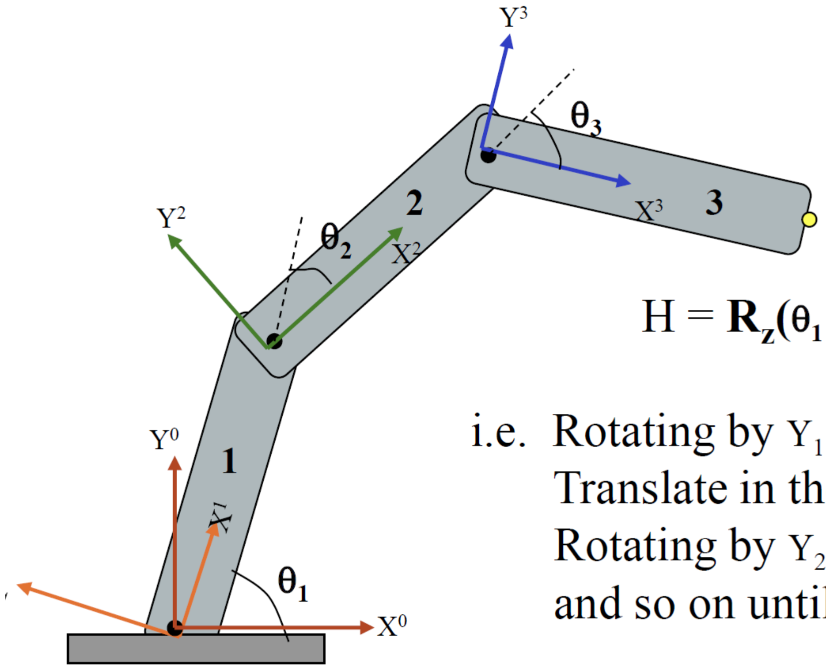
Kinematics and IK

REF: Howie An Introduction to Robot Kinematics.pdf

Example Problem:

You are have a three link arm that starts out aligned in the x-axis. Each link has lengths l_1, l_2, l_3, respectively. You tell the first one to move by Y_1, and so on as the diagram suggests. Find the Homogeneous matrix to get the position of the yellow dot in the X^0Y^0 frame.



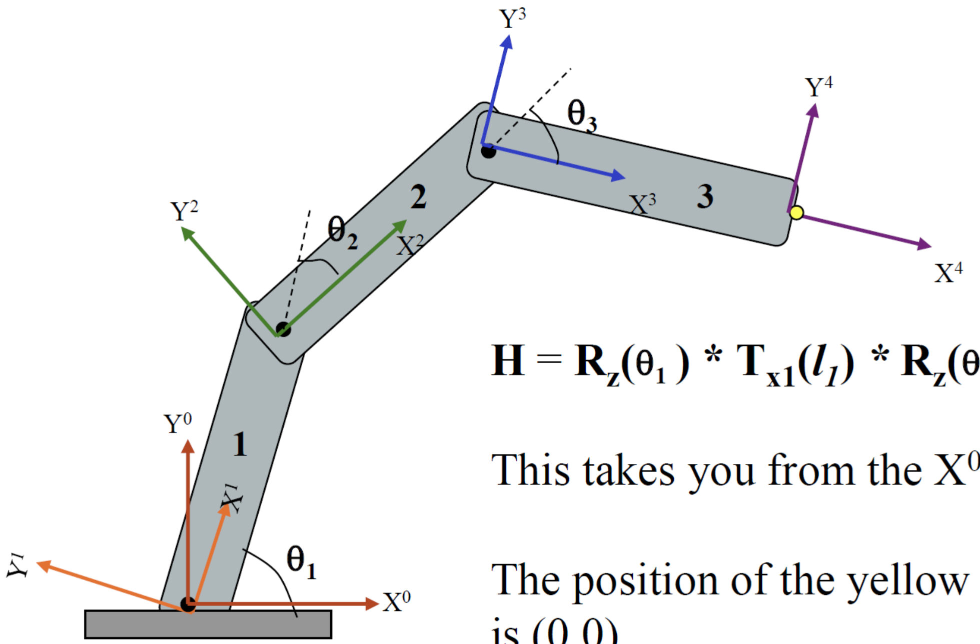
$$H = \mathbf{R_z(\theta_1) * T_{x1}(\mathit{l_1}) * R_z(\theta_2) * T_{x2}(\mathit{l_2}) * R_z(\theta_3)}$$

i.e. Rotating by Y_1 will put you in the X^1Y^1 frame.
Translate in the along the X^1 axis by l_1.
Rotating by Y_2 will put you in the X^2Y^2 frame.
and so on until you are in the X^3Y^3 frame.

