HW 3 Fourier Series Analysis 5431 Spring

Due February 16, 2015

February 6, 2015
Do the problems by hand and **check** the results whenever possible. However,

you may wish to verify your results with MATLAB solutions to the problems when appropriate. You can use symbolic MATLAB to check results if you wish

`Read Chapter 8 on WEB - Fourier Series Harman`

Problem 1 15 Points
Integrate the following by parts:

(a)

$$\int e^x \cos x \; dx$$

(b)

$$\int e^x \sin x \; dx$$

(c)

$$\int e^x e^{ix} \; dx$$

What is the relationship between the integrals in Parts a and b and the integral of Part c? How do we get the integrals of Parts a and b from the result in Part c?

Problem 2 20 Points

A square wave of amplitude A and period T as shown in Figure 8.4 in Harman Chapter 8 can be defined as

$$f(t) = \begin{cases} A, & 0 < t < \dfrac{T}{2}, \\ -A, & -\dfrac{T}{2} < t < 0, \end{cases}$$

with $f(t) = f(t+T)$, since the function is periodic.

(a) Do the math and prove that the Fourier Series can be written as

$$f(t) = \frac{4A}{\pi} \sum_{n=1}^{\infty} \frac{\sin[(2n-1)\omega_0 t]}{(2n-1)},$$

where $(2n-1)$ is introduced to assure that only odd terms are included in the summation. The sine waves that make up the Fourier series for the odd square wave are

$$f(t) = \frac{4A}{\pi} \left[\sin(\omega_0 t) + \frac{\sin(3\omega_0 t)}{3} + \cdots \right],$$

so the series consists not only of sine terms, as expected, but also odd harmonics appear. This is due to the rotational symmetry of the function since the wave shapes on alternate half-cycles are identical in shape but reversed in sign. Such waveforms are produced in certain types of rotating electrical machinery.

(b) Go through Example 8.5 in Harman Chapter 8 and fill in the details to show that the exponential form of the Fourier series as in Equation 8.30 leads to the same result as the trig series.

Problem 3 30 Points
Do Example 8.7, Page 391 in Fourier Analysis Chapter 8, in detail using the Complex series. Consider Figure 8.7 which shows the Spectrum of the trigonometric series and the reconstruction of $f(t)$.

1. Let A=2,tau=pi, T=2*pi To define Amplitude, pulse width, period. Plot the function $f(t)$ using MATLAB code similar to the following:

```
N = 21;             % Select arbitrary summation limit or use input(N odd)
w0 = 2*pi/T;        % w0=1, f=1/2pi, T=2pi fundamental frequency (rad/s)
t = -pi:0.01:3*pi;  % declare time values- Plot two cycles
% Compute yce, the Fourier Series in complex exponential form
% The complex values for n=1,2,... are 1/2 the trig values a_n
c0 = A*tau/T;        % dc bias Here c0=2*(1/2)=1
yce = c0*ones(size(t));% initialize yce to c0-This adds the dc value to all
%  NOTE:   sinc= sin(pi*x)/(pi*x) function using MATLAB. So use w0/pi
for n = -N:2:N,
cn =(A)*(tau/T)*sinc(n*(w0/pi)*tau/2) ;    % 1/(j*n*pi)
yce = yce + (cn*exp(j*n*w0*t));  % Fourier Series computation
end
figure(1),plot(t,yce),grid; % plot f(t) truncated exponential FS
xlabel('t (seconds) Period is 2*pi'); ylabel('y(t)');
ttle = ['Truncated Exponential Fourier Series with N = ',...
         num2str(N)];  % Convert N from number to ASCII
title(ttle);           % Put on a title
```

2. Plot the spectrum of the complex series as in Example 8.6 but with negative and positive components

```
  figure(2)                            % put next plots on figure 2
stem(0,c0);                          % plot c0 at nwo = 0
hold;
for n = -N:2:N,                      % loop over series index n
    cn =(A)*(tau/T)*sinc(n*(w0/pi)*tau/2) ;    % 1/(j*n*pi)
  %cn = (1/2)*((4*A)/(j*n*pi));     % Fourier Series Coefficient
  stem(n*w0,abs(cn))                 % plot |cn| vs nwo
end
for n = -N+1:2:N-1,                  % loop over even series index n
  cn = 0;                            % Fourier Series Coefficient
  stem(n*w0,abs(cn));                % plot |cn| vs nwo
end
xlabel('w (rad/s)'),ylabel('|cn|')
ttle = ['Amplitude Spectrum with N = ',num2str(N)];
title(ttle),grid, hold;
```

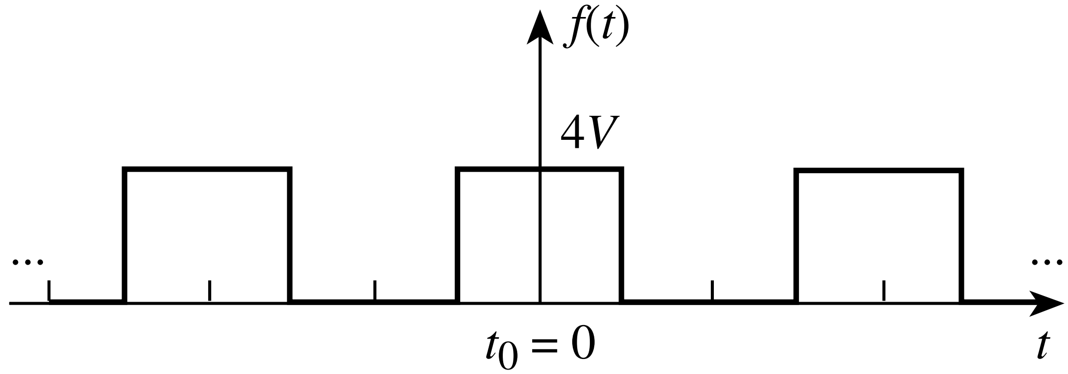

Figure 1: Computer Clock Signal

Problem 4 35 Points
Fourier series of clock signal Consider the computer clock signal shown in the Figure, with a pulse rate of 8 million pulses per second ($f_c = 8$ Megahertz) and amplitude of 4 volts and a pulse width of 0.05 microseconds. NOTE: The figure does not show the signal to scale since the duty cycle is 0.4.

1. Find the Fourier series by hand calculation using the basic definitions of the coefficients.

2. Using MATLAB plot the Fourier amplitude spectrum in Hertz of the clock signal as discrete values up to 80 Megahertz.

3. Using MATLAB Plot $f(t)$ in seconds from the Fourier Series you calculated for 6 periods of the pulses. Use 20 terms in the series. Turn in the program and the plots. See the guidelines for MATLAB programs in the Syllabus.

4. Using **stem** plot, plot both the negative and positive frequencies from -48 Mhz to 48 MHz. This is the range from components $-6, -5, -4, \cdots 0, 1, 2, \cdots 6$ since the spacing is 8 Mhz.

Hint: You can write the series for many pulse trains by letting the period be T, the pulse width be τ and the amplitude be A and then computing the Fourier Series. Then, just plug in the numbers. Go over Example 8.7 in Harman carefully.