INERTIAL FRAME OF REFERENCE

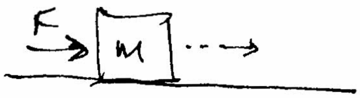
CH 3
HANDOUT 1

NO ACCELERATION

$\vec{F} = m\vec{a}$ $\quad$ $\vec{F}$ IS THE SUM OF "REAL" FORCES

SI UNITS

$\vec{F}_1 = 1\,kg \times 1\,m/s^2 = 1\,N$ (Newton) $= 0.2248\,lbf$



$v = at$ m/sec

UNITS	SI	US
MASS	Kg	LbM = 0.45359237 Kg
FORCE	NEWTON	Lbf

I BUY MEAT IN PARIS. DO I ORDER 1 NEWTON OF SAUSAGE OR 1 kg OF SAUSAGE?

ON THE SCALE, THE QUANTITY IS 1 KG
SO I AM BUYING A MASS OF FOOD.
THE SCALE ACTUALLY MEASURES

$W = mg$ BUT IS CALIBRATED IN Kg
BECAUSE $g = 9.80\,m/s^2$ IS CONSTANT!

IN THE US - POUNDS, OUNCES, ETC ARE USED FOR BOTH MASS AND WEIGHT! THE VALUES ARE NOT THE SAME. (lbm vs lbf)

IN SPACE $W=0$ IF $g=0$ BUT $m\neq 0$,
SO $\bar{F}=m\bar{a}$ STILL APPLIES.

THE SAUSAGE ON THE MOON WOULD WEIGH

$$1\,\text{Kg} \times 1.62\,\text{m/s}^2 = \underline{1.62\,\text{N}}$$

EATING IT ON THE MOON IS JUST AS FILLING!

INERTIA (SLUGGISHNESS IN LATIN) DEFINES NEWTONS FIRST LAW - A BODY IN MOTION WITH NO NET FORCE ON IT REMAINS IN MOTION.

(DIRECTION AND SPEED i.e. $\vec{V}$ m/s UNCHANGED)

SO MASS PLAYS TWO ROLES - GRAVITATIONAL AND INERTIAL. FOR LINEAR MOTION, THE INERTIA IS MEASURED BY MASS (m).

US, UNITS $\quad g = 32.174\,\text{FT/s}^2$

HERE IS THE PROBLEM

SI	US
density = kg/m^3	$slugs/ft^3$
$\rho_{Al} = 2700\,kg/m^3$	$\rho_{Al\,mass} = 5.3\ slugs/ft^3$
$= 2.7\,g/cm^3$	$\rho_{Al\,weight} = 170\ lb/ft^3$
	$(5.3\ slugs/ft^3 \times 32\,ft/s^2)$

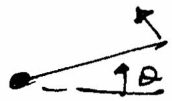
$$\rho_{Al\,weight} = 170\,lb/ft^3 \times \frac{16\,oz}{lb} \cdot \frac{1\,ft^3}{1728\,in^3}$$

$$= 1.58\ oz/in^3$$

(KLAFTER p112 Ex 3.2.3)

ROTATION ABOUT A FIXED AXIS

$\vec{\theta}, \vec{\omega}, \vec{\alpha}$ ARE VECTORS



$$|\omega| = \left|\frac{d\theta}{dt}\right| \ \text{rad/sec}$$

$$|\alpha| = \left|\frac{d\omega}{dt}\right| = \left|\frac{d^2\theta}{dt^2}\right| \ \text{rad/sec}^2$$

$$|\Delta\theta| = |\omega| t \ \text{radians}$$

LINEAR

$\vec{X}, \vec{V}, \vec{a}$ ARE VECTORS

IN ONE DIMENSION

$$v = \frac{dx}{dt} \ \text{m/sec}$$

$$a = \frac{d^2x}{dt^2} \ \text{m/sec}^2$$

$$d = vt \ \text{meters}$$

$$x = \int_0^t v(t)\,dt \ \text{etc.}$$

$$s = r\theta \ \text{so} \ \Delta s = r\Delta\theta \quad \left(s = 2\pi r \text{ if } \theta = 2\pi\right)$$

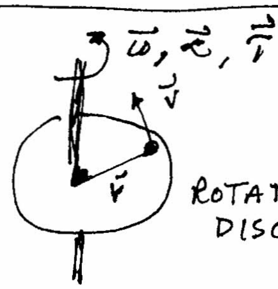
$$|v| = \frac{ds}{dt} = r\frac{d\theta}{dt} \quad \text{TRANSLATION}$$

$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

AS VECTORS

$$\vec{\omega} = \frac{d\vec{\theta}}{dt}$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$



ROTATING DISC

$$\vec{F} = \frac{d}{dt} m\vec{v} = m\vec{a}$$

m constant

WATCH IT

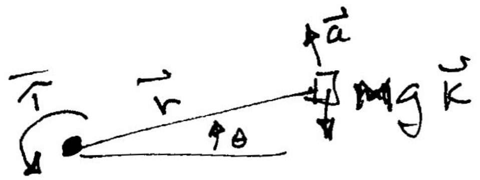
$$\frac{d\vec{v}}{dt} = \underbrace{\frac{d\vec{\omega}}{dt}}_{=\vec{\alpha}} \times \vec{r} + \vec{\omega} \times \underbrace{\frac{d\vec{r}}{dt}}_{=\vec{v}}$$

BECAUSE DIRECTION OF $\vec{r}$ IS CHANGING

TORQUE —

FROM NEWTONS LAW $\vec{F} = m\vec{a}$, $\vec{F}$ and $\vec{a}$ ARE IN THE SAME DIRECTION.

FOR A ROTATING MASS AT A DISTANCE FROM AN AXIS, THE TURNING FORCE OR <u>TORQUE</u> IS $\perp$ TO THE LEVER ARM



$\vec{\tau} = \vec{r} \times \vec{F}$ so if $\vec{r} \perp \vec{F}$; $\underline{|\tau| = |rF|}$

since $|F| = m|a|$ as a SCALAR

$$|\tau| = m\frac{dv}{dt} r = m\frac{d\omega}{dt} r \cdot r = \underbrace{mr^2}_{I}\alpha$$

I = MOMENT OF INERTIA

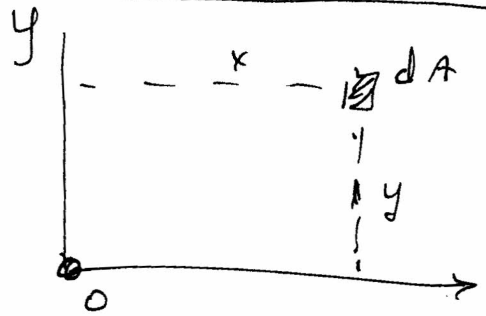
$\tau = I\alpha$ Newton.meter

SO IF WE KNOW I (J in KLAFTER) AND DESIRED α, A MOTOR WITH APPROPRIATE TORQUE CAN BE CHOSEN.