The position of the yellow dot relative to the X^3Y^3 frame is $(l_1, 0)$. Multiplying H by that position vector will give you the coordinates of the yellow point relative the the X^0Y^0 frame.

Slight variation on the last solution:

Make the yellow dot the origin of a new coordinate X^4Y^4 frame



$$\mathbf{H} = \mathbf{R}_z(\theta_1) * \mathbf{T}_{x1}(l_1) * \mathbf{R}_z(\theta_2) * \mathbf{T}_{x2}(l_2) * \mathbf{R}_z(\theta_3) * \mathbf{T}_{x3}(l_3)$$

This takes you from the X^0Y^0 frame to the X^4Y^4 frame.

The position of the yellow dot relative to the X^4Y^4 frame is (0,0).

$$\begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \\ \mathbf{Z} \\ \mathbf{1} \end{bmatrix} = \mathbf{H} \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \end{bmatrix}$$

Notice that multiplying by the (0,0,0,1) vector will equal the last column of the H matrix.

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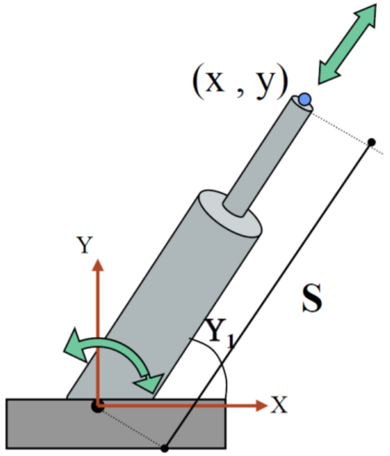
A Simple Example

Revolute and Prismatic Joints Combined



Finding **Y**:

$$\theta = \arctan(y, x)$$

More Specifically:

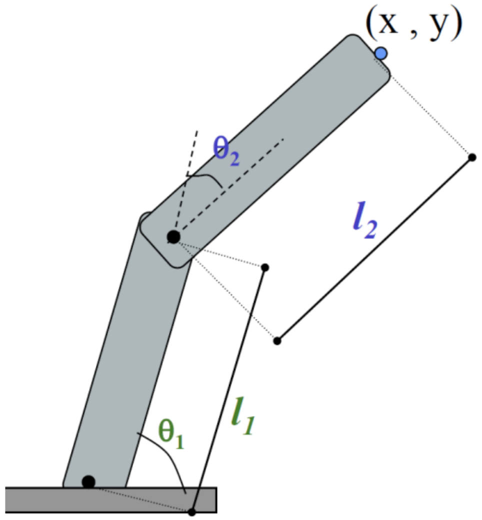
$$\theta = \arctan2(y, x)$$

arctan2() specifies that it's in the first quadrant

Finding **S**:

$$S = \sqrt{(x^2 + y^2)}$$

Inverse Kinematics of a Two Link Manipulator

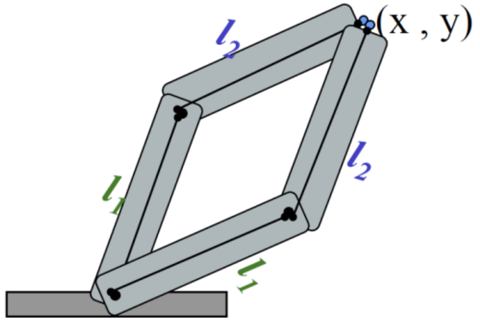


Given: l_1, l_2, x, y

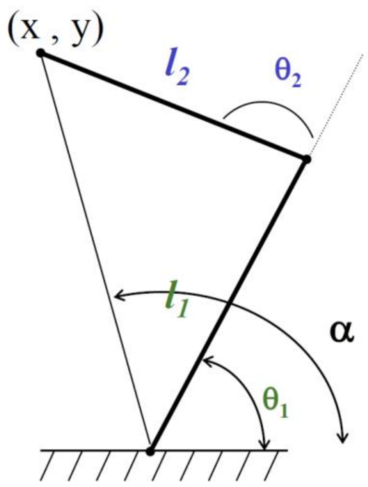
Find: Y_1, Y_2

Redundancy:

A unique solution to this problem does not exist. Notice, that using the "givens" two solutions are possible. Sometimes no solution is possible.



