CONTROL TECHNIQUES

1. OPEN LOOP (P204)
   OFTEN USED WITH STEPPER MOTORS

2. CLOSED-LOOP (P 204 – 244)
   FEEDBACK CONTROL - MOST USED

3 ADAPTIVE AND OPTIMAL CONTROL P246–248

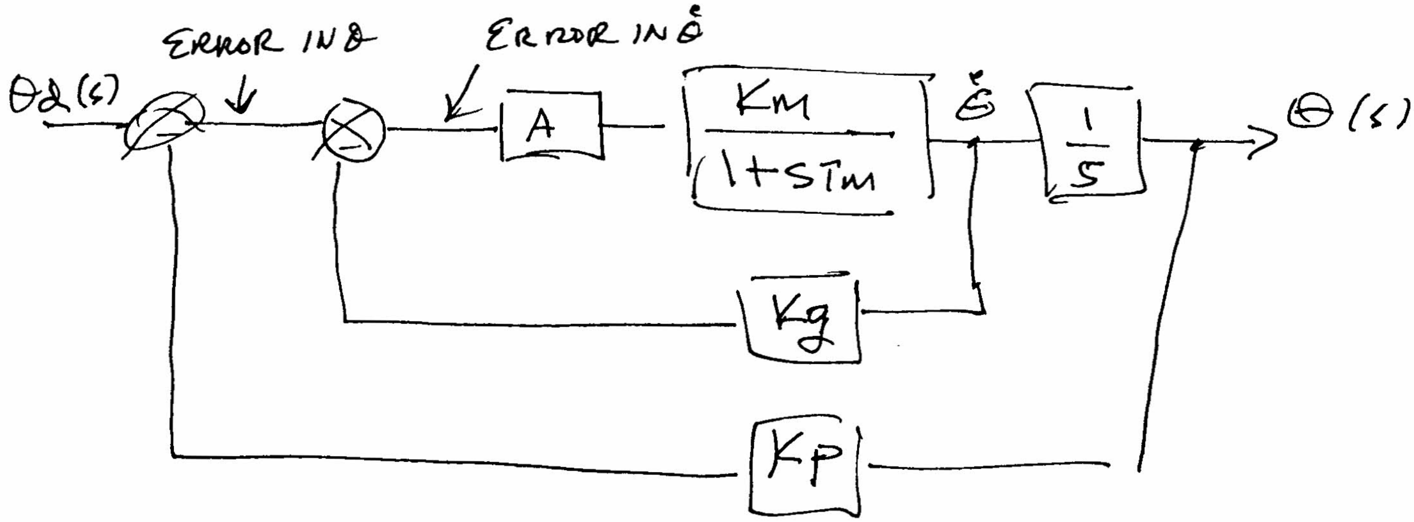
- CONTROL PARAMETERS CHANGE ACCORDING TO INERTIAL LOAD VARIATION

- MODEL BASED CONTROL – COMPARE RESPONSE TO MODEL PREDICTION AND ADJUST PARAMETERS

- EXAMPLE WIDELY USED – AUTOMATIC GAIN CONTROL IN RADIO

- OPTIMAL CONTROL (P 247)
  CONTROL TO OPTIMIZE (MAXIMIZE) A PERFORMANCE PARAMETER WHILE MEETING CONSTRAINTS
  I.E. FIND INPUTS THAT MAXIMIZE SPEED OF MOTION WITH LIMITATION ON DC SUPPLY VOLTAGE TO POWER MOTORS.

KLAFTER SHOWS THAT MOTOR CAN BE APPROXIMATED AS SECOND ORDER SYSTEM



DEAL WITH THE MOTOR AND TACH

$$T_{im}(s) = \frac{G_{im}}{1+G_{im}K_g} = \frac{\frac{AK_m}{1+s\tau_m}}{1+\frac{AK_mK_g}{1+s\tau_m}} = \frac{AK_m}{s\tau_m+(1+AK_mK_g)}$$

NOW FEEDBACK KP - POSITION

$$T(s) = \frac{\theta(s)}{\theta_d(s)} = \frac{T_{im}\cdot 1/s}{1+T_{im}1/s\cdot K_P} = \frac{T_{im}}{s+T_{im}K_P}$$

$$= \frac{AK_m}{s[s\tau_m+(1+AK_mK_g)]+AK_mK_P} =$$

$$\frac{AK_m/\tau_m}{s^2+s(1+AK_mK_g)/\tau_m+\frac{AK_mK_P}{\tau_m}} \quad 4.2.4$$