See KLAFTER P259 EX: 4.6.1

P107 TORQUE

MOMENT OF INERTIA SINGER p217



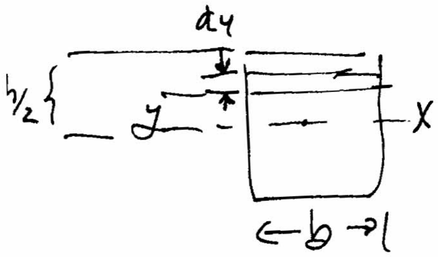
$$\begin{cases} I_x = \int y^2 dA & \text{ABOUT X AXIS} - y \text{ is moment arm} \\ I_y = \int x^2 dA \end{cases}$$

SECOND MOMENT OF AREA

WE WANT <u>MASS</u> MOMENTS OF INERTIA
SINGER p243

$$I_m = \int \rho^2 dm$$

1. consider rectangle (SINGER p220)



$$I_x = \int y^2 dA = \int_{-h/2}^{h/2} y^2 b\,dy =$$

$$b \left. \frac{y^3}{3} \right|_{-h/2}^{h/2} = \boxed{\frac{bh^3}{12}}$$

FOR MASS MOMENT OF INERTIA WE NEED MASS PER UNIT VOLUME

The WOULD BE

$$I = \sum m_i r_i^2$$

RESNICK p 268

This IS THE PRODUCT of masses and distance squared from axis of rotation

SOLID BODY

$$I = \int r^2 \, dm$$

$dm = \rho \, dv$

if ρ is constant

cylinder

$$dv = 2\pi r \, dr \cdot L$$

$$dm = 2\pi L \rho r \, dr$$

$$I = \int r^2 dm = 2\pi L \int_0^R \rho r^3 dr$$

$$= 2\pi L \rho \frac{R^4}{4} = \frac{\pi L R^4}{2} \rho$$

BUT $M = \underbrace{(\pi R^2) L \cdot \rho}$ is mass

SO $I = \frac{1}{2} M R^2$ KLAFTER J_z

FAG 3.2.9
P110

CORRECTION TO 1ST EDITION

center is given by

$$J = \tfrac{1}{12}Ml^2 \tag{3.2.16}$$

The center of mass is located in the middle of the bar at $l/2$ and has been designated as the y axis in Figure 3.2.9. Therefore, by Eq. (3.2.15) we obtain

$$\begin{aligned} J_{y'} &= \tfrac{1}{12}Ml^2 + M(l/2)^2 \\ &= \tfrac{1}{3}Ml^2 \end{aligned} \tag{3.2.17}$$

ERROR PRINT 1

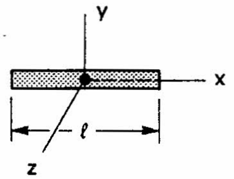
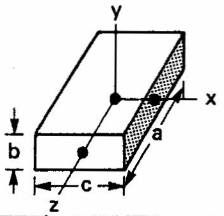
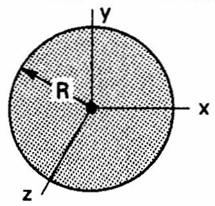
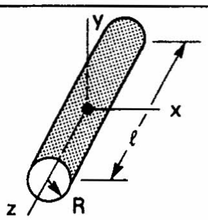
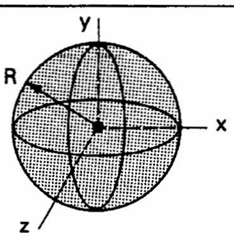
Figure		J_x	J_y	J_z
	Slender Bar	0	$\frac{1}{12}M\ell^2$	$\frac{1}{12}M\ell^2$
	Rectangular Parallelepiped	$\frac{1}{12}M(a^2 + b^2)$	$\frac{1}{12}M(a^2 + c^2)$	$\frac{1}{12}M(b^2 + c^2)$
	Thin Circular Disk	$\frac{1}{4}MR^2$	$\frac{1}{4}MR^2$	$\frac{1}{2}MR^2$
	Right Circular Cylinder	$\frac{1}{12}M(3R^2 + \ell^2)$	$\frac{1}{12}M(3R^2 + \ell^2)$	$\frac{1}{2}MR^2$
	Sphere	$\frac{2}{5}MR^2$	$\frac{2}{5}MR^2$	$\frac{2}{5}MR^2$

Figure 3.2.9 Centroidal moments of inertia for some common shapes.

Example 3.2.2 can also be used to illustrate a common error used in computing the moment of inertia about an axis. Although Eq. (3.2.13) correctly defines the moment of inertia of a point mass about an axis it cannot always be applied to obtain the moment of inertia of a body of arbitrary shape. Consider, for example, that we approximate the body as a point mass physically located at its center of gravity and then use this point to define the distance of the body from its axis of rotation. For the case of the rod shown in Figure 3.2.10, if we had used the center of gravity to compute the moment of inertia about the y' axis, the result would be

$$J_{y'} = \tfrac{1}{4}Ml^2 \tag{3.2.21}$$

Comparing this with the correct result of Eq. (3.2.17) shows that an error of 25% on the low side has been made. This error could cause serious problems in that the payload of a robot may be incorrectly calculated, thereby causing the system to be unable to perform adequately.

Based on the previous discussions, it should be obvious that the point-mass approximation of Eq. (3.2.13) should not be used arbitrarily to compute the inertia of an object. In some cases this approximation is sufficient. However, one must ensure that the error introduced does not produce misleading values. A more conservative approach is to decompose the body into elementary shapes as shown in a table of centroidal moments (e.g., those in Figure 3.2.9) and then use the parallel axis theorem [Eq. (3.2.15)] to compute the inertia of the object in question. Example 3.2.3 illustrates this procedure.

EXAMPLE 3.2.3: CALCULATION OF INERTIA FROM ELEMENTARY SHAPES

Al: $0.098\,\text{lb/in}^3$ $2.7\,\text{g/cm}^3$

$M = \frac{W}{g}$

Weight $= \rho \times$ Volume $= Mg$

Figures 3.2.11 and 3.2.12 show a simplified parallel-jaw type gripper which has been modeled by three rectangular parallelepipeds, each consisting of a length, width, and height dimension. The density of the material, aluminum, is 1.56 oz/in.^{3}. For the particular application being analyzed, the gripper is free to rotate about two perpendicular axes (z and y) as shown (i.e., the roll and pitch axes). Note that the z axis goes through the center of the gripper, while the y axis is some distance from the back surface. For the dimensions shown, compute the moment of inertia about both the z and y axes.

From Figure 3.2.9 we identify the axes associated with each rectangular member as shown in the exploded view of the gripper given in Figure 3.2.12. The dimensions a, b, and c correspond to the formulas given in Figure 3.2.9, and we identify the components by the subscript top, side, and bottom to delineate the members.

The contribution to the moment of inertia about the z axis is computed by first determining the moment of inertia of each member about the centroidal axis parallel to the z axis of the complete gripper and then using the parallel axis theorem. By summing the moment of inertia of the three members referenced to the z axis, we find the total moment of inertia about the z axis. Equations (3.2.22) through (3.2.24) show the value of each of the three members referenced to the z axis of the gripper.

