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$$\sum_i \vec{F}_i = \frac{d}{dt}\vec{P} = m\frac{d\,v(t)}{dt} = ma$$ (1)

$\bar{P}$ is linear momentum

$$\sum \vec{r}_i \times \vec{F}_i = \frac{d\,\bar{J}_{cm}}{dt}$$

J_{cm} is angular momentum of center of mass

The force $\vec{F}_i$ acts on vehicle at point $\vec{r}_i$ in a coordinate system aligned with center of mass (c.m)

ALSO $\vec{J}_{c.m.} = \sum \vec{r}_i \times m_i (\bar{\omega} \times \vec{r}_i)$ angular momentum

$\bar{\omega}$ is angular velocity $\bar{\omega} = \omega_x \bar{x} + \omega_y \bar{y} + \omega_z \bar{z}$

$\vec{r}_i = x_i \bar{x} + y_i \bar{y} + z_i \bar{z}$

OR $\vec{r}_i = x_i \vec{i} + y_i \vec{j} + z_i \vec{k}$ USING $\vec{i}, \vec{j}, \vec{k}$ UNIT VECTORS

$$\bar{\omega} \times \bar{r}_i = \begin{vmatrix} \bar{x} & \bar{y} & \bar{z} \\ \omega_x & \omega_y & \omega_z \\ x_i & y_i & z_i \end{vmatrix}$$

2

$$= \bar{x} \begin{vmatrix} \omega_y & \omega_z \\ y_i & z_i \end{vmatrix} - \bar{y} \begin{vmatrix} \omega_x & \omega_z \\ x_i & z_i \end{vmatrix} + \bar{z} \begin{vmatrix} \omega_x & \omega_y \\ x_i & y_i \end{vmatrix}$$

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$$= \bar{x} \underbrace{(\omega_y z_i - \omega_z y_i)}_{\alpha} - \bar{y} \underbrace{(\omega_x z_i - \omega_z x_i)}_{\beta} + \bar{z} \underbrace{(\omega_x y_i - \omega_y x_i)}_{\gamma}$$

Now $\bar{r}_i \times m_i (\bar{\omega} \times \bar{r}_i)$ in $\bar{x}$

$$m_i \begin{vmatrix} \bar{x} & \bar{y} & \bar{z} \\ x_i & y_i & z_i \\ \omega_y z_i - \omega_z y_i & \beta & \gamma \end{vmatrix} = \bar{x} (y_i \gamma - z_i \beta)$$

So

$$J_x = m_i \{ y_i [(\omega_x y_i - \omega_y x_i) - z_i (\omega_x z_i - \omega_z x_i)] \}$$

$$m_i \{ (y_i^2 + z_i^2) \omega_x - x_i y_i \omega_y - x_i z_i \omega_z \}$$

Sum over points

We write this as

$$\bar{J}_{cm} \doteq \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = I\vec{\omega}$$

$I \doteq$ Inertia tensor

so $I_{xx} = \sum_i m_i (y_i^2 + z_i^2)\omega_x$

etc

For the coordinate system on the vehicle and rigid, I is constant.

Hw compute I_{yy}, I_{zz}

$I = \int r^2 dm$ INERTIA

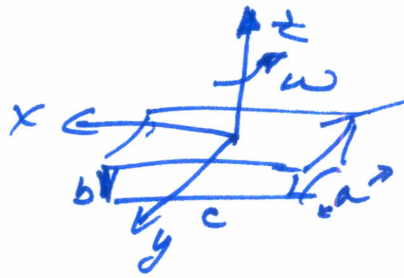
$J = I\vec{\omega}$ ANGULAR VELOCITY

$\vec{\tau} = I\vec{\alpha}$ Torque

$= I\dot{\vec{\omega}}$

$= I\ddot{\vec{\theta}}$

$$I_z = \int (x^2 + y^2)\, dm$$



AREA = ac
volume = abc m^3

$$dm = \rho\, dV$$
$$= \rho\, dx\,dy\,dz$$

see www.latech.edu

$$\rho = \frac{m}{\text{volume}} \quad \text{constant}$$
$$= kg/m^3$$

HERE $V = a \cdot b \cdot c$
$\rho abc = m$ (mass)

$$I_z = \iiint \rho r^2\, dx\,dy\,dz$$

$$\underset{z}{\int_{-b/2}^{b/2}} \underset{y}{\int_{-a/2}^{a/2}} \underset{x}{\int_{-c/2}^{c/2}} \rho (x^2+y^2)\, dx\,dy\,dz$$

$$= \rho b \int_{-a/2}^{a/2} \int_{-c/2}^{c/2} (x^2+y^2)\, dx\,dy$$

$$= \rho b \int_{-a/2}^{a/2} \left[\frac{x^3}{3} + x y^2 \right]_{-c/2}^{c/2} = \rho b \int_{-a/2}^{a/2} \left(\frac{c^3}{12} + c y^2 \right) dy$$

$$= \rho b c \left\{ \frac{c^2}{12} y + \frac{y^3}{3} \right]_{-a/2}^{a/2} = \boxed{\frac{\rho abc}{12} (c^2 + a^2)}$$

1. LOOK AT OUR BOOK
2. LOOK AT DIFFERENTIAL EQUATION

IN CHAPTER 2, CAROL DEFINED A ROBOT WITH WHEELS FOR SIMULATION

Pg 41 – Define x, y, z and size
WHERE AND HOW BIG

XML CODE

P44 FIGURE

P46 ADD WHEELS — POSITION + ANGLE ($\pi/2$)
SIZE
FIGURE 47

P 48-49 CASTER LIKE TURTLEBOT

P 49-52 ADD COLOR AND DEFINE AS SOLID (COLLISIONS)

P53 LET WHEELS MOVE
. DEFINED IN ROS

P54-55 DEFINE PHYSICS-DYNAMICS
INERTIA OF EACH COMPONENT

P57 GAZEBO

WHAT WE CAN DO
BULLET POINTS