Now we see this as

$$T_2(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad 2^{ND} \text{ DEGREE}$$

define $\omega_n^2 = A K_m K_p / \tau_m$ so

$$T(s) = \frac{1/K_p \, \omega_n^2}{s^2 + s(1 + A K_q K_m)/\tau_m + \omega_n^2} \qquad \underline{4.2.4}$$

so $2\zeta\omega_n = (1 + A K_q K_m)/\tau_m$ AND THE DAMPING RATIO

$$\zeta = \frac{1}{2} \frac{(1 + A K_q K_m)/\tau_m}{\sqrt{A K_m K_p / \tau_m}} = \frac{1}{2} \frac{(1 + A K_q K_m)}{\sqrt{A K_m K_p \tau_m}}$$

FOR THE 2^{ND} ORDER SYSTEM

$\zeta < 1$ = UNDERDAMPED ; CRITICALLY DAMPED, $\zeta = 1$
OR OVER DAMPED = $\zeta > 1$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_{1,2} = \frac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2}$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$\zeta = 1$ Real, equal
$\zeta < 1$ complex
$\zeta > 1$ Real, unequal

CONTROL SUMMARY - KLAFTER CH 4 p 208

CONSIDER EQ 4.25 PHILLIPS + HARBOR

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \text{AND} \quad \frac{T(s)}{s}; \text{ STEP}$$

$$t(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin(\beta\omega_n t + \theta) \qquad 0 < \zeta < 1$$

$$\beta = \sqrt{1-\zeta^2} \quad \theta = \tan^{-1}(\beta/\zeta); \quad \tau = 1/\zeta\omega_n \text{ sec.}$$

SEE THE CURVES FOR VALUES OF ζ. (PHILLIPS)

EQ. 4.2.6 (KLAFTER)

$$\zeta \simeq \frac{A^{1/2} K_m^{1/2} K_g}{\sqrt{K_p T_m}}$$

a) INCREASING K_g (VELOCITY feedback) INCREASES DAMPING

b) INCREASING POSITION FEEDBACK K_p DECREASES ζ.

COMPARE FIGURES 4.3.9, 4.3.10, 4.3.11, 4.3.12, 4.3.13

PID $\quad E(s) = \Theta_d(s) - \Theta(s)$

$$M(s) = K_p E(s) + sK_D E(s) + K_I \frac{E(s)}{s} \qquad \text{ACTUATOR SIGNAL}$$

IF $\Theta_d(s) = \frac{\Theta_d}{s}$ CONSTANT (STEP INPUT)

$$m(t) = K_p[\theta_d - \theta(t)] - K_p \frac{d\theta(t)}{dt} + K_I \int [\theta_d - \theta(t)] dt$$

P: K_p (F 4.3.9) LARGER K_p - FASTER RISE; ERROR $\propto \frac{1}{K_p}$

PD: K_D (F 4.3.10) LARGER K_D - SLOWS RESPONSE IF $\theta(t)$ is INCREASING.

$K_D = 0.02$ CRITICAL DAMPING

PID (F 4.3.13) $K_p = 20$ NO OVERSHOOT

The inverse Laplace transform is not derived here (see Problem 4.8); however, assuming for the moment that the poles of $G(s)$ are complex, the result is

$$c(t) = 1 - \frac{1}{\beta}e^{-\zeta\omega_n t}\sin(\beta\omega_n t + \theta) \tag{4-20}$$

where $\beta = \sqrt{1-\zeta^2}$ and $\theta = \tan^{-1}(\beta/\zeta)$. In this response, $\tau = 1/\zeta\omega_n$ is the time constant of the exponentially damped sinusoid in seconds (we can usually ignore this term after approximately four time constants). Also, $\beta\omega_n$ is the frequency of the damped sinusoid.

We wish now to show typical step responses for a second-order system. The step response given by (4-20) is a function of both ζ and ω_n. If we specify ζ, we still cannot plot $c(t)$ without specifying ω_n. To simplify the plots, we give $c(t)$ for a specified ζ as a function of $\omega_n t$. A family of such curves for various values of ζ is very useful and is given in Figure 4.4 for $0 \le \zeta \le 2$. Note that for $0 < \zeta < 1$, the response is a damped sinusoid. For $\zeta = 0$, the sinusoid is undamped, or of sustained amplitude. For $\zeta \ge 1$, the oscillations have ceased. It is apparent from (4-20) that for $\zeta < 0$, the response grows without limit. We consider only the case that $\zeta \ge 0$ in this chapter. A MATLAB program that calculates some of the step responses of Figure 4.4 is given by

```
zeta = [0.2 0.5 1 2];
for k = 1:4
 G =tf ([1],[1 2*zeta(k) 1]);
   step(G)
   hold on
end
hold off
```