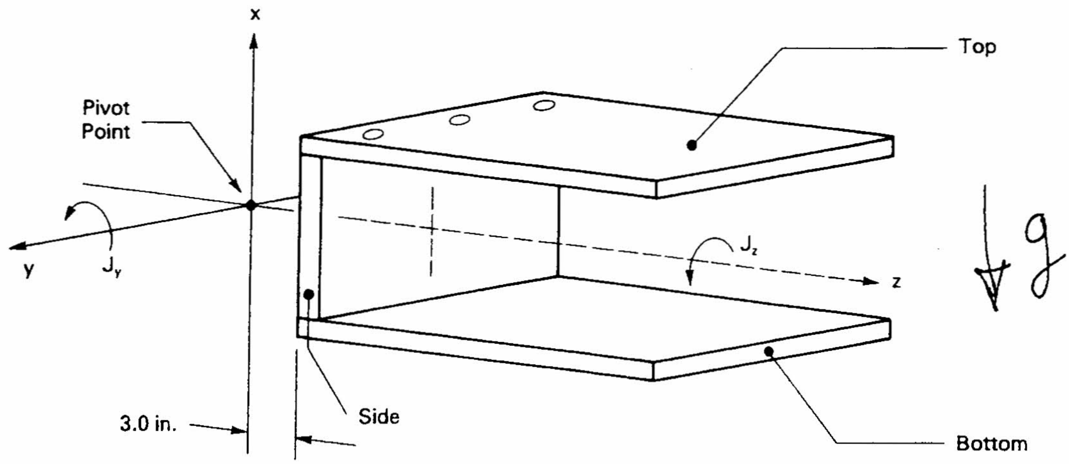


Figure 3.2.11 Parallel-jaw gripper model.

SEE F3.2.12

$$J_{z_{top}} = \tfrac{1}{12}M_{top}(b^2 + c^2) + M_{top}\, r_{zt}^2 \tag{3.2.22}$$

$$J_{z_{bot}} = \tfrac{1}{12}M_{bot}(b^2 + c^2) + M_{bot}\, r_{zb}^2 \tag{3.2.23}$$

$$J_{z_{side}} = \tfrac{1}{12}M_{side}(a^2 + c^2) \tag{3.2.24}$$

Note that the parallel axis theorem was not needed to compute the contribution from the side member since its centroidal y axis was coincident with the z axis of the gripper. Therefore, the total moment of inertia about the z axis is given by

$$J_{z_{total}} = J_{z_{top}} + J_{z_{bot}} + J_{z_{side}} \tag{3.2.25}$$

Utilizing the actual dimensions given in Figure 3.2.12 yields

WEIGHT

$$J_{z_{total}} = 0.075\text{3 oz-in.-s}^2 = \frac{\text{WEIGHT}}{g} = \text{MASS MOMENT} \tag{3.2.26}$$

ERROR PRINT

$J_{mass} \times g$ OR $\frac{J_{weight}}{g}$

The moment of inertia about the y axis of the gripper is computed in a similar manner. In this case, however, the parallel axis theorem must be used for all three members since none of the centroidal axes under consideration are coincident with the y axis. Equations (3.2.27) through (3.2.29) define the moments of inertia due to each plate about the y axis.

$$J_{y_{top}} = \tfrac{1}{12}M_{top}\,(a^2 + b^2) + M_{top}\, r_{yt}^2 \tag{3.2.27}$$

$$J_{y_{bot}} = \tfrac{1}{12}M_{bot}\,(a^2 + b^2) + M_{bot}\, r_{yb}^2 \tag{3.2.28}$$

$$J_{y_{side}} = \tfrac{1}{12}M_{side}\,(a^2 + b^2) + M_{side}\, r_{ys}^2 \tag{3.2.29}$$

$$J_{y_{total}} = J_{y_{top}} + J_{y_{bot}} + J_{y_{side}} \tag{3.2.30}$$

Again substituting the dimension values of Figure 3.2.12 yields

$$J_{y_{total}} = 0.525 \quad \text{oz-in.-s}^2 \tag{3.2.31}$$

ERROR PRINT

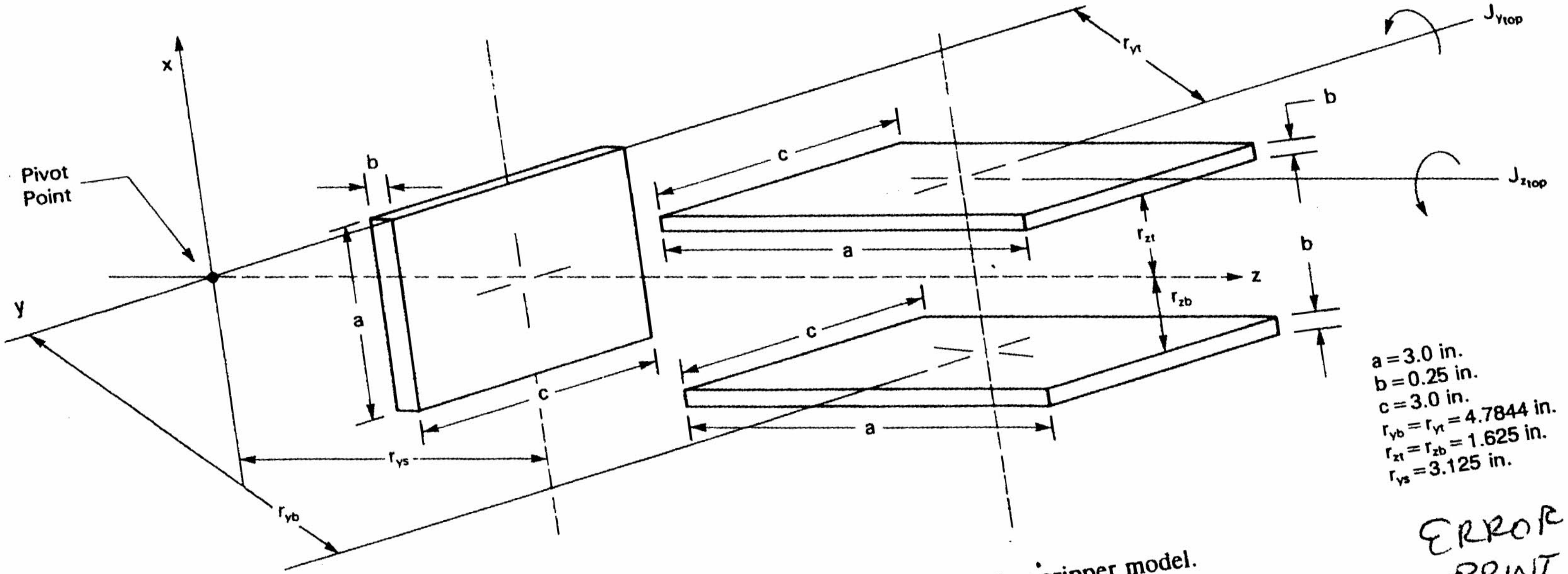


Figure 3.2.12 Exploded view of the parallel-jaw gripper model.

ERROR PRINT 1

[illegible] LITS P113 –

2/10/2010 (1)

DETERMINE THE MOMENT OF INERTIA OF A GRIPPER

WE KNOW

$$J = \int_{\text{volume}} \rho r^2 \, dV \qquad 3.2.14 \text{ p109}$$

PARALLEL AXIS

$$J = J_{\text{center of gravity}} + M r^2 \qquad 3.2.15$$

Take the BACK PLATE (SIDE)

TABLE FIGURE 3.2.9

$$J_{cg} = \frac{1}{12} M (a^2 + b^2) = \frac{1}{12} M (3.0^2 + 0.25^2)$$

$$m = 1.56 \text{ oz/in}^3 \times (3.0 \times 0.25 \times 3.0 \text{ in}^3)$$

The PLATE is $r_{4S} = 3.0$ in FROM PIVOT POINT

So

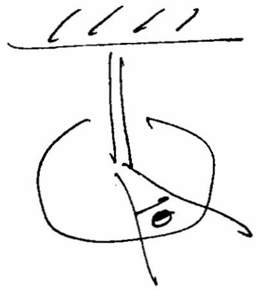
$$J_{side} = \frac{1}{12} M_{side} (a^2 + b^2) + M_{side} r_{4S}^2 \qquad \text{Eq } 3.2.29$$

This is ROTATION AROUND y AXIS in Fig 3.2.11 P113

WATCH WEIGHT = M × g VS. MASS MOMENT OF INERTIA!