The Geometric Solution



Using the Law of Cosines:

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin \bar{\theta}_1}{l_2} = \frac{\sin(180-\theta_2)}{\sqrt{x^2+y^2}} = \frac{\sin(\theta_2)}{\sqrt{x^2+y^2}}$$

$$\theta_1 = \alpha - \bar{\theta}_1$$

$$\alpha = \arctan 2\left(\frac{y}{x}\right)$$

Using the Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$(x^2+y^2) = l_1^2 + l_2^2 - 2l_1l_2\cos(180-\theta_2)$$

$$\cos(180-\theta_2) = -\cos(\theta_2)$$

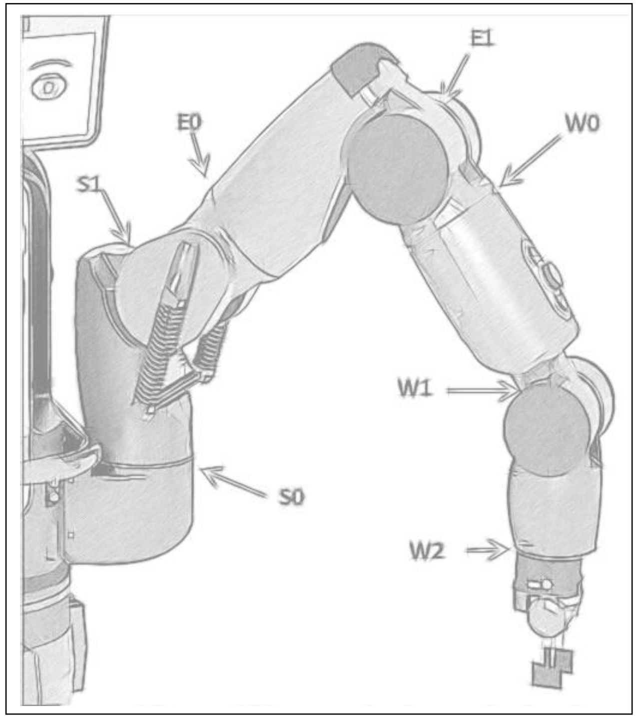
$$\cos(\theta_2) = \frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}$$

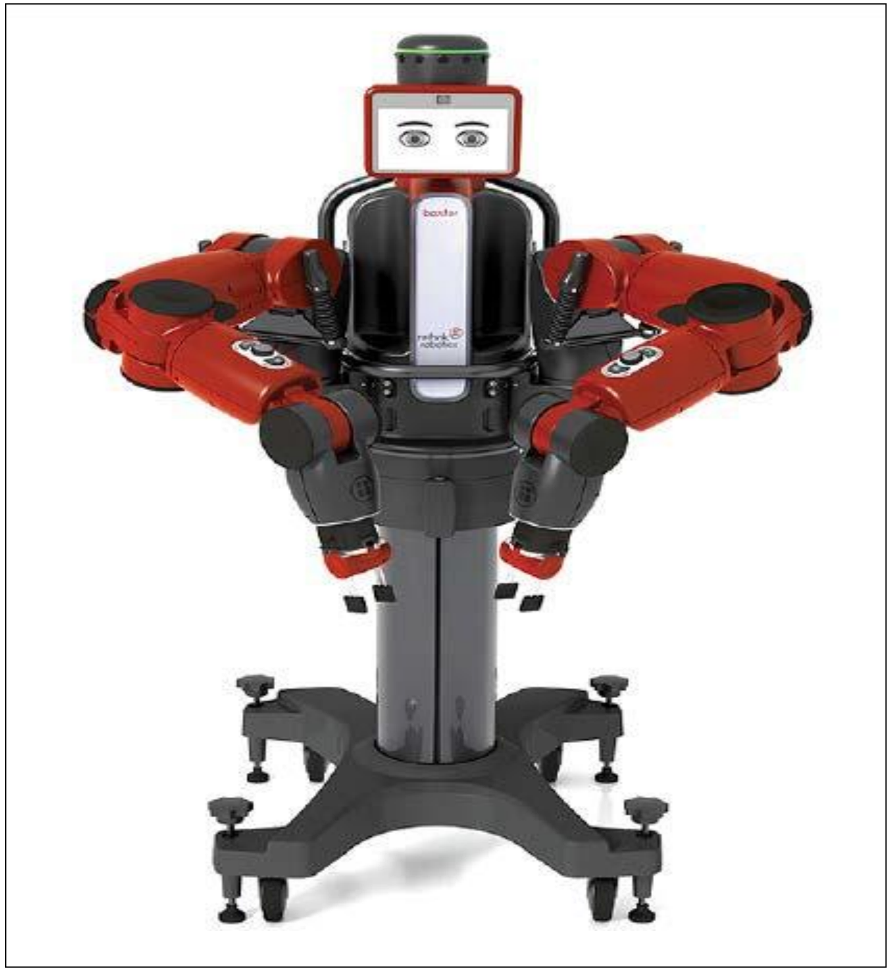
$$\theta_2 = \arccos\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)$$

