servomotor.

EXAMPLE 4.6.1

SPEC

Suppose that we wish to move a robot joint a total distance of π radians in 400 ms using a stepper motor. The joint "sees" a reflected load inertia $J_L = 0.004$ oz-in.s^2. (The load is coupled to the motor shaft through a 10:1 gear train.) It is proposed to use a stepper with the following specifications: SEE P122

SPEC1

π rad in 400ms

MOTOR

$$J_M = 0.003 \text{ oz-in.-s}^2$$

$$\text{step angle} = 1.8^\circ \text{ (200 steps/rev)}$$

$$\text{rms torque} = \text{rated continuous torque} = 7 \text{ oz-in.}$$

$$\text{maximum step rate} = \text{slew rate} = 4000 \text{ steps/s}$$

In addition, a triangular velocity profile is to be assumed (see Figure 4.6.6). Here t_a and $(t_g - t_a)$ are the acceleration and deceleration times, respectively, and are assumed to be equal in this case. The acceleration and distance curves resulting from this velocity are also shown in parts b and c of this figure and ω_{pk} and α_{pk} are the peak angular velocity and peak angular acceleration of the motor shaft, respectively. The problem is to determine whether the given motor will be able to meet all the motion requirements for a joint move of this type.

WILL IT WORK?

Because the load is coupled to the motor shaft through a 10:1 speed reducer, the *motor* must move 10π radians (i.e., 10 times the distance of the actual joint output). From Figure 4.6.6b and c, it is seen that

$$\omega_{pk} \frac{t_g}{2} = 10\pi$$

✓ here $t_g = 2t_a$

and

$$\alpha_{pk} = \frac{\omega_{pk}}{t_a}$$

(+ acceleration)

Since $t_a = t_g/2 = 200$ ms, ω_{pk} and α_{pk} needed to make this move are found to be

$$\omega_{pk} = 50\pi = 157.1 \text{ rad/s} = \frac{10\pi \times 2}{0.4\,s}$$

$$\alpha_{pk} = \frac{157.1}{0.2} = 785.5 \text{ rad/s}^2 = \frac{50\pi}{0.2}\ r/s^2$$

The corresponding peak acceleration and deceleration torques in this case

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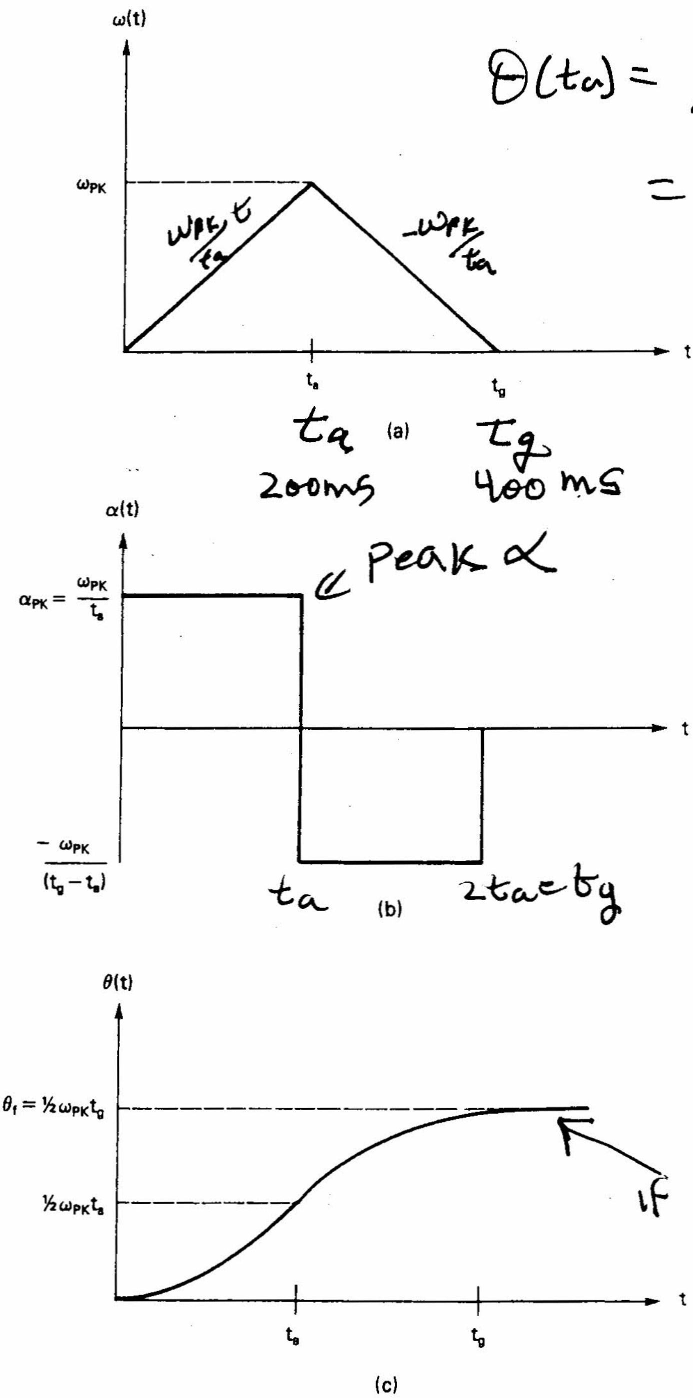


