EXAMPLE 5.5 Automobile proportional speed control

Consider the automobile traveling on a straight, hilly road shown in Figure 5.20. There are three forces acting on the automobile: the forward thrust produced by the engine and transmitted through the tires (or the braking force if negative) (F_e), the aerodynamic force (including wind) (F_w), and the tangential component of gravity as the automobile climbs and descends hills (F_h). Applying Newton's second law, the equation of motion of the automobile can be written

$$m\ddot{x} = F_e - F_w - F_h$$

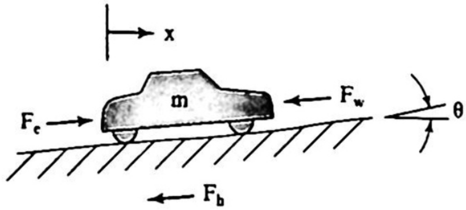

FIGURE 5.20: Automobile on a hilly road

where m represents the mass of the automobile and x the distance traveled. F_e must have upper and lower bounds. The upper bound is the maximum force that the engine can transmit through the wheels to the road, and the lower bound is the maximum braking force. We will assume that $-2000 \le F_e \le 1000$, with units of lb, and that the mass is 100 slugs.

The aerodynamic force is the product of the drag coefficient (C_D), the automobile's frontal area (A), and the dynamic pressure (P), where

$$P = \frac{\rho V^2}{2}$$

and ρ represents the air density and V the sum of the automobile speed and wind speed (V_w). Assume that

$$\frac{C_D A\rho}{2} = 0.001$$

and that the wind speed varies sinusoidally with time according to the rule

$$V_w = 20\sin(0.01\,t)$$

so that the aerodynamic force can be approximated by

$$F_w = 0.01\,(\dot{x} + 20\,\sin(0.01\,t))^2$$

Next, assume that the road angle varies sinusoidally with distance, according to the rule

$$\theta = 0.0093\sin(0.0001x)$$

Then the hill force is

$$F_h = 30\sin(0.0001x)$$

We will control the automobile speed using the simple proportional control law

$$F_c = K_e\,(\dot{x}_{desired} - \dot{x})$$

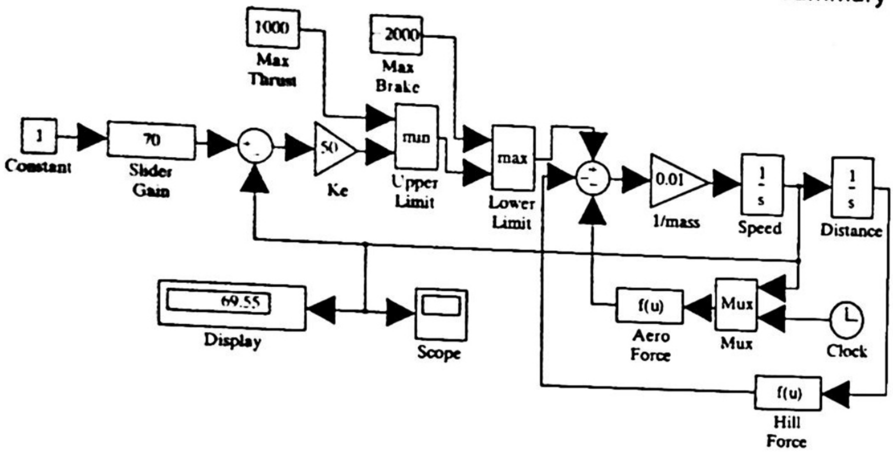

FIGURE 5.21: Automobile model with proportional speed control

Here, F_x is the commanded engine (or braking) force, $\dot{x}_{desired}$ is the commanded speed (ft/sec), and K_e is the feedback gain. Thus, the command engine force is proportional to the speed error. The actual engine force (F_e) is, as stated earlier, bounded from above by the maximum engine thrust from below by the maximum braking force. We choose $K_e = 50$.

A Simulink model of this system is shown in Figure 5.21. We will simulate the motion of the car for 1000 sec.

The input to the proportional controller is the desired automobile speed in ft/sec. This is implemented with a Slider Gain block (from the Math block library) with a constant input. Double-click the Slider Gain block to open a slider window, which allows you to vary the desired speed while the simulation runs.

The proportional controller consists of a Sum block that computes the speed error (the difference between the commanded speed and the actual speed) and a Gain block.

The upper and lower limits on engine force are imposed using MinMax blocks. The constant blocks labeled `Max thrust` and `Max Brake`, together with the Min and Max blocks, are used here to illustrate the use of those blocks. This part of the model could be replaced with a Saturation block from the Discontinuities block library. (Why don't you try that?)

The nonlinear hill and aerodynamic forces are computed by Fcn blocks. The **Expression** field of the block dialog box for the Fcn block labeled Aero Force contains `0.001*(u[1]+20*sin(0.01*u[2]))^2`. The **Expression** field for the Fcn block labeled Hill Force contains `30*sin(0.0001*u[1])`.

A Display block serves as a speedometer (which indicates ft/sec), and the speed is plotted using a Scope block.

This model is a good example of a slightly stiff system. To observe the effect of this stiffness, run the model using solver ODE45, then repeat the simulation using ODE15S. See Chapter 13 for a discussion of stiff systems.

5.5 SUMMARY

In this chapter, we discussed using Simulink to model continuous systems. We started with scalar, linear, time-invariant systems, and progressed to vector linear systems using the state-space concept. Finally, we discussed modeling nonlinear continuous systems.