MEASUREMENT

KLAFTER
p 116-117 ②



BAR ACTS LIKE A SPRING

$$J \frac{d^2\theta}{dt^2} = -k\theta$$ <u>IF</u> θ IS SMALL

$(\sin\theta \approx \theta)$

The 2ND ORDER DE HAS SOLUTIONS FOR NO APPLIED FORCE

$$J\ddot{\theta}(t) + k\theta = 0 \qquad (\theta(0), \dot{\theta}(0)) =$$

$$\lambda^2 + K/J = 0 \quad \lambda = i\sqrt{K/J} \qquad (\theta(0), 0)$$

$$\ddot{\theta} = A\cos\sqrt{K/J}\,t + B\sin\sqrt{K/J}\,t \qquad \omega = \sqrt{\frac{K}{J}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{K/J}$$ (OSCILLATION) in HERTZ

$$T_{period} = \frac{1}{f}$$

PHYSICAL PENDULUM

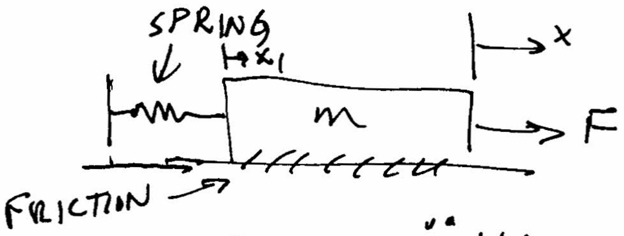
SEE HANDOUT

KLAFTER CHAPTER 3 ~~INTRO~~ REVIEW

(1)

CH 3 HANDOUT 2

IN CHAPTER 3, WE STUDY THE MODELING OF PHYSICAL SYSTEMS



P103

$F(t) = m\ddot{x}(t)$ NEWTON $M = \frac{W}{g}$ $\left(\frac{lbs}{ft/s^2}\right)$

$F = KX_1(t)$ LINEAR SPRING

FRICTIONS (P105)

a) STATIC (STARTING OR STICTION)

$$F_s(t) = \pm F_s \Big|_{\dot{x}(t)=0} \; ; \quad \vec{F} = \mu_s \vec{N}$$

b) COULOMB FRICTION

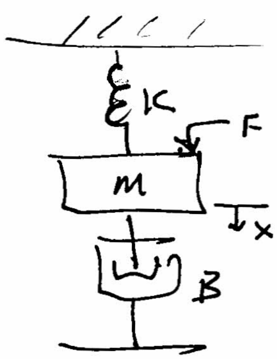
$$F(t) = F_c \,\text{sgn}(t)$$

OPPOSES CONSTANT VELOCITY MOTION

c) VISCOUS FRICTION

$F(t) = B\dot{x}(t)$ DEPENDS ON VELOCITY

FOR 2ND ORDER SYSTEM



$$m\ddot{x}(t) + \beta\dot{x}(t) + Kx(t) = f(t)$$

HARMAN P221

ROTATIONAL SYSTEMS –

REPLACE M WITH J (OR I) – MOMENT OF INERTIA

$$T = J\ddot{\theta}(t) = J\alpha(t)$$

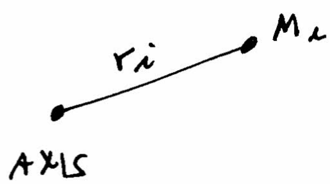
WE EXPECT

$$J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = T_{\text{Applied}}$$

$$J = \iiint_{\text{volume}} \rho r^2 \, dV \qquad \text{IN GENERAL } J = cMR^2, \quad 0 < c < 1.$$

simple cases of particles

$$J = \sum_{i=1}^{n} M_i r_i^2$$



P 108

Torque is in DIRECTION NORMAL TO PLANE OF FORCE AND RADIUS VECTOR

$$\overline{T} = \vec{r} \times \vec{F} \qquad |T| = rF\sin\phi$$

NOTICE THAT WHEN WE DEAL IN 3 DIMENTIONS $\vec{X}, \dot{\vec{X}}, \ddot{\vec{X}}, \vec{\theta}, \dot{\vec{\theta}}, \ddot{\vec{\theta}}, \vec{F}, \vec{T}$ etc are VECTORS

TO AVOID VECTOR NOTATION, WORK (CHANGE IN ENERGY) CAN BE USED! $\frac{1}{2}mv^2$, $\frac{1}{2}J\omega^2$

$$\text{Power} = \frac{dW}{dt} \qquad P(t) = F(t)v(t) \quad \text{Linear}$$

$$P(t) = T(t)\omega(t) \quad \text{Rotation}$$

P 118-119

A FEW DEFINITIONS

REVIEW 3

KLAFTER
P117-119
p167

CONSIDER EQUIVALENTS

X-Y-Z	ROTATION	ELECTRICAL
F	T	i
$\dot{X}$	ω	v
m	J	C
K	K	$\frac{1}{L}$
B	B	$\frac{1}{R} = G$

FORCE/CURRENT ANALOG

The Roles of Energy and Work are similiar

P119

LINEAR	ROTATION
$E_K = \frac{1}{2} m V_2$	$E_K = \frac{1}{2} J\omega^2$
$E_P = \frac{1}{2} K X^2$	$E_P = \frac{1}{2} K\theta^2$

Work $= \int F(s)ds$ $\quad W = T\theta$

$= Fs$
If F is constant in direction s

(Δ Kinetic Energy) + Δ Potential Energy
EQ 3.2.36
P118

Power $= dw/dt$ $\quad$ WATTS $\rightarrow \frac{\text{KWatts-HOURS}}{\text{Time}}$

$P(t) = F(t)\,V(t)$ $\quad P(t) = T(t)\,\omega(t)$

MOTION CONVERSION (CH3)

FROM THE ACTUATOR (MOTOR) TO THE END EFFECTOR THE ROBOT HAS A "TRANSMISSION". THIS IS GEARS, COUPLERS AND OTHER MECHANICAL DEVICES (1)

P 119

THE PURPOSE MAY BE:

CHANGE ROTATIONAL DIRECTION

CHANGE AXIS

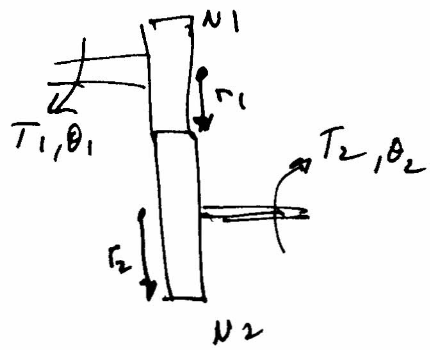
REDUCE OR INCREASE TORQUE OR SPEED

CHANGE ROTARY TO LINEAR MOTION

THE MOST COMMON IS GEARS

SEE HANDOUT

THE # OF TEETH DETERMINE THE RELATIONSHIPS SEE FIG 3.3.1



$WORK_{in} = WORK_{out}$

(1) $T_1\theta_1 = T_2\theta_2$

OF TEETH PROPORTIONAL TO r

$$\frac{N_1}{N_2} = \frac{r_1}{r_2} = \frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1}$$

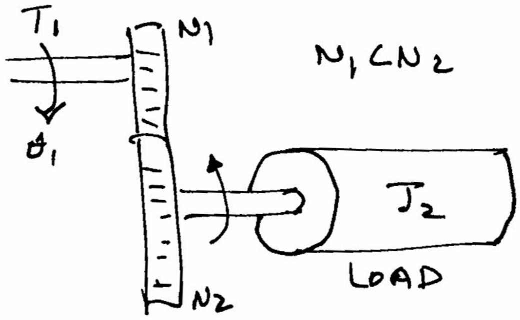
SO Let $N_1 < N_2$

speed ω_2 IS REDUCED

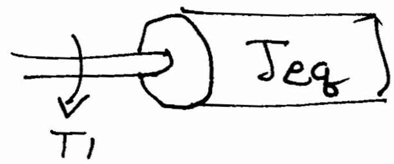
Torque T_2 IS <u>INCREASED</u>

SO IF $N_1 < N_2$, SPEED AT SHAFT 2 IS LESS BUT TORQUE IS GREATER.

