**Solving Systems of Linear Equations Matrix Operations**

A system of linear equations may be represented in matrix form. Thus the system of two equations :

x - 2y =-1

3x + 4y = 17

may be represented by Equation M:

$$\left(\begin{array}{c} 1 -2 \\ 3 4 \end{array}\right)\left(\begin{array}{c}x\\y\end{array}\right)=\left(\begin{array}{c} -1 \\17 \end{array}\right)$$

Performing the matrix multiplication in Equation M, we get:

$$\left(\begin{array}{c} 1x -2y \\ 3x 4y \end{array}\right)=\left(\begin{array}{c} -1 \\17 \end{array}\right)$$

and since the matrix on the left equals the matrix on the right, the corresponding
elements are equal, from which it follows that *x* + *2y* = 14 and *2x* - *y* = 5; these are
the equations with which we started, thereby justifying the statement that we may
represent a system of linear equations in matrix form.

Let us write Equation M in the form: AX = C

Multiply both sides by the inverse of A giving A-1AX = A-1C

But since A-1A = I, this becomes IX = A-1C

We know that IX = X, therefore X = A-1C

From this we see that the value of the Xmatrix may be obtained by computing A-1C. Next find the inverse of this matrix A-1, so we may write:

$$\left(\begin{array}{c}x\\y\end{array}\right)=\left(\begin{array}{c} 0.4 0.2 \\ -0.3 0.1 \end{array}\right)\left(\begin{array}{c}-1\\17\end{array}\right)$$

Performing the multiplication gives:

$$\left(\begin{array}{c}x\\y\end{array}\right)=\left(\begin{array}{c} 2\\ 3\end{array}\right)$$

From which we see that *x* = 3 and *y* = 2

This may have left you less than impressed; you could have solved the two simultaneous equations in your head. The method is now applied to more challenging problems. In the next exercise, solve:

2x + 3y - 2z = 15

3x - 2y + 2z =-2

4x - y + 3z = 2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | A  |  | B  | I  | C  | D  | E  |  |
| 1  |  |  | Using Matrix functions to solve  |  |  |
|  |  |  | a System of Linear Equations  |  |  |
| 3  |  |  |  |  |  |  |  |  |  |
| 4  | Matrix of Coefficients (A)  | Matrix of Constants (C)  |
| 5  | 2  |  | 3  |  |  | -2  | 15  |  |  |
| 6  | 3  |  | -2  |  |  | 2  | -2  |  |  |
| 7  | 4  |  | -1  | I,  | 3  | 2  |  |  |
| 8  |  |  |  | ,  |  |  |  |  |
| 9  |  |  | A-1 |  |  | A-1C =X  |  |
| 10  | 0.190476  | 0.333333,  | -0.09524  | 2  | x  |  |
| 11  | 0.047619  | -0.666671  | 0.47619  | 3  | y  |  |
| 12  | -0.2381  | -0.66667  |  | 0.619048  | -1  | z  |  |
| 13  |  |  |  |  |  |  |  |  |  |
| 14  | Reconstructed equations us x=2,y=3, z=-1 |  |  |
| 15  | 4  |  | 9  |  |  | 2  | 15  |  |  |
| 16  | 6  |  | -6  |  |  | -2  | -2  |  |  |
| 17  | 8  |  | -3  | ,  | -3  | 2  |  |  |

Figure

1. Open a workbook enter the text in rows 1, 2, 3, 9 and 14.
2. In A5:C7 enter the coefficients of the equations, and in D5:D7 enter the constants.
3. The next step is to compute the inverse A-1of the matrix of coefficients. Select the range A10:C12, enter the formula =MINVERSE(A5:C7) and press **Ctrl+Shift+Enter**.
4. The final step to find the solutions is to compute A-1C. Select D10:D12, enter the formula =MMULT(A10:C12, D5:D7) and press **Ctrl+Shift+Enter**.

The solutions is found x=2,y=3, z=-1. You can check that these agree with the system of equations.

e) Use Insert|Name to name the cells D5:D7 as x, y and z*,* respectively.

f) The formulas in row 15 are:

 A15: =A5\*x

 B15: =B5\*y

 C15: =C5\*z

 D15: =SUM(A15:C15) Use AutoSum to make this formula.

Copy these formulas down to row 16 and 17. The values in D 15:D17 agree with
those in D5 :D7, thus confirming that we have solved the system of equations.

You now have a worksheet that may be used to find the solutions of any set of linear equations in three unknowns, providing the equations are independent and consistent. If the equations are not independent (for example, the third equation is twice the first plus the second) then the Amatrix will be singular - it will have no inverse. In this case Microsoft Excel will return the #NUM! error value when MINVERSE is used with a singular matrix. The MDETERM function may also be used to test the *A* matrix; MDETERM(array) returns a zero value for a singular matrix. If the determinant of *A* is very small it may be difficult to solve the set of equations.