Redundant since θ_2 could be in the first or fourth quadrant.

Redundancy caused since θ_2 has two possible values

$$\theta_1 = \arctan 2(y, x) - \arcsin\left(\frac{l_2\sin(\theta_2)}{\sqrt{x^2+y^2}}\right)$$


E1
E0
W0
S1
W1
S0
W2

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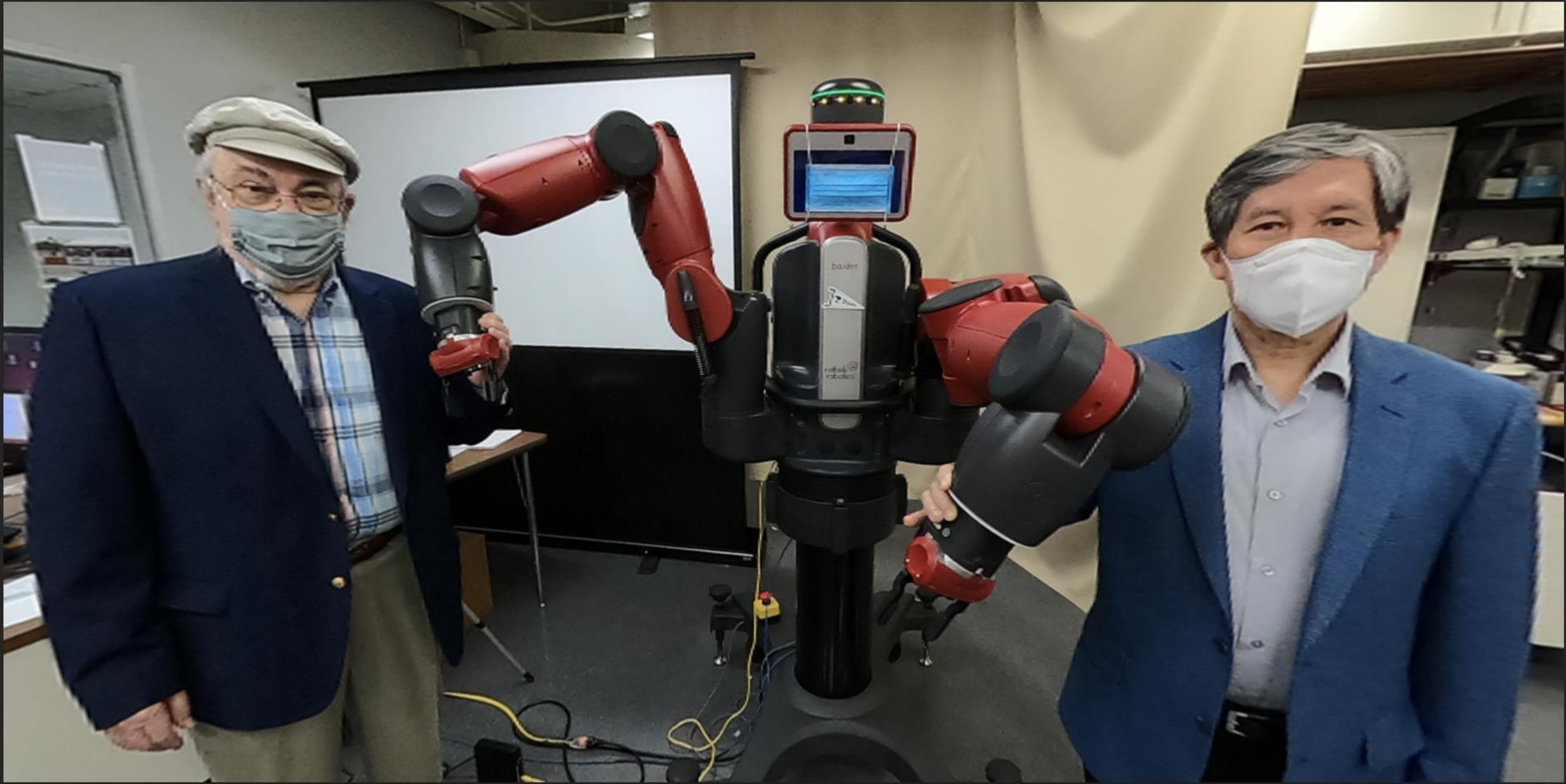
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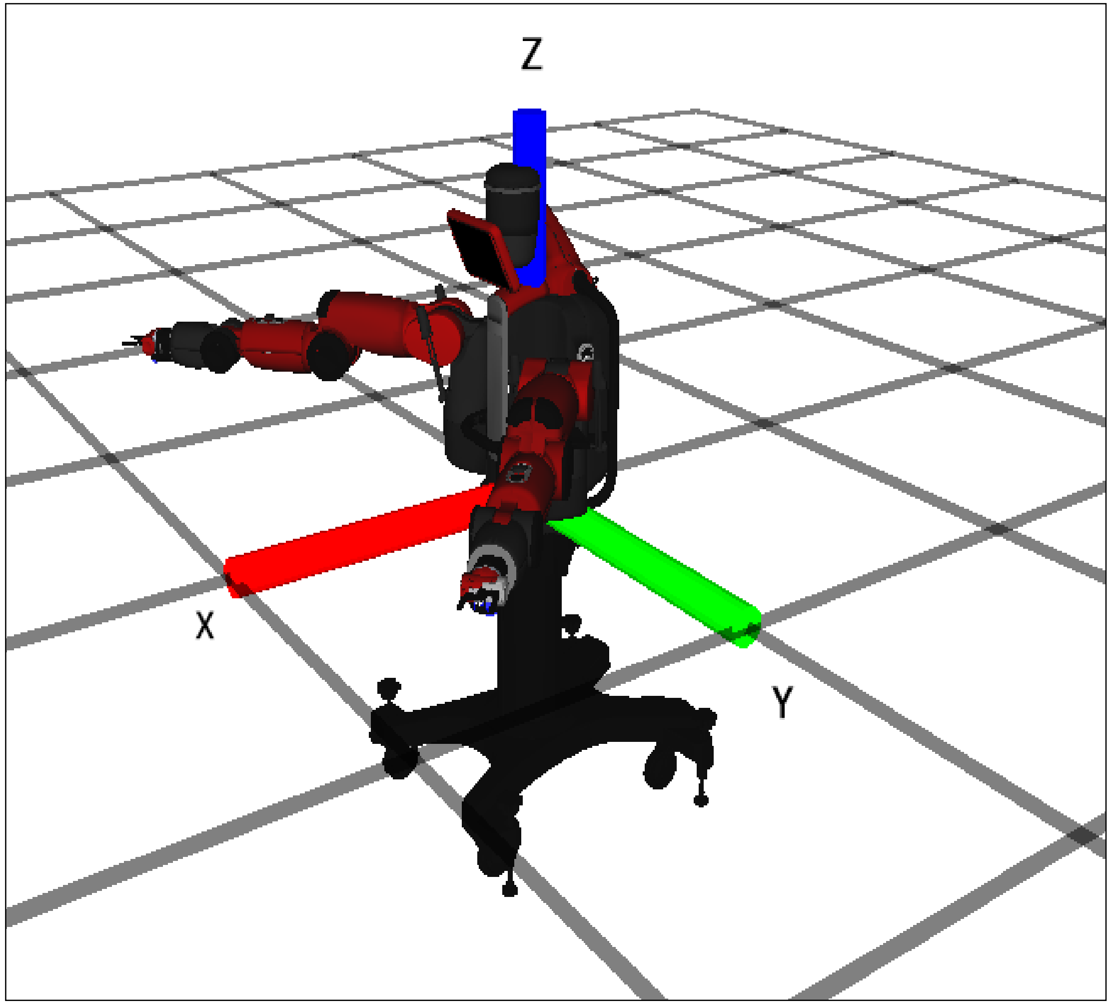
Z
X
Y