HW9 Filter Homework CENG 5431 Spring 2015 April 27

April 21, 2015

Problem 1 20 Points

See Handouts Chapter 19 Ambardar Convert the analog filters to digital filters $H(z)$ using the impulse invariant method.

(a) $H(s) = \dfrac{1}{s+2}, \quad S = 2$ Hz.

(b) $H(s) = \dfrac{2}{(s+1)} + \dfrac{2}{(s+2)}, \quad t_s = 1$ second .

Do the problems by hand (10 points) and then use MATLAB (10 points) to prove your answers. Use either c2d command with 'impulse" or the impinvar command. At the MATLAB command prompt do >> help c2d or impinvar. Note that using impinvar scales the result by $1/S$. Do not forget to get the sys form using the command tf.

```
SYS = tf(NUM,DEN) creates a continuous-time transfer function SYS with
   numerator NUM and denominator DEN.
```

Problem 2 30 Points

Since we associate transfer functions in the z-domain and the s-domain by the relationship

$$H_d(z) = H(s)|_{s=(1/T)\ln z},$$

then $s = (1/T_s)\ln z$ and the *bilinear transform* is used as

$$s = \frac{1}{T_s}\ln z \approx \frac{2}{T_s}\frac{z-1}{z+1} \tag{1}$$

where T_s is the time between samples for the discrete filter. One consideration is that the cutoff frequency Ω_c of the discrete filter H_d is *warped* from the desired value of $\omega_c T_s$ where ω_c is the cutoff frequency of the analog filter $H(s)$. The relationship is

$$\Omega_c = 2\tan^{-1}\frac{\omega_c T_s}{2}. \tag{2}$$

Consider the two-pole Butterworth filter with transfer function

$$H(s) = \frac{\omega_c^2}{s^2 + \sqrt{2}\,\omega_c s + \omega_c^2} \tag{3}$$

from which it is desired to construct a digital filter using the bilinear transformation. Let the parameters be

$$\omega_c = 2 \text{ rad/sec}, \qquad T_s = 0.2 \text{ sec}.$$

Do the following :

1. Determine the digital filter $H_d(z)$ that corresponds to the Butterworth analog filter using the bilinear transform by hand using direct substution of Equation 1 into Equation 3. Then check your answer with MATLAB bilinear command - note that MATLAB normalizes the coefficient of z^2 in the denominator as $z^2 + az + b$.

2. Determine the discrete cutoff frequency from Equation 2.

3. Using MATLAB, compare the magnitude of the frequency response of $|H(\omega)|$ versus ω $(-4 \le \omega \le 4)$ and H_d versus Ω over the same range of Ω and discuss the differences. Note that we expect the digital frequency range in Ω to be $-\pi \le \Omega \le \pi$.

Problem 3 20 Points

Given the signal

$$x(t) = 1 + \cos t + \cos 5t$$

Do the following :

1. Remove the $1 + \cos t$ term with a highpass filter using Example 10.6 and Example 10.8 of the Kamen and Heck handout page 551.

2. Plot the signal $x(t)$ and the output $y(t)$ and discuss the results briefly.

Problem 4 30 Points
Let

$$H(z) = \frac{1 - z^{-2}}{1 - 0.9z^{-1} + 0.81z^{-2}}.$$

Do the following (30 points):

1. Write the difference equation corresponding to $H(z)$.

2. Plot the frequency response in the range $-6 \leq \Omega \leq 6$. First, find the transfer function in z using the MATLAB command **freqz**. Then use the **plot** to plot the magnitude (**abs**) versus Ω.

   ```
   To plot a range of frequencies that are not normalized by freqz from 0 to 1
   try the following MATLAB code after defining the numerator a and denominator b
   coefficients as vectors and the frequency range w:

   Hw=freqz(b,a,w);
   figure(1)  % Plot magnitude of result from freqz over range in w
   plot(w,abs(Hw)),grid
   title('Plot of H(z) HW9 Problem 4')
   ```

3. What is the highest digital radian frequency allowed? What kind of filter is this in terms of its passband - lowPass?- etc.?