$$\Theta(t_a) = \int_0^{t_a} \omega_{PK} \, t/t_a \, dt$$

$$= \frac{\omega_{PK}}{t_a} \, t^2/2 \Big|_0^{t_a} = \frac{\omega_{PK} t_a}{2}$$

$$\frac{d\omega}{dt} = \alpha \qquad \omega = \int \alpha \, dt$$

$$\int v \, dt + \Theta(t_a)$$

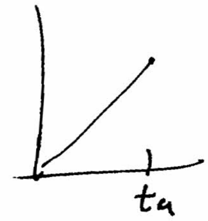
$$\alpha = \omega = 0 \qquad \theta = \Theta(t_g)$$

Figure 4.6.6. Assumed velocity, acceleration, and position profiles for Example 4.6.1. Shown are the motor shaft angular: (a) velocity; (b) acceleration; (c) position.

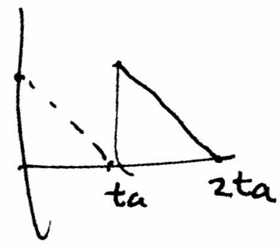
See Figure 4.616 p 260

$$\theta(t_a) = \int_0^{t_a} \omega_{pk} \, t/t_a \, dt = \frac{\omega_{pk}}{t_a} \left\{ t^2/2 \right\}_0^{t_a}$$

$$= \omega_{pk} \, t_a/2$$



$$\underset{t \geq t_a}{\theta(t)} = \int_{t_a}^{2t_a} \omega_{pk} \left(1 - \frac{t - t_a}{t_a} \right) dt$$



$$= \omega_{pk} \left[t - \frac{t^2}{2t_a} + t \right] \Bigg|_{t_a}^{2t_a}$$

$$\omega(t) = -\frac{\omega_{pk}}{t_a}(t - t_a) + \omega_{pk}$$

$$\omega(\cancel{t_a}) = \omega_{pk}$$

$$\omega(2t_a) = -\frac{\omega_{pk}}{t_a}(t_a) + \omega_{pk}$$

$$= 0$$

$$= \omega_{pk} \left\{ \left[2t_a - 4t_a/2 + 2t_a \right] - 2t_a + t_a/2 \right\}$$

