Geometric Series P 287
5. Series of functions P 288
6. Power Series P 290

Taylor Series (6.3, P292)
After a review of Taylor series, we can visit Euler's formula again using results in Table 6.3 P295. Consider the Taylor series

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \cdots$$

Collecting terms using the fact that $i^2 = -1$, $i^3 = -i$, and so on yields

$$\begin{aligned} e^{i\theta} &= \left(1 - \frac{(\theta)^2}{2!} + \frac{(\theta)^4}{4!} - \cdots\right) + i\left(\theta - \frac{(\theta)^3}{3!} + \frac{(\theta)^5}{5!} - \cdots\right) \\ &= \cos\theta + i\sin\theta \end{aligned} \tag{4}$$

Notice that in all the Taylor series examples, if the argument such as θ in the expansions for sin and cos only a few terms may be needed to satisfy the precision requirements of a problem. For example, if θ is 0.05 radians (about 3 degrees),

$$\sin(0.05) = 0.0500$$

to four decimal places. In other words, $\sin x \approx x$ when x is small.

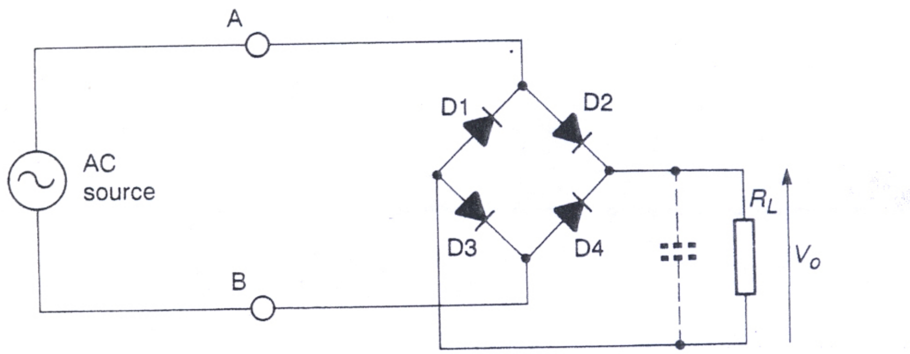

Figure 1: Caption for FullWaveRectscan0001

EXAMPLE Series Output of a Full Wave Rectifier

When the diodes are reversed biased by the voltage V_p on the capacitor, the voltage across the resistor and capacitor are equal so that

$$i = \frac{v}{R} = C\frac{dv}{dt}$$

The differential equation

$$\frac{dv}{dt} + \frac{v}{CR} \text{ with initial conditions } v(0) = V_P$$

The solution is $v(t) = V_p e^{\lambda t}$ with λ the solution to the *characteristic equation* $\lambda + 1/RC = 0$ as described in Harman p216. The decay in the voltage from the peak is computed by expanding the exponential and evaluating the first terms at $t = T/2$ to yield

$$V_{min} = V_p \exp^{(\frac{-t}{RC})} \approx V_p \left(1 - \frac{T/2}{RC}\right) = V_p \left(1 - \frac{1}{2fRC}\right) \quad (5)$$

The figures illustrate the problem. The idea is to determine the ripple.

COGSDELL

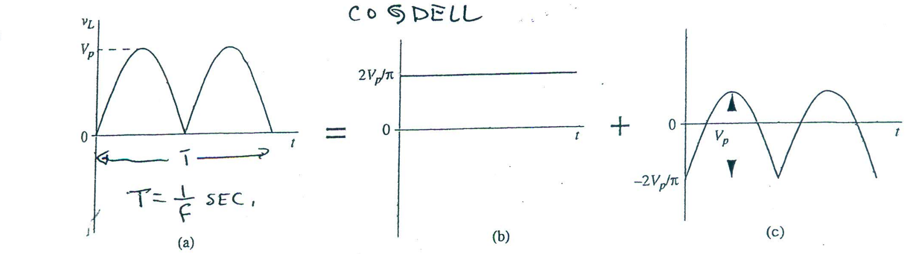

Figure 2.11 The ripple is the undesirable portion of the output. (a) Rectifier output; (b) dc component; (c) ripple component.

