BODE/TRACK

plots of the magnitude and phase of the frequency transfer function versus frequency will usually reveal where the problem lies and compensation can then be added to correct it (these graphs are referred to as *Bode plots*).

4.4.1 Bode Plots

Let us illustrate the frequency-domain approach through the use of an example. We begin by considering the tach or velocity portion of the position servo shown in Figure 4.3.5, consisting of an amplifier and servomotor. Assuming that the amplifier bandwidth is 1000 Hz (i.e., $\tau_A = 1/6280$) and that the motor in Example 4.3.1 is used with an inertial load of 0.007 oz-in.-s² [i.e., J_T in Eq. (4.3.5) is 0.0108 oz-in.-s²], the tach open-loop transfer function is given by

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$\tau_A = 1\,ms$

$\tau_m = 0{,}14\,sec$

K

$\propto s^3 + \cdots + 1$

$$GH(s) = \frac{14.1AK_g}{(1 + s/6280)(1 + s/44.14)(1 + s/439.73)} \tag{4.4.1}$$

The frequency transfer function (FTF) is obtained from this equation by substituting $j\omega$ for s, where ω, the radian frequency (having the units rad/s) is equal to $2\pi f$ (f in hertz). The magnitude of the FTF, expressed in decibels [i.e., $20 \log_{10}(|\text{FTF}|)$] and its corresponding phase angle both drawn versus $\log_{10}\omega$ are called Bode plots. This is shown in Figure 4.4.1 for the tach open-loop FTF.

In Figure 4.4.1a both the straight-line approximation and the continuous plots are given for $AK_g = 1$. The former is obtained by following a set of simple rules:

1. The FTF is placed in Bode form as shown in Eq. (4.4.2):

$$GH(j\omega) = K_{\text{Bode}} \frac{\prod_{i=1}^{M} (1 + j\omega/\omega_{z_i})}{\prod_{k=1}^{N} (1 + j\omega/\omega_{p_k})} \tag{4.4.2}$$

   where ω_{z_i} and ω_{p_k} are called the "break" frequencies corresponding to each of the M zeros and N poles of $GH(s)$, respectively. Note that the transfer function in Eq. (4.4.1) is already in Bode form.

2. The magnitude of $GH(j\omega)$ expressed in dB is then

$$\text{dB} = 20 \log (K_{\text{Bode}}) + \sum_{i=1}^{M} 10 \log \left[1 + \left(\frac{\omega}{\omega_{z_i}}\right)^2\right] - \sum_{k=1}^{N} 10 \log \times \left[1 + \left(\frac{\omega}{\omega_{p_k}}\right)^2\right] \tag{4.4.3}$$

   Where "log" implies "log to the base 10." Note that multiple poles and zeros are permitted so that all of the ω_{p_k}'s and/or ω_{z_i}'s need not be distinct.

$$H(j\omega) = H(s)\Big|_{s=j\omega}$$

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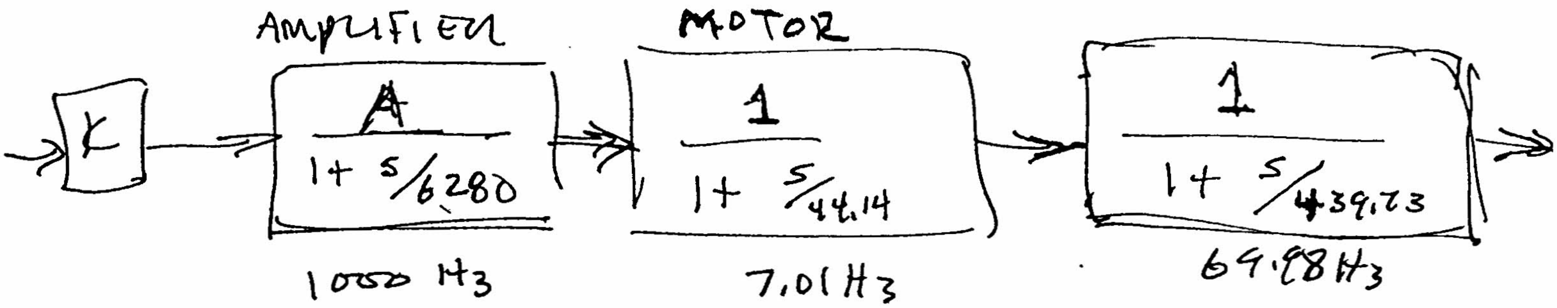
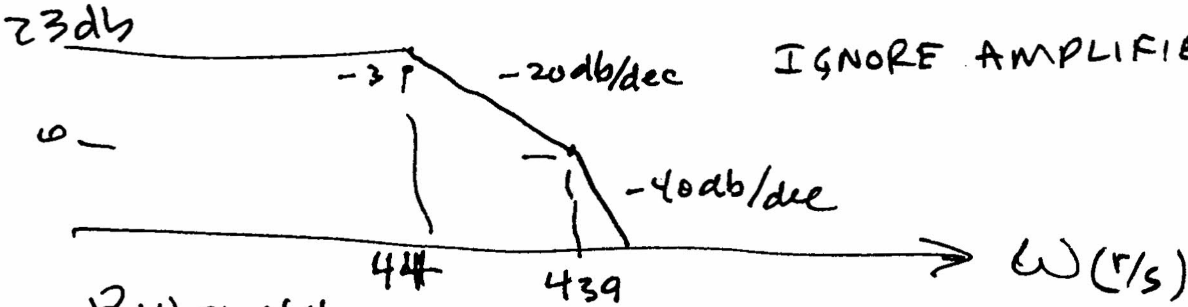
EQ 4.4.3 FROM 4.4.1; MOTOR WITH VELOCITY FEEDBACK