HOW ABOUT INERTIA OF THE LOAD AS SEEN BY SHAFT 1?



AT SHAFT 1



WE KNOW $T_1 \approx \frac{N_1}{N_2} T_2 = \frac{N_1}{N_2} \alpha_2 J_2$

SO $T_1 = \alpha_1 J_{eq} = \frac{N_1}{N_2} \alpha_2 J_2$ LOAD THAT SHAFT 1 SEES

So $J_{eq} = \left(\frac{N_1}{N_2}\right)\left(\frac{\alpha_2}{\alpha_1}\right) J_2 = \left(\frac{N_1}{N_2}\right)^2 J_2$ so $\underline{\underline{J_{eq} < J_2}}$

(THIS IS LIKE A TRANSFORMER CHANGING ELECTRICAL IMPEDANCES)

SEE FIGURE 3.3.3 J_1, T_1, θ_1 on input side and FRICTION B_1. J_2, T_2, θ_2, B_2 ON OUTPUT

SO Reduce load on motor if $N_1 < N_2$ BUT ARM MOVES SLOWER!

$$T_{total} = \left\{J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2\right\} \ddot{\theta}_1(t) + \left(B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2\right) \dot{\theta}_1(t) + T_F$$

$$T_F = F_{c1} \operatorname{sgn} \dot{\theta}_1 + \frac{N_1}{N_2} F_{c2} \operatorname{sgn} \dot{\theta}_2$$

KLAFTER P119-123 GEARS, etc Gears 1

SEE FIGURE 3.3.1 P121

BRIEFLY – IDEAL GEARS

WE ASK

WHAT IS TORQUE TRANSFER?

WHAT IS SPEED TRANSFER?

WHAT IS INERTIA TRANSFER?

EVERY THING DEPENDS ON GEAR RATIO

$$TR = \frac{N_1}{N_2} = \frac{\text{input teeth}}{\text{output teeth}} \qquad \text{Fig 3.3.1}$$

See equation 3.3.4

$$\frac{N_1}{N_2} = \frac{\text{radius}_1}{\text{radius}_2} = \frac{\text{Torque}_1}{\text{torque}_2} = \frac{\theta_2}{\theta_1} = \frac{\overset{\omega}{\text{speed}_2}}{\text{speed}_1} = \frac{\alpha_2}{\alpha_1}$$

So $N_1 < N_2$; $T_1 < T_2$ but $\omega_2 < \omega_i$

So the smaller gear rotates faster but Torque output increases

$$T_2 = T_1 \frac{N_2}{N_1} \qquad N_2 > N_1$$

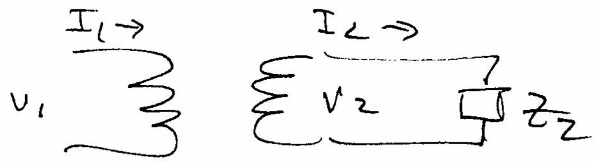
IN A CAR 1st GEAR 2.97:1 ENGINE ~~2.97~~ revolutions for 1.0 in TRANSMISSION

4th GEAR 1.00 : 1

So 1st gear $\underset{\text{Engine} \to}{} \frac{\omega_2}{\omega_1} = \cancel{\frac{2.97}{4}} = \frac{1}{2.97} = \frac{N_1}{N_2}$ (TORQUE IS HIGH Speed is low)

CONSIDER A TRANSFORMER

KLAFTER G2
P122



$N_1 : N_2$

$V_1 I_1 = V_2 I_2$ IDEALLY SO $\frac{V_1}{V_2} = \frac{I_2}{I_1}$

NOTE INVERSION

FARADAY'S LAW

$$V_1(t) = N_1 \frac{d\phi_m(t)}{dt}$$

$$V_2(t) = N_2 \frac{d\phi(t)}{dt}$$

SO $\frac{V_1}{V_2} = \frac{N_1}{N_2} = TR$ AND $\frac{I_1}{I_2} = \frac{N_2}{N_1}$

SO IMPEDANCE

$$Z_1 = \frac{\bar{V}_1}{\bar{I}_1} = \frac{TR \cdot \bar{V}_2}{I_2/TR} = TR^2 \frac{V_2}{I_2} = TR^2 Z_2$$

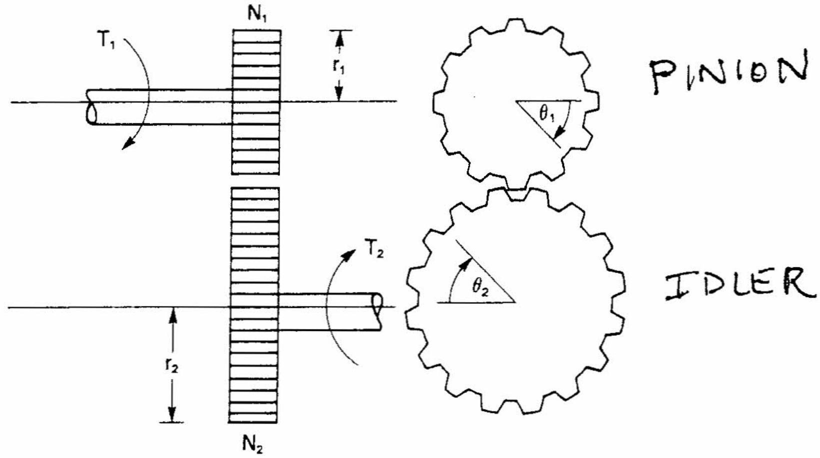
IN ELECTRONICS TR^2 CAN BE CHOSEN TO MATCH INPUT AND OUTPUT IMPEDANCE

SEE KLAFTER EQ 3.3.6 + 3.3.7

$$J_{eq} = \left(\frac{N_1}{N_2}\right)^2 J_{LOAD}$$

INERTIA AT INPUT IS LESS IF $N_1 < N_2$

THE TORQUE EQUATION DEPENDS ON TR^2 ALSO EQ 3.3.8a P123.



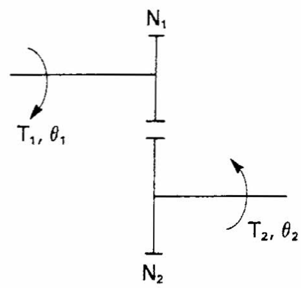


Figure 3.3.1 Ideal gear train with parameters.