$$= \omega_{pk} (t_a/2)$$

So $\theta(2t_a) = \theta(t_g) = \omega_{pk}(t_a/2) + \omega_{pk}(t_a/2)$

$$= \underline{\omega_{pk} \, t_g/2} \checkmark$$

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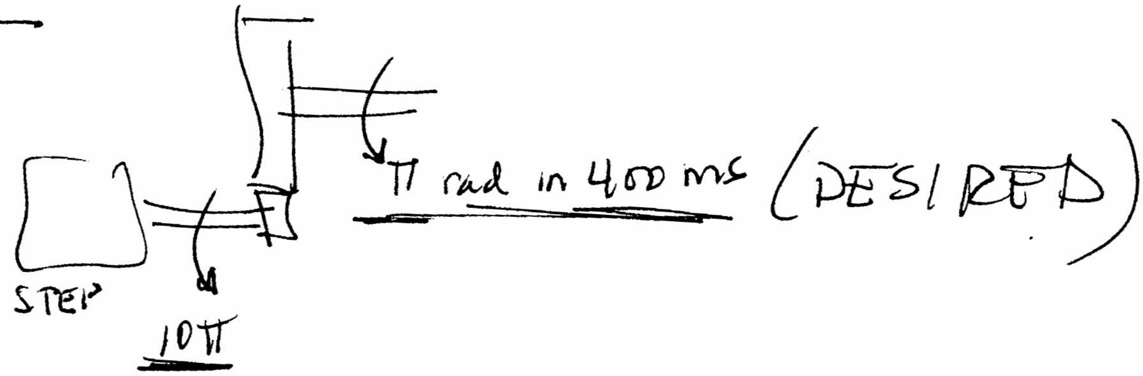
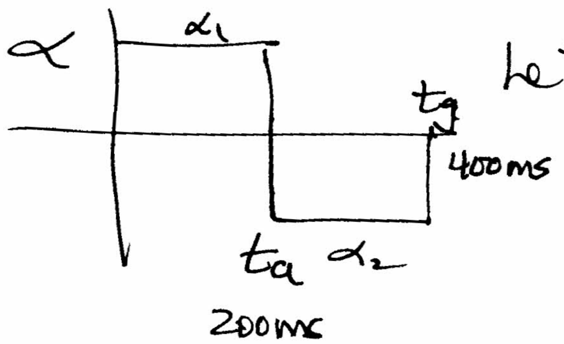


So move 180° (π rad) in 400 ms using accelleration ½ time and then decellerate ½ time



Let $\alpha_1 = \frac{\omega_{pk}}{t_a}$

$$\alpha_2 = \frac{-\omega_{pk}}{(t_g - t_a)}$$

So for the motor to turn joint π, motor rotates 10π in the time t_g

What is average velocity $0 - t_a$

$$\frac{\frac{1}{2}(t_a)\,\omega_{pk}}{t_a}$$

velocity $t_a - t_g$ $\quad \frac{-\frac{1}{2} t_a \omega_{pk}}{(t_g - t_a)}$ $\quad t_g - t_a = t_a$ here

So $\theta_1 = \frac{1}{2}\omega_{pk} \cdot t_a$ $\quad \theta_2 = \frac{1}{2}\omega_{pk} t_a$

So for motor to move 10π P260 (5)

$$10\pi = \frac{1}{2}\omega_{pk} t_a + \frac{1}{2}\omega_{pk} t_a$$
$$= \frac{1}{2}\omega_{pk}(2t_a) = \frac{1}{2}\omega_{pk}(t_g)$$

So motor moves 10π in 400 ms (t_g)

$$\omega_{pk} = \frac{10\pi \times 2}{.4\ sec} = 50\pi = rad/sec$$

Then of course for t_a time

$$\alpha_{pk} = \frac{\omega_p}{t_a} = \frac{50\pi}{0.2} = 250\pi\ rad/s^2$$

(1) So Torque

$$T_{accel} = (J_L + J_M)\,\alpha_{pk}$$
$$= \{(0.004 + 0.003)\ oz\text{-}in\text{-}s^2\}\ 250\pi\ rad/s^2$$

