HW 6 Sampling, DFT & FFT CENG 5431 April 1st

March 23, 2015

Problem 1 20 points
Let a filter have the frequency response of

$$H(\omega) = \frac{10}{10 + j\omega}.$$

(a) Define the output $y(t)$ to the input signal $x(t) = 2 + 2\cos(10t + \pi/2)$ showing $y(t)$ with magnitude and phase.(5 points)

(b) What is the bandwidth of the filter? Give the units of the Bandwidth. (5 points)

(c) Plot the bode plot for the filter and determine the attenuation in dB at the bandwidth value and do the following on the graphs of the plot:

(i) Put on a title for the plot - "Bode diagram for 1st order filter".

(ii) On the figure menu bar, use TOOLS -Data Cursor and click on the cutoff points in amplitude and phase to annotate the points on the graph.

Problem 2 20 Points

(a) Compute the analytical (By hand) DFT of the sequence $f = \{1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\}$ Like Harman's Example 11.7 but not quite. Reduce the exponential results to the complex values such as $a + ib$.

(b) Write a MATLAB script that computes the DFT directly from the definition (See Harman Equation 11.5) with the inputs

(i) N, the number of sample points;

(ii) f, the vector of sample points.

(c) Test the routine of Part 1 on the sequence $f = \{1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\}$ and compare to the results of Part a.

(d) Compute by hand the IDFT of the sequence for the element $x[2]$. Combine the complex values to yield the real values of f if you did it correctly. Be patient - this is a good exercise in manipulating complex numbers.

Problem 3 20 Points
Write a MATLAB program to compute the DFT using the MATLAB FFT function and the inverse IDFT using MATLAB function IFFT of the sequence in problem 1 to check your hand calculations and compare the results to the DFT result in Problem 1.

Problem 4 10 Points
Your system samples a sinusoidal signal

$$x(t) = \cos(2\pi 800t)$$

with $f_s = 600$ samples per second. After sampling the reconstructed signal appears as a sinusoid at what frequency in Hertz?

Problem 5 10 Points
A 3-minute song is sampled using 16 bits at 44,100 Hz for 2 channels. How many bytes are stored? Show all the conversions.

(a) How many bytes are stored? Show all the conversions with their units.

(b) Considering the size of the storage medium needed for Part a, research the MP3 format for compression and see how much storge space can be saved if the values are stored in MP3 format.

Problem 6
20 Points- Easy Questions

(a) The signal

$$x(t) = 3\cos(2\pi 404t + \pi/4) + \cos(2\pi 660t - \pi/5)$$

is sampled at 20kHz. How many samples would be stored after 60 ms?

(b) If $x(t) = 4\cos(2\pi 250t + 2\pi/7)$, what is the period of this signal?

(c) For CDs, the sampling rate is 44,100 samples per second. How often (in seconds) must the ADC sample the signal?

(d) A DFT of a signal is generated with $f_s = 5000$ samples/second for 250 samples. The results for the first 10 frequencies are

$$F[k] = 10, 0, 0, 2 + j4, 0, 1, 0, 0, 0, 2 - j4.$$

Determine the magnitude and frequency in Hertz of the sinusoids in the signal.