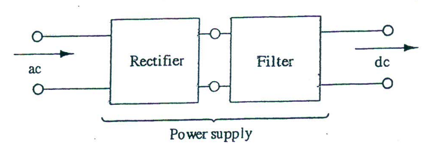

Figure 2.12 A filter is used to reduce the ripple.

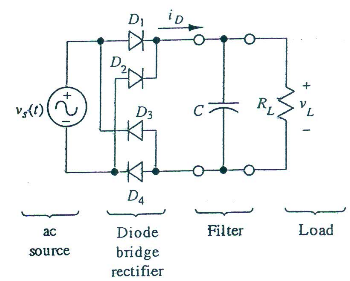

Figure 2.13 Full-wave bridge rectifier with a capacitor filter.

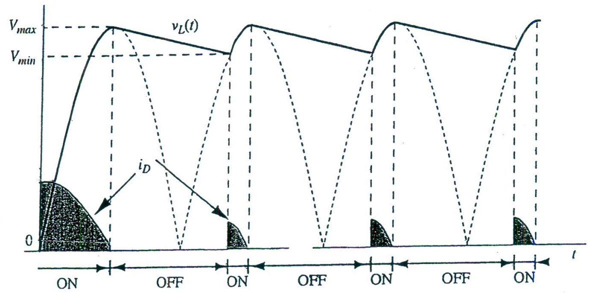

Figure 2.14 Waveforms for the full-wave rectifier with a capacitor filter.

Figure 2: Caption for 5131FullWaveCogdell0001

The input wave in is a 120 volt rms, 60 Hertz sine wave so that $v(t) = 120\sqrt{2}\sin(2\pi 60t)$. The time constant for the decay is

$$\tau = RC = 100 \times 2000 \times 10^{-6} = 200\ ms$$

which is long compared to the period of the sine wave at $T = 1/60 =$ 16.67 ms. Using the values in the series yields a value of about 0.04 for the exponent at $t = T/2$ which indicates that the approximation to V_{min} is reasonable.

EXAMPLE 2.3 **Filtered full-wave rectifier**

Find the ripple and dc voltage out of the filtered full-wave rectifier in Fig. 2.16.

f = 60Hz

$T = \frac{1}{60} = 16.67\,mV$

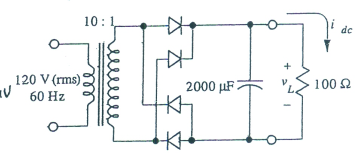

Figure 2.16 Full-wave rectifier circuit with a capacitor filter.

SOLUTION:

The input ac to the rectifier has an rms voltage of ~~12 V~~ 120 at 60 Hz. Thus, the peak value is $12\cancel{0}\sqrt{2} = \cancel{17.0}$ V. The time constant is

170V

OR 120V × $\sqrt{2}$ ~170 VOLTS

$$\tau = R_L C = 100 \times 2000 \times 10^{-6} = 200 \text{ ms} \tag{2.14}$$

which is long compared to the period of 16.7 ms. Thus the approximate analysis is valid. The maximum voltage across the load is $12\cancel{0}\sqrt{2} = \cancel{17.0}$ V. The minimum voltage is given by Eq. (2.11):*

170

$$V_{\min} \approx 170\left[1 - \frac{1}{2 \times 60(0.2)}\right] = 163V = 170\left(1 - 0.04\right) \tag{2.15}$$

170

Thus, the ripple voltage is $\cancel{17.0} - 163 = \cancel{0.7}\,7$ V peak-to-peak and the dc (time-average) voltage at the load is the average between the maximum and minimum, ~~16.6~~ 166 V. The average dc current in the load is ~~16.6~~ 166V/100Ω V divided by the load resistance, or ~~166 mA~~. 1.66 AMPS

WHAT IF? What if the full-wave rectifier is replaced by a half-wave rectifier?[3]

* OUR EQUATION 5

Figure 3: Cogdell FullWave Ex2_14