BASICALLY IGNORE THE AMPLIFIER – IT IS HIGH FREQUENCY

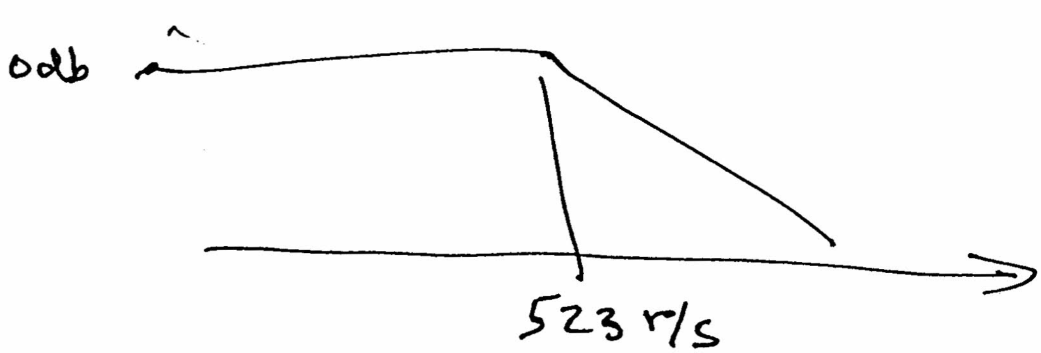
$$db = \underbrace{20\log(K_{bode})}_{23db} - \sum_{i=1}^{m} 10\log\left[1 + \left(\frac{\omega}{\omega_{pk}}\right)^2\right]$$