Finally, noting that since the two gear radii do not vary with time, if Eqs. (3.3.2) and (3.3.3) are differentiated with respect to time, their relationship still holds but with respect to $\dot{\theta}$ (i.e., the angular velocity ω*) or $\ddot{\theta}$ (i.e., the angular acceleration, α). Using this concept, we may write

$$\frac{N_1}{N_2} = \frac{r_1}{r_2} = \frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1} \tag{3.3.4}$$

Equation (3.3.4) can be used to investigate various properties of the "ideal gear train." For example, assume that the speed of both shafts is known and that the speed of shaft 1 is greater than the speed of shaft 2; then we know that the number of teeth on gear 2 is greater than that on gear 1. In addition, we also

*Although not shown explicitly, the reader should keep in mind that ω, α, and T are functions of time.

know the ratio N_1/N_2. Finally, if the torque on shaft 1 is known, we can compute the torque on shaft 2 by

$$T_2 = T_1 \frac{N_2}{N_1}$$

This particular relationship shows the speed reduction and torque multiplication property of a gear train.

A commonly used definition in motion conversion is that of the *coupling ratio*. Loosely defined, this ratio is the angular movement of the input compared to the load. For a rotational system, a coupling ratio of 2:1 defines a gear train in which two turns of the input shaft produce a single rotation of the output. Note that in this case the coupling ratio is the inverse of the tooth ratio, TR, which we define as (N_1/N_2).

It is interesting to note that the ideal gear train is similar to the ideal electrical transformer. In fact, one may transform a mechanical system containing a gear train into an analogous electrical network containing a transformer. In Section 3.5.4 we discuss this in more detail.

X former

Employing the same concepts that were used to develop Eq. (3.3.4), the transfer relationship between the input and output shafts of a compound gear train (i.e., one consisting of more than two gears) may be derived.

Gear trains can be used to change "mechanical loads" in a manner that is similar to using a transformer to reduce or increase electrical impedances. For example, if a pure inertial load is placed on the output of a gear train as shown in Figure 3.3.2a, the input torque required to accelerate that load is given by

$$T_1 = \frac{N_1}{N_2} \alpha_2 J_2 \tag{3.3.5}$$

We may ask the question: What inertia is "seen" by the input shaft? Or in other words: What inertial load applied to the input shaft produces the same torque requirement as that of the original load? Figure 3.3.2b shows this equivalent system.

Assuming that T_1 accelerates an inertial load J_{eq} at an angular acceleration of α_1, we may write

$$\alpha_1 J_{eq} = \frac{N_1}{N_2} \alpha_2 J_2 \tag{3.3.6}$$

Using the relationships of Eq. (3.3.4), we may solve for the equivalent inertia J_{eq}:

$$J_{eq} = \left(\frac{N_1}{N_2}\right)^2 J_2 \tag{3.3.7}$$

For speed reduction and torque multiplication at the output of the gear train, the ratio N_1/N_2 is less than 1. The reflected inertia at the input shaft is seen to be less than that on the load.

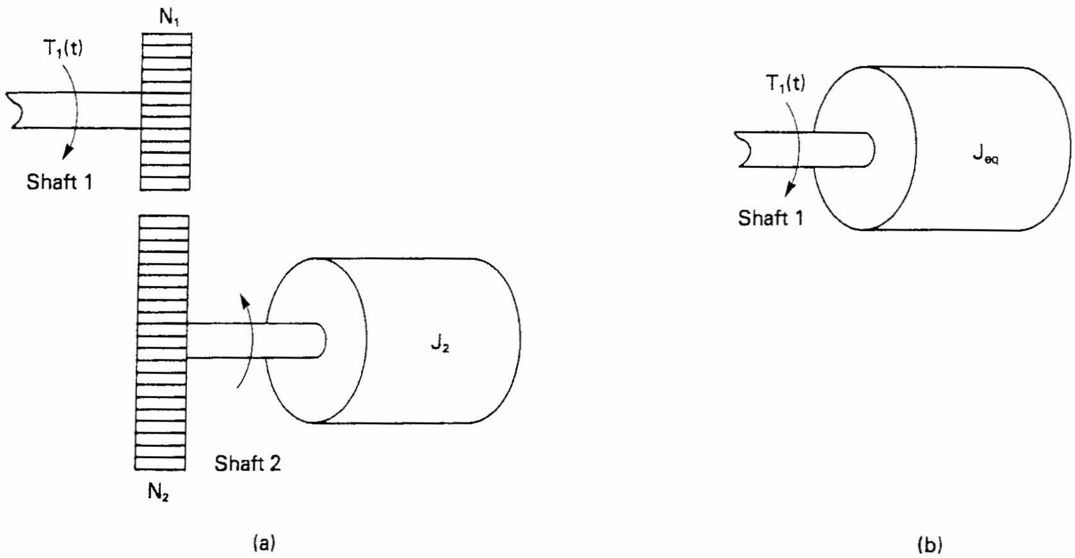


Figure 3.3.2 (a) Gear train with inertial load; (b) equivalent system.

Besides inertia, both the reflected viscous and Coulomb frictions are reduced by a gear train. Figure 3.3.3 shows two gears each having an inertia and friction on their shafts. The total torque as seen by the input shaft is given by Eqs. (3.3.8a) through (3.3.8c).

INPUT

$$T_{\text{total}} = (J_1 + \mathrm{TR}^2 J_2)\ddot{\theta}_1 + (B_1 + \mathrm{TR}^2 B_2)\dot{\theta}_1 + T_f \tag{3.3.8a}$$

$$T_f = F_{c1}\,\mathrm{sgn}\,(\theta_1) + \mathrm{TR}\,F_{c2}\,\mathrm{sgn}\,(\theta_2) \tag{3.3.8b}$$

$$\mathrm{TR} = \frac{N_1}{N_2} \tag{3.3.8c}$$

$T_{\text{friction}} = B\dot{\theta}(t)$

COULOMB 3.2.10

Note that both inertia and viscous friction are reduced (or increased) by the factor (N_1/N_2) squared, whereas Coulomb friction is reduced by the factor (N_1/N_2). Note also that Eq. (3.3.8a) is nonlinear.

By making TR less than 1, it is seen from Eqs. (3.3.8a) through (3.3.8c) that the gear train is effective in reducing the reflected inertial and viscous loads that must be accelerated by a motor (or other actuator). This is an attractive feature since the actuator does not actually have to produce the high torque needed at the output to drive the load but rather, a reduced value. Thus the actuator's size and torque capability can be significantly smaller than that required to drive the load directly. In robotic applications where large inertial loads must be accelerated, this property of reducing the inertia is often utilized in order to reduce significantly the size, weight, volume, and cost of the various joint actuators.

REASON

THE ADVANTAGES OF HARMONIC DRIVE GEARING KLAFTER P125

96

Because it consists of only three simple parts, Harmonic Drive gearing offers design engineers the freedom to integrate drive components directly into machines or equipment. Harmonic Drive is a pure torque couple with all concentric elements and requires less space and less bulky support structures than conventional gearing.

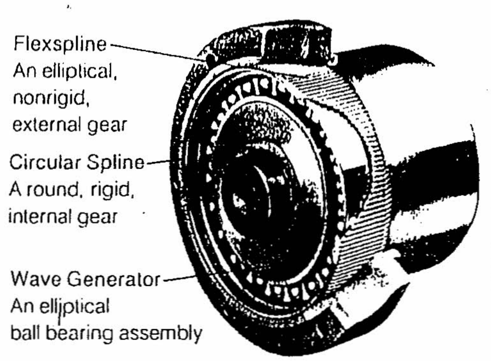
Harmonic Drive's precision and performance are ideal in applications requiring accurate positioning or precise motion control.