OK

$$= 5.50\ oz\text{-}in < 7\ oz\text{-}in$$

(2) $$\omega_{pk} = \frac{\text{Total move velocity}}{\text{single step}} = \frac{50\pi\ r/s}{0.031416\ (1.8°)} = 5000\ s/sec$$

$$= \frac{rads/sec}{rad/step}$$

TOO FAST TO MEET 400MS move

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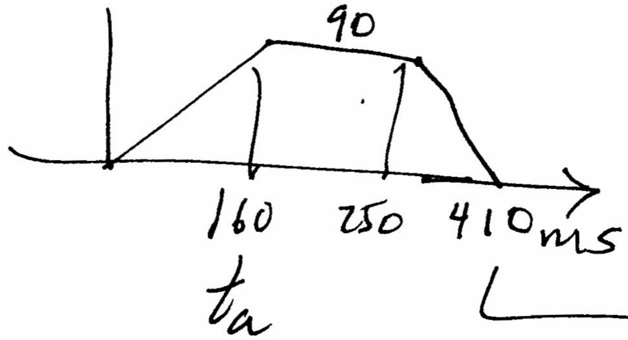
SOLUTION



Relax
400 ms

* VISCOUS DAMPING and friction are ZERO.

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are equal.* Thus

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$$T_{\text{accel}} = (J_L + J_M)\alpha_{\text{pk}} = 5.5 \text{ oz-in.}$$ < 7 oz-in

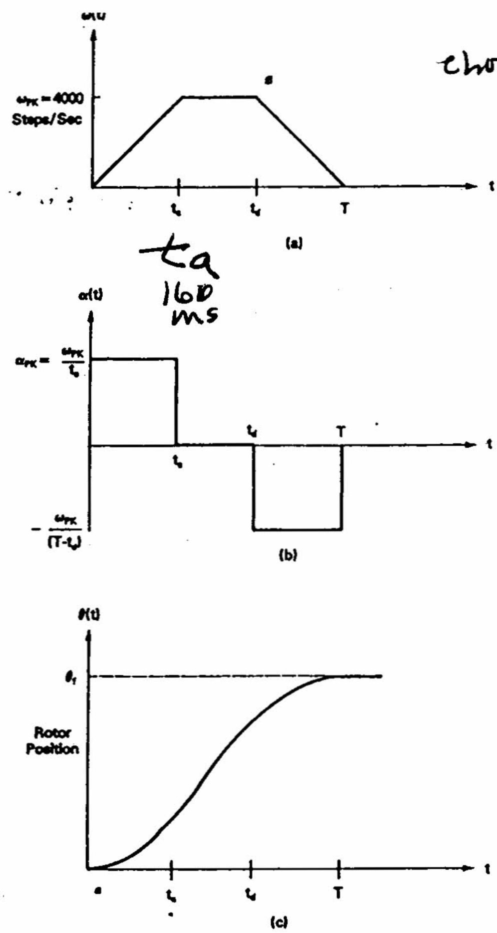
Since the acceleration curve in Figure 4.6.6b is piecewise constant, the rms and peak torques are equal.* Consequently, it is seen that the proposed motor is adequate from a torque point of view. However, a single step is 1.8° or 0.031416 rad. The resultant peak angular velocity is therefore

$$\frac{\text{rads/sec}}{\text{rads/STEP}} \qquad \omega_{\text{pk}} = \frac{157.1}{0.031416} = 5000 \text{ steps/s}$$ TOO FAST

Clearly, the speed requirement exceeds the maximum slew rate of the motor by 25%. In fact, it is probable that if we use it for the proposed application, steps will be dropped and accuracy will suffer.

There are two things that can be done to meet the requirements of the problem. 1 One involves using a motor that has the same torque rating but a higher slew rate. 2 The other necessitates relaxing one of the specs. For example, suppose that it is permissible to make the move in slightly more than 400 ms. Then the trapezoidal velocity profile shown in Figure 4.6.7 could be employed. Using this profile with $\omega_{\text{pk}} = 4000$ steps/s and the acceleration and deceleration times still assumed to be equal, it is found that the acceleration torque is still 5.5 oz-in, $t_a = 160$ ms and the overall move time $T = 410$ ms (see Problem 4.23). Thus only a small time penalty results from using a constant-velocity segment of 90 ms during the move.

if w_p = 5000 steps/sec choose faster motor or slow This one down to ≤ 4000 steps/sec



Solution here let motor

Figure 4.6.7. Alternate set of profiles (assuming a trapezoidal velocity) for Example 4.6.1. Shown are the motor shaft angular: (a) velocity; (b) acceleration; (c) position.