IGNORE AMPLIFIER BW

BW ≈ 44 r/s

IN CLOSED LOOP P229 – EXTEND BANDWIDTH



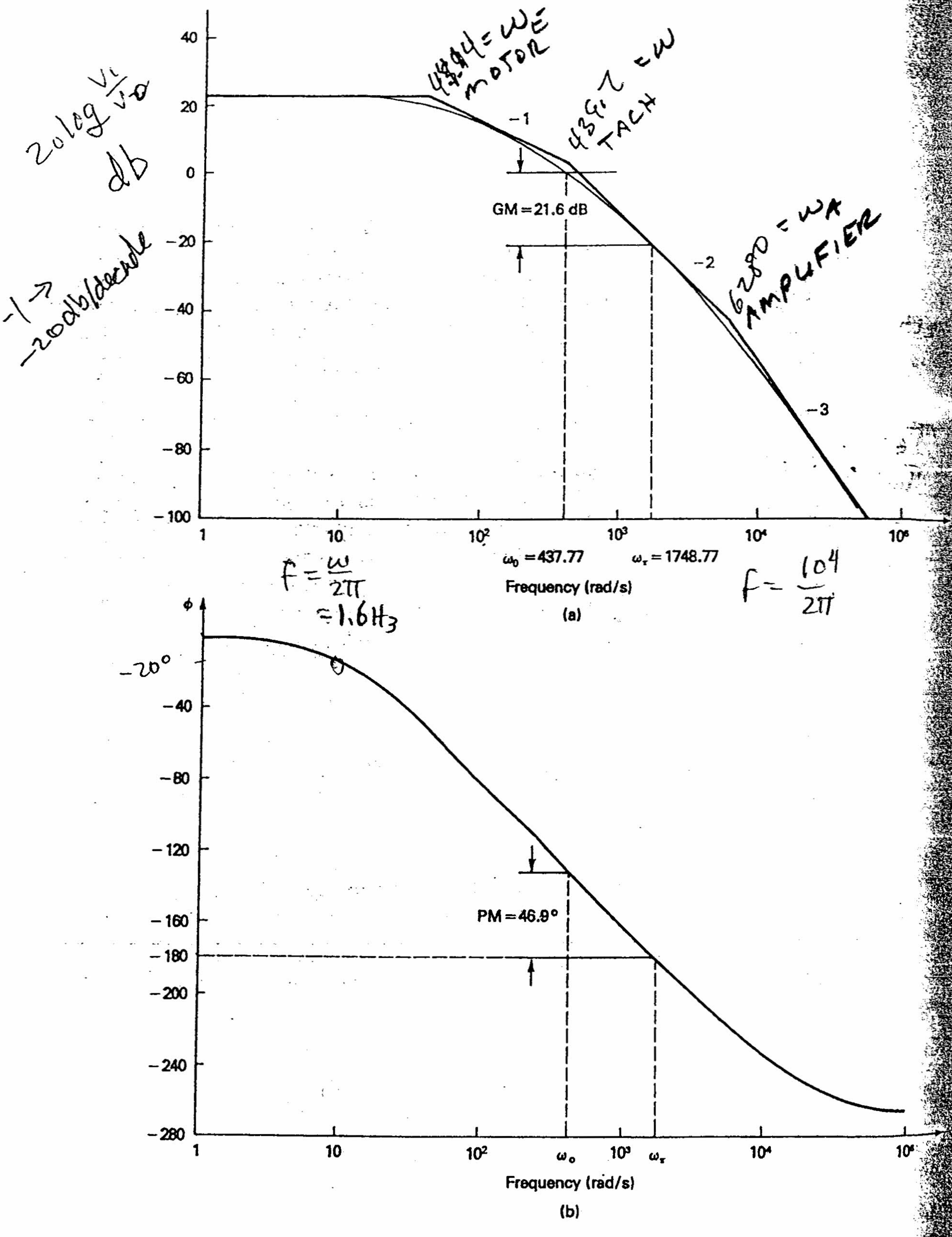


Figure 4.4.1. Bode plots for the open-loop tach described by Eq. (4.4.1) with $AK_g = 1$ and $K_g = 1$. The gain and phase margins are also shown: (a) magnitude (in dB) vs. $\log_{10} \omega$; (b) phase angle ϕ (in degrees) vs. $\log_{10}\omega$.

P230 DISCUSSION OF BANDWIDTH
AND TRACKING ERROR

MOTOR BW 44.14 r/s (7.03 Hz)

CLOSED LOOP BW 336.7 r/s (53.6 Hz)

THIS IS ABOUT AN 8-FOLD INCREASE

EFFECT OF LIMITED BANDWIDTH

SYSTEM WILL BE SLUGGISH AND
THE RESPONSE WILL LAG THE INPUT
COMMAND

SEE PAGE 231 FIGURES $T_1 = 0.5$ sec / $T_2 = 2$ sec
$BW_1 = 2$ r/s $BW_2 = 0.5$ r/s

1/2

Recalling that the break frequencies occur at the poles or zeros of the transfer function, the approximate tach closed-loop transfer function can be obtained from this figure. Thus

$$G_{\mathrm{tach}}(s) = \frac{1}{(1 + s/523.1)^2(1 + s/6280)} \tag{4.4.7}$$

Note that the double pole at 523.1 occurs because the slope of the straight-line plot changes from 0 to -2 at this frequency. It is important for the reader to understand that the continuous curve shown in Figure 4.4.3 is what one would get for the FTF of Eq. (4.4.7). However, it is *still an approximation of the actual closed tach loop response*.

4.4.4 Bandwidth and Tracking Error Considerations

Often, the *bandwidth* of a system is defined as the frequency where the response is down from its peak value by 3 dB. Using this definition, it is seen from Figure 4.4.3 that the closed-loop tach bandwidth is about 336.7 rad/s or 53.6 Hz. Note that the motor bandwidth was 44.14 rad/s or 7.03 Hz. Thus it is observed that closing the loop has significantly increased the bandwidth of the system. This is, of course, a well-known result.

A more important point to remember is that the bandwidth limits the maximum speed with which a system can respond to an input signal. That is, if the input requires a rapid change (with respect to time), the system must have sufficient bandwidth to follow (or "track") this command. Otherwise, the response will significantly lag behind the command input, thereby producing a large "tracking error."

An example of such behavior is shown in Figure 4.4.4. Here the effect of moving the single system pole is observed. In Figure 4.4.4a, the pole is located at $s = -2$ (bandwidth $= 2$ rad/s), whereas in Figure 4.4.4b it is at $s = -0.5$ (bandwidth $= 0.5$ rad/s). Note that the tracking error is much smaller for the first case (where the bandwidth is larger).