Low or Zero Backlash
Natural gear preload and almost pure radial tooth engagement allow standard Harmonic Drive gearing to operate with essentially zero backlash for the entire gear life without preload adjustments or significant wear.

Efficiencies as High as 90%
Measured on actual shaft-to-shaft losses rather than tooth losses (as with other gearing), standard Harmonic Drive gearing efficiencies are normally in the 80 – 90% range.

Simple Support Requirements
Since torque is transmitted by pure couple, Harmonic Drive gearing does not generate radial loads and, therefore, can be used with much simpler bearings and less structural support than other forms of gearing.

HDC CUP COMPONENT GEAR SET



High Single-Stage Ratios From 50:1 Up
Depending on size, Harmonic Drive products offer ratios from 50:1 (60:1 for standard products) to as high as 320:1. Using compound drives, much higher ratios can be achieved.

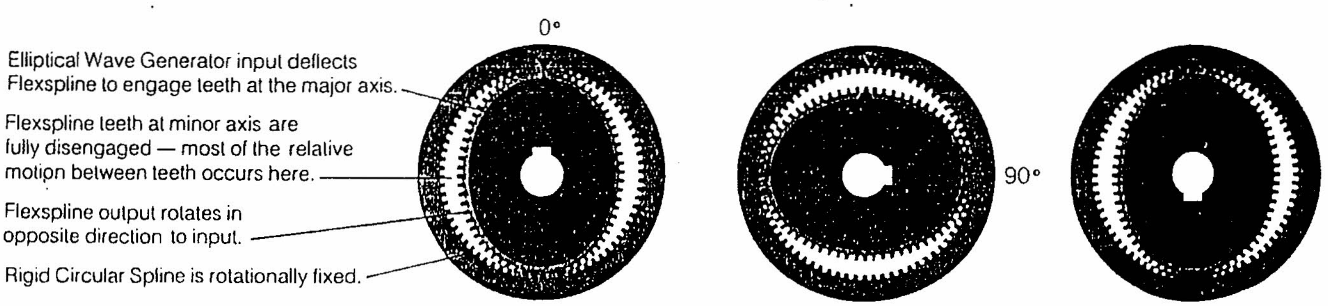
Torque Equal to Drives Twice as Large
Pound for pound, no other mechanical power transmission can compare with Harmonic Drive gearing for torque capacity.

Excellent Positional Accuracy and Repeatability
Harmonic Drive gearing's design ensures that approximately 10% of the total teeth are engaged at any point in time, minimizing the effect of tooth-to-tooth error. Accuracies as fine as 30 arc seconds are achievable in some sizes. Repeatabilities are in the arc second range.

Design Flexibility
Harmonic Drive gearing allows designers multiple input/output possibilities in speed reduction and speed increasing applications. Concentric shafting makes it ideal for differential designs.

Long Life and High Reliability
Proven in years of industrial, military, and other applications, Harmonic Drive gearing has an average life of over 15,000 hours at rated loads. In addition, the tooth mesh is unaffected by the impact of stepping motors or frequent starts, stops, and reversals.

PRINCIPLE OF OPERATION

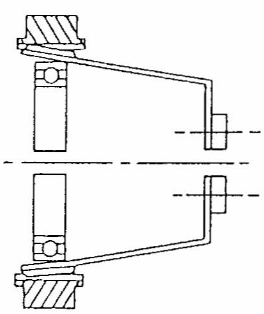


Note: The amount of Flexspline deflection has been exaggerated in the diagrams in order to demonstrate the principle. Actual deflection is much smaller than shown and is well within the material fatigue limits for infinite service life.

The teeth on the nonrigid Flexspline and the rigid Circular Spline are in continuous engagement. Since the Flexspline has two teeth fewer than the Circular Spline, one revolution of the input causes relative motion between the Flexspline and the Circular Spline equal to two teeth. With the Circular Spline rotationally fixed, the Flexspline rotates in the opposite direction to the input at a reduction ratio equal to one-half the number of teeth on the Flexspline.

This relative rotation may be seen by examining the motion of a single Flexspline tooth over one-half an input revolution. The tooth is fully engaged when the major axis of the Wave Generator input is at 0°. When the Wave Generator's major axis rotates to 90°, the tooth is fully disengaged. Full reengagement occurs in the adjacent Circular Spline tooth space when the major axis is rotated to 180°. This motion repeats as the major axis rotates another 180° back to 0°, thereby producing the two tooth advancement per input revolution.

All tabulated Harmonic Drive gear reduction ratios assume output through the Flexspline with the Circular Spline rotationally fixed. However, any drive element may function as the input, output, or fixed member.



Zero Backlash

All Harmonic Drive cup-type gearing products have zero backlash at the gear mesh.

Under most circumstances, this zero backlash lasts beyond the expected life of the drive. This unusual characteristic is due to the unconventional tooth path combined with a slight cone angling of the teeth caused by deflection of the cup walls. Together, these factors produce preload and ensure very little sliding and no relative motion between teeth at the points where most of the torque is transferred.

While a small amount of backlash occurs at the oldham input coupling, because of the high ratios involved, this backlash becomes negligible when measured at the output. Even this backlash can be eliminated by coupling directly to the Wave Generator. Please consult the factory for methods and guidelines.

CHAPTER 3 – VARIOUS MECHANISMS
VERY BRIEFLY P 124 – 156

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EFFICIENCY P 144 – 145

$$\eta = \frac{\text{POWER OUT}}{\text{POWER IN}} = \frac{\text{WORK OUTPUT}}{\text{WORK INPUT}}$$

Let's say the efficiency of the transmission is 22%; $\eta = 0.22$

IDEALLY WE CALCULATED IN oz–inches

$$\frac{\text{INPUT } T}{\text{OUTPUT } T} = \frac{6.66}{500} \quad \text{Ratio is } \frac{N_1}{N_2} = \frac{1}{75}$$

BUT
$$T_{Total} = \left(J_1 + \frac{(N_1/N_2)^2 J_2}{\eta}\right)\ddot{\theta} + \cdots$$

So $J_{ideal\,out} = 75 \times 6.66$ oz-in $= 500$ oz-in

but $J_{actual} = .22 \times 500 = 110$ oz in

So MOTOR TORQUE T_1 is

$$\frac{6.66}{.22} = \underline{\underline{30.27 \text{ oz in}}} \quad \approx 5\times \text{ as much}$$

<u>TO get 500 oz-in</u>

EFFICIENCY

P 145–146

GENERALLY

$$\eta = \frac{100 \cdot P_{out}}{P_{in}} = \frac{100\ P_{out}}{P_{out} + \text{LOSSES}}$$

TRANSFORMERS

$\eta \gtrsim 90\%$

EFFECT IS TO REQUIRE MORE POWER (OR TORQUE) INPUT FOR A GIVEN OUTPUT

KLAFTER P 145

FIGURE 3.4.2

HE SHOWS THAT FOR 500 oz in output

η	INPUT T
100%	6.66 oz-in
22%	30 oz in

See Eq 3.4.3a

See TABLE 3.4.1 P146

3.4.1 Efficiency

Efficiency η is defined as the ratio of the output power to the input power, or the ratio of the work output to the work input over the same period of time. For an ideal mechanism, the efficiency is 1 or 100%. In the case of real components, the work output is less than the work input, with the difference being dissipated in friction. Equation (3.4.1) defines efficiency.