The two poles of the transfer function $G(s)$ in (4-18) occur at

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

For $\zeta > 1$, these poles are real and unequal, and the damped sinusoid portion of $c(t)$ is replaced by the weighted sum of two exponential functions; that is,

$$c(t) = 1 + k_1e^{-t/\tau_1} + k_2e^{-t/\tau_2} \tag{4-21}$$

where $\tau_1 = 1/(\zeta\omega_n + \omega_n\sqrt{\zeta^2-1})$, $\tau_2 = 1/(\zeta\omega_n - \omega_n\sqrt{\zeta^2-1})$ are the two system time constants. For $\zeta = 1$, the poles of $G(s)$ are real and equal, so that

$$c(t) = 1 + k_1e^{-t/\tau} + k_2te^{-t/\tau},\ \tau = 1/\omega_n$$

For $0 < \zeta < 1$, the system is said to be *underdamped,* and for $\zeta = 0$ it is said to be *undamped.* For $\zeta = 1$, the system is said to be *critically damped,* and for $\zeta > 1$, the system is *overdamped.*

For a linear time-invariant system,

$$C(s) = G(s)R(s) \tag{4-22}$$

For the case that $r(t)$ is a unit impulse function, $R(s) = 1$ and

$$c(t) = \mathcal{L}^{-1}[G(s)] = g(t) \tag{4-23}$$

where $g(t)$ is the *unit impulse response,* or *weighting function,* of a system with the transfer function $G(s)$. Then, by the convolution integral (see Appendix B), for a general

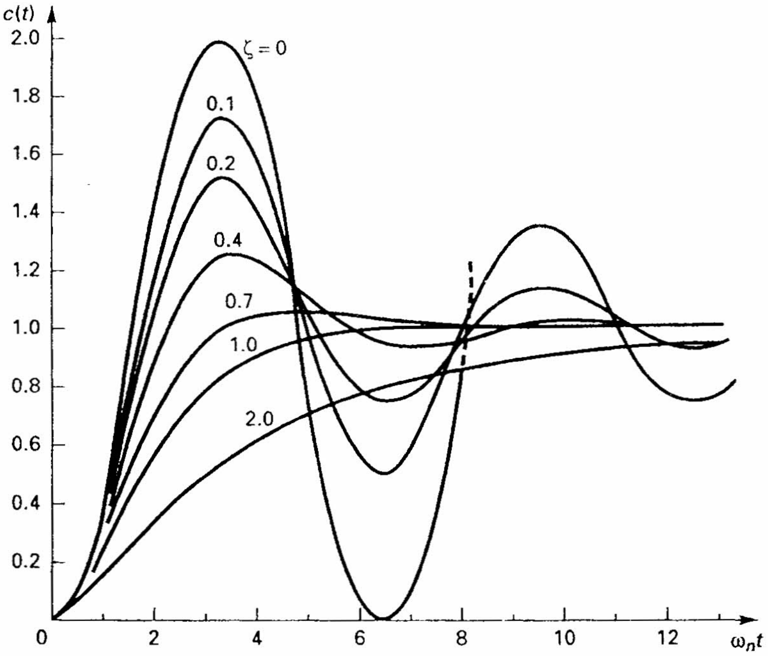


Figure 4.4 Step response for second-order system (4-20).

input $r(t)$,

$$c(t) = \int_0^t g(\tau)r(t-\tau)\,d\tau \tag{4-24}$$

from (4-22). [In (4-24), τ is the variable of integration and is not related to the time constant.] Hence, all response information for a general input is contained in the impulse response $g(t)$.

Recall also from Section 4.1 that an initial condition on a first-order system can be modeled as an impulse function input. While the initial condition excitation of higher-order systems cannot be modeled as simply as that of the first-order system, the impulse response of any system does give an indication of the nature of the initial-condition response, and thus the transient response, of the system. The unit-impulse response of the second-order system (4-18) is given in Figure 4.5. This figure is a plot of the function

$$c(t) = \mathcal{L}^{-1}\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right) = \frac{\omega_n}{\beta}e^{-\zeta\omega_n t}\sin\beta\omega_n t = g(t) \tag{4-25}$$

Compare Figure 4.4 with Figure 4.5 and note the similarity of the information. In fact, the unit impulse response of a system is the derivative of the unit step response (see Problem 4.9). The impulse response of the second-order system can also be considered to be the response to certain initial conditions, with $r(t) = 0$ (see Problem 4.9).

TEST MOTOR

$V_{arm} = R_a I_a + \omega(t) K_E$ Klafter p 211 4.3.2

Snyder p 74

NOTE AT STALL

$V_{arm} = 24V$ $I_{arm} = 10A$

$R_{arm} = \frac{24}{10} \Omega = 2.4 \Omega$

So Test at ω_{max} where $\ddot{\theta} = 0$

$T_q = (J_m + J_L) \cdot 0 + B\omega(t) + T_f$

If Back emf = V_{arm} $(I = 0)$

$V_{arm} = \omega_{max}(t) K_E$ $\omega = 3000 rpm$

or $K_E = \frac{24V}{3000} = 8 \times 10^{-3}$ V/rpm

NO TORQUE [IF T=0, I=0]