Amplifier saturation and sluggish performance are possible consequences of large tracking error. Moreover, in robotic applications, this quantity is often monitored in order to determine whether the manipulator is actually moving (i.e., has it hit an unforeseen obstacle or is the load too large to handle?). If the joint normally exhibits a large tracking error because of insufficient bandwidth in the servo loop, the criterion for automatically halting the manipulator motion must be relaxed. This can have serious safety consequences and can also permit damage to electrical and/or mechanical components. In addition, if the tracking error of all of the joints of the robot is not the same, it will be virtually impossible for the robot to move its end effector in a straight line.

In more mathematical terms, if it is required that at least a certain percentage of the total energy contained in a signal be "passed" by a system in order to reproduce it faithfully at the output, and the frequency at which this occurs is $f_{\%}$

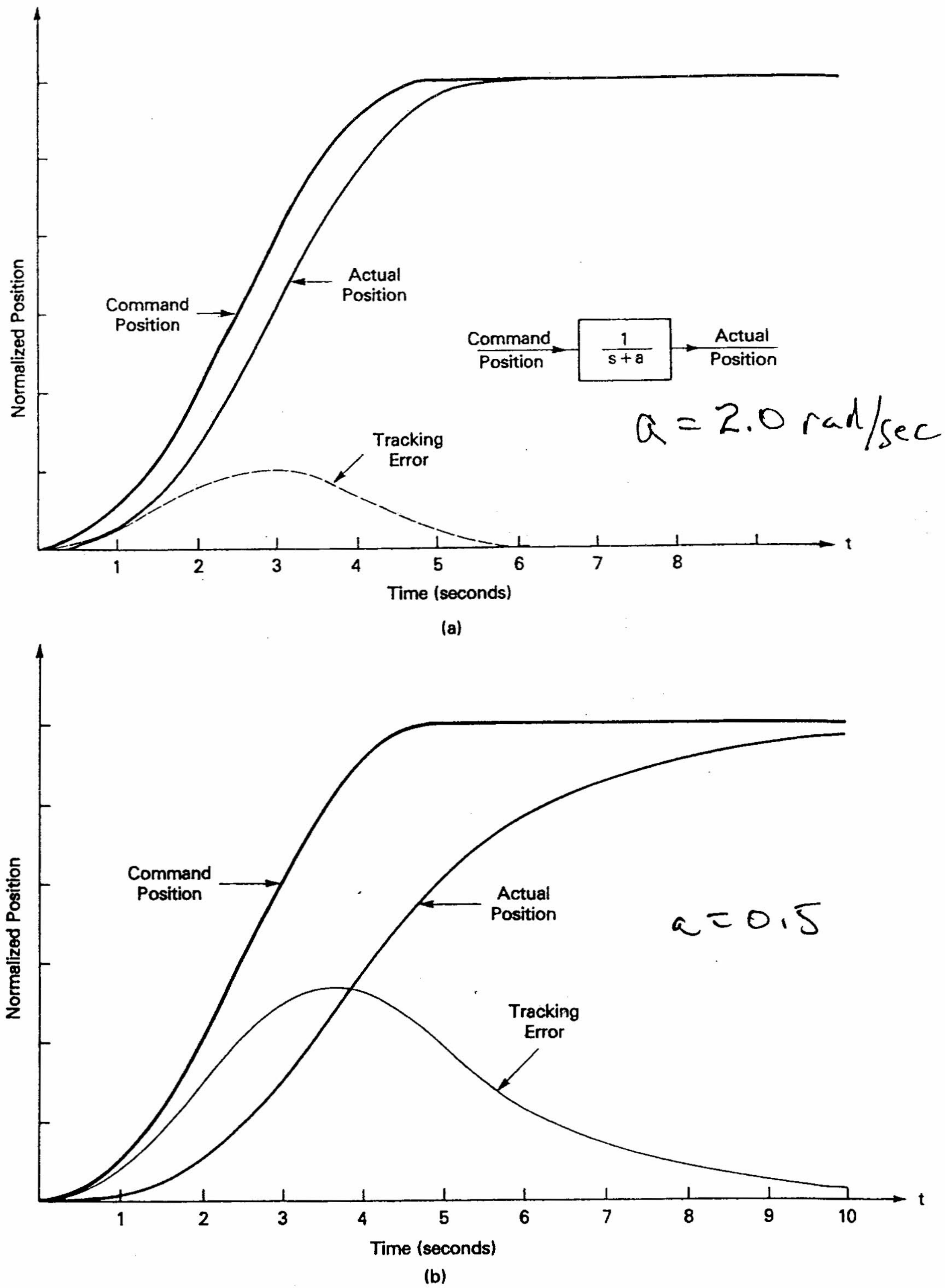


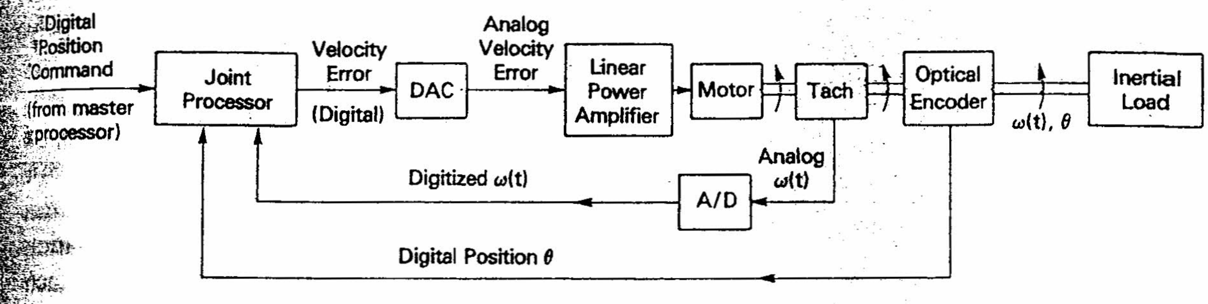
Figure 4.4.4. Illustration of the effect of system bandwidth on tracking error: (a) a = 2; (b) a = 0.5.

DIGITAL CONTROL
APPENDIX C P729