$$\eta = \frac{\text{power out}}{\text{power in}} = \frac{\text{work output}}{\text{work input}} \tag{3.4.1}$$

For the case of a gear train, we may restate Eq. (3.4.1) as the ratio of the actual output torque divided by the ideal output torque. Thus for a gear train having a tooth ratio TR of N_1/N_2, with $N_2 > N_1$ so that torque multiplication results, we obtain

$$\eta = \frac{\text{actual output torque}}{\text{input torque/TR}} \tag{3.4.2}$$

Figure 3.4.1 shows a transmission consisting of a right-angle gear train having a tooth ratio of $\frac{1}{15}$ and a set of antibacklash gears having a ratio of $\frac{1}{5}$. A plot of the input versus the output torques for the assembly is shown in Figure 3.4.2. Note that the actual overall transmission has a measured efficiency of only 22% as compared with an ideal performance of 100%. Although it is possible for efficiency to be a function of speed, a first approximation would consider it to be dependent only on the forces encountered by the gear teeth, which are primarily frictional in nature. Since these forces are directly proportional to torques, the efficiency of

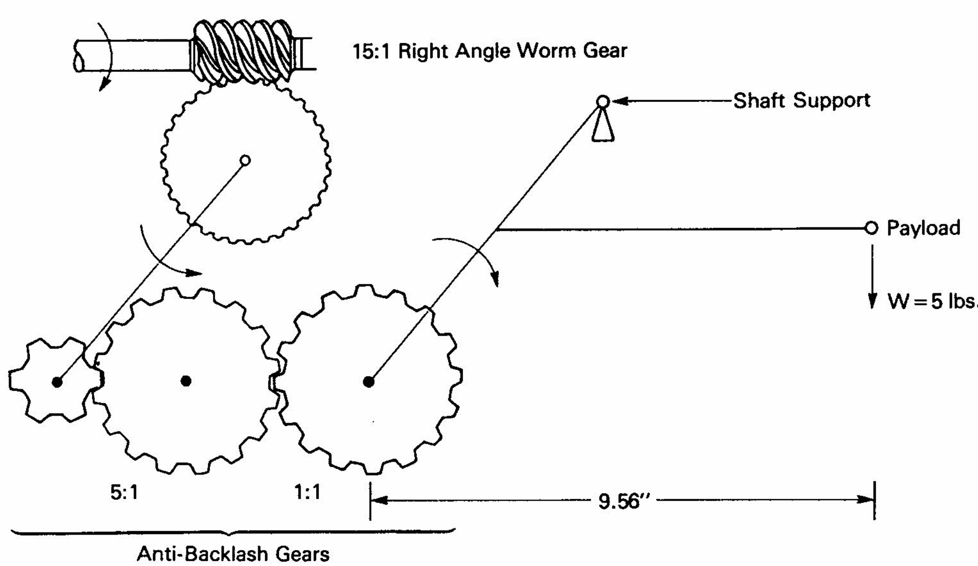

Figure 3.4.1 Complex gear assembly.

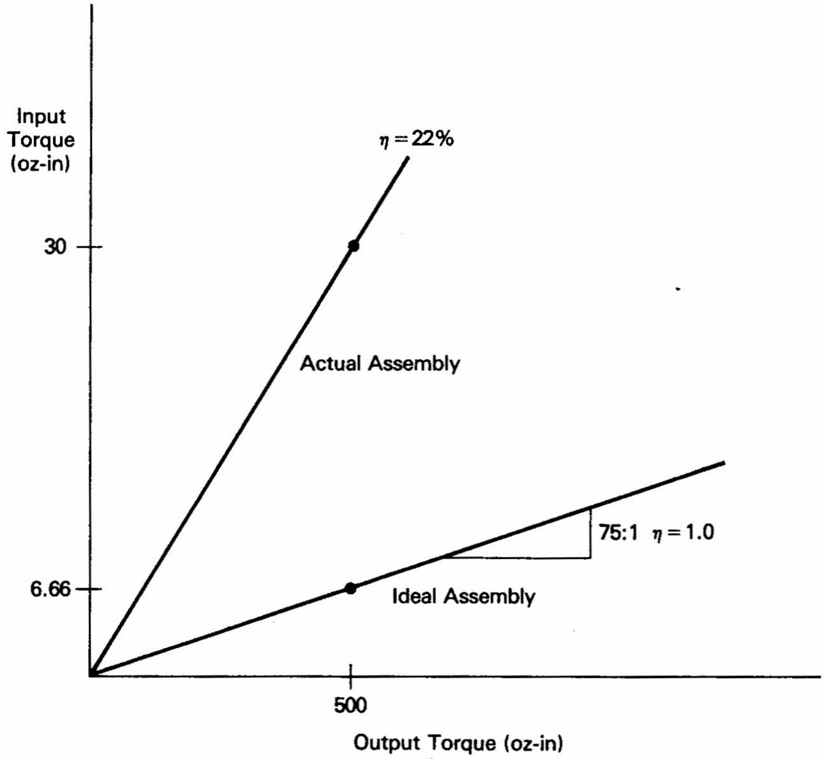

Figure 3.4.2 Actual and theoretical torque transfer curves.

the transmission can be approximated by measuring the resulting torque on the output for static torques applied to the input shaft.

The efficiency of any mechanical device becomes important in sizing actuators. It is no longer safe to assume that the output loads are reflected to the input shaft by a function of the gear ratio as defined in Eqs. (3.3.8a) through (3.3.8c), but one must now include the efficiency. These equations then become

$$T_{\text{total}} = \left(J_1 + \frac{\text{TR}^2}{\eta} J_2\right)\ddot{\theta}_1 + \left(B_1 + \frac{\text{TR}^2}{\eta} B_2\right)\dot{\theta}_1 + T_f \tag{3.4.3a}$$

$\eta \leq 1.0$

$$T_f = F_{c1}\,\text{sgn}(\theta_1) + \frac{\text{TR}}{\eta} F_{c2}\,\text{sgn}(\theta_2) \tag{3.4.3b}$$

$$\text{TR} = \frac{N_1}{N_2} \tag{3.4.3c}$$

These equations reveal that any efficiency less than 1 (i.e., 100%) will increase the torque required to accelerate a given inertial load or overcome an external torque load. It is important to note that efficiency does not affect the actual transfer ratio of the gears (or other transmission device) in terms of displacement, velocity, or acceleration, but greatly affects any torque-related property.

EFFICIENCY P144-145

$$\eta = \frac{\text{POWER OUT}}{\text{POWER IN}} = \frac{\text{WORK OUTPUT}}{\text{WORK INPUT}}$$

Let's say the efficiency of the transmission is 22%; $\eta = 0.22$

IDEALLY WE CALCULATED IN oz-inches

$$\frac{\text{INPUT } T}{\text{OUTPUT } T} = \frac{6.66}{500} \quad \text{Ratio is } \frac{N_1}{N_2} = \frac{1}{75}$$

BUT $T_{TOTAL} = \left(J_1 + \frac{(N_1/N_2)^2}{\eta} J_2\right)\ddot{\theta} + \cdots$

So $J_{ideal\,OUT} = 75 \times 6.66$ oz-in $= 500$ oz-in

but $J_{actual} = .22 \times 500 = 110$ oz in

So MOTOR TORQUE T_1 is

$$\frac{6.66}{.22} = \underline{\underline{30.27 \text{ oz in}}}$$

≈ 5X as much TO get 500 oz-in