monitoring velocity and position. An optical encoder (or some other digital position sensor; see Section 5.3.2.2) mounted on either the motor shaft or output (i.e., workpiece or tool tip) is used to produce a digital representation of the current position. Velocity information is provided through the use of an analog tachometer similarly mounted. Since the information from the latter device must be converted into digital data in order to be utilized by the joint processor, an analog-to-digital (A/D) converter may have to be employed.

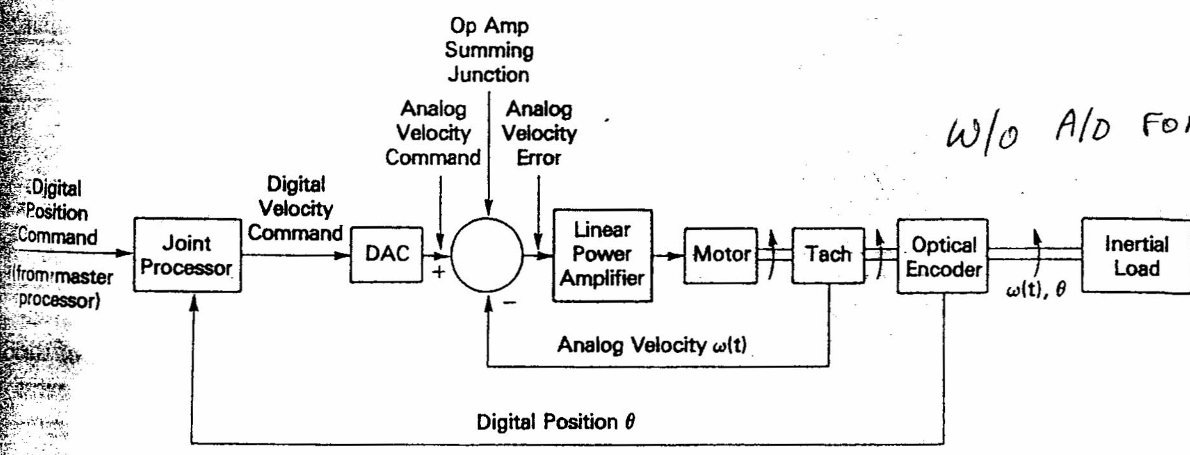
For the joint servo in a robot, the command signal from the master computer is digital in form. As mentioned previously, the joint processor uses this input together with the position and/or velocity information to produce an error signal which is used either directly or indirectly as the input to the servo (power) amplifier. Two approaches are possible.

In the first case, the joint processor uses *both* the position and velocity information to produce a velocity error signal, which is in turn, used as the drive



(a)

W/O A/D FOR ω



(b)

Figure 4.5.1. Two realizations of a joint position servo utilizing digital position and analog velocity information: (a) both ω and θ are digitized and fed back to the joint processor to obtain the velocity error signal; (b) only the digitized value of θ is used to produce the velocity command. Then, this is compared to the analog value of ω to obtain the